

The Large Scale Structure of the Universe: Theory *vs.* Observations *

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I. Introduction

Observations indicate that the Universe is homogeneous and isotropic on scales $\geq 100 - 200$ Mpc., whereas on smaller scales, its fundamental units – galaxies – cluster together to form groups, clusters and even superclusters, with effective mass scales ranging from $10^{12}M_{\odot}$ (groups) to $10^{15}M_{\odot}$ (superclusters). There are also indications that most of the Universe consists of large empty regions – voids, which are virtually devoid of any matter. Although many statistical indicators are used to describe the observed clustering of galaxies²⁹, the best researched and perhaps most robust indicator of galaxy clustering is the two point galaxy - galaxy correlation function $\xi(r)$, and its angular counterpart $w(\theta)$. Recent investigations¹ indicate that $w(\theta)$ remains positive till $\sim 50 - 100h^{-1}$ Mpc. indicating that galaxies continue to cluster even on such large scales (see fig. 1). (We have chosen the Hubble parameter to be $H_0 = 100 \times h \text{ km.s}^{-1} \text{ Mpc}^{-1}$). The decreasing amplitude of $w(\theta)$ indicates that clustering is getting weaker with scale, so that one is justified in assuming the Universe to be fairly homogeneous on scales $\geq 100h^{-1}$ Mpc.

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A detailed knowledge of the observed clustering of galaxies, as well as their gravity induced random motions, provides us with a sensitive test against which to probe models of structure formation. In this review, I shall attempt to confront theory with observations in order to try and constrain models of galaxy formation, dwelling in some detail on the currently popular Cold Dark Matter model.

I shall mainly concern myself with the following three sets of observations with which to try and constrain theory:

- 1) The observed isotropy of the Cosmic Microwave Background Radiation,
- 2) The clustering of galaxies measured by the two point correlation function $\xi(r)$,
- 3) The large scale peculiar velocity field of galaxies.

II. The Cosmic Microwave Background Radiation

A remarkable feature of the Cosmic Microwave Background Radiation (CMBR) is that it is accurately described by a thermal distribution of photons with temperature² $\simeq 2.7^\circ K$ (Fig 2a). The CMBR also appears to be smooth to better than one part in ten thousand on virtually all angular scales (after one has subtracted out a visible dipole component caused presumably by the motion of our galaxy with respect to the Hubble flow).

The observational upperlimits on the CMBR anisotropy are summarised in fig. 2b, the most stringent constraints on $\frac{\Delta T}{T}$ ($= \frac{T-T_0}{T_0}$, $T_0 \approx 2.7^\circ K$) arising on arc minute scales are : $\frac{\Delta T}{T} < 1.5 \times 10^{-5}$ at $\theta = 7.15'$, and are provided by observations at the Owens Valley Radio Observatory³ (OVRO). Since a comoving scale λ subtends an angle $\theta = 34.4''(\Omega h) \times \lambda_{Mpc} \equiv 65.4''(\Omega^{\frac{2}{3}} h^{\frac{1}{3}})(\frac{M}{10^{12}M_\odot})^{\frac{1}{3}}$, it follows that masses of the order $10^{13}M_\odot \leq M(\theta) \leq 10^{17}M_\odot$ - covering the entire mass range from rich groups to superclusters of galaxies⁴ are covered by angular scales ranging between : few arc min $\leq \theta \leq 3^\circ$. As a result, the absence of fluctuations in the CMBR on these angular scales constrains the form of the density perturbation spectrum on scales of 2 - 100 Mpc, and is consequently an important check on theories of galaxy formation.*

On scales $\geq 3^\circ$ the main contribution to the CMBR arises because of primordial gravitational perturbations on the surface of last scattering $\frac{\delta T}{T} \simeq \frac{1}{3} \frac{\delta \phi}{c^2}$ (the *Sachs-Wolfe*

* Since recombination is not instantaneous, fluctuations on very small angular scales $\leq \Delta\theta \sim 10(\Omega h^2)^{-\frac{1}{2}}$ arc min, are wiped out because of the finite thickness of the last scattering surface .

effect). Anisotropies on such large scales when detected are likely to be particularly revealing because they probe scales that were larger than the Horizon size at recombination. Consequently, any information concerning the microwave anisotropy on such scales, also carries with it implicit information regarding the primordial form of the fluctuation spectrum which might have been generated by physical processes occurring at the very beginning of the big bang, such as inflation.

III. Non-baryonic models of galaxy formation

Since most scenario's of galaxy formation predict a definite, albeit small amplitude of density fluctuations at the surface of last scattering, present observational constraints on $\frac{\Delta T}{T}$ effectively rule out a host of cosmological models including many in which structure formation proceeds via gravitational instability, such as baryonic models of galaxy formation, the hot dark matter model with an early epoch of galaxy formation, and a low density cold dark matter model⁵. Models which are ruled out also include those in which structure forms nongravitationally due to the propagation of shocks driven by primordial explosions⁶ or by the collapse of superconducting cosmic strings⁷. In the latter class of models, a sizeable amount of energy is injected into the intergalactic medium during the formation of shocks, leading to large distortions in the microwave spectrum which have not been observed.

Models which survive the stringent CMBR constraints include the standard Cold Dark Matter (CDM) model, as well as the cosmic string scenario of galaxy formation. Since Alex Vilenkin will discuss string models of galaxy formation at this meeting, I shall mainly focus on reviewing the present status of the CDM model *vis a vis* observations.

The Cold Dark Matter model presupposes that most of the matter in the Universe today, exists in the form of nonrelativistic (*ie.* cold), weakly interacting, non - baryonic matter, which decoupled from the rest of the baryonic matter in the Universe at some early epoch, soon after the big bang.

The need for incorporating nonbaryonic forms of matter into the standard big bang model, arises because of the necessity to generate large enough perturbations by the present epoch, to account for the presence of galaxies, from sufficiently small initial values of the density contrast $\frac{\delta\rho}{\rho}$, without violating the CMBR constraints on small - arc minute - scales. This is clearly a difficult task in baryonic models in which perturbation growth takes place only after the cosmological recombination of hydrogen,

which occurs at redshifts ~ 1100 . (Before recombination, radiation pressure caused by Thompson scattering, effectively prevents the growth of all perturbations having wavelenths smaller than $\sim 180h^{-1}\text{Mpc}$. – the horizon size at recombination). After recombination, density perturbations grow linearly with the scale factor of the Universe – $a(t)$, until the Universe becomes curvature dominated, after which time perturbation growth is strongly suppressed²⁰ (if $\Omega \leq 1$).

The maximum permitted growth for perturbations is therefore $\frac{\delta}{\delta_{\text{rec}}} \leq \frac{a_0}{a_{\text{rec}}} \simeq 1100$ [for $\Omega \leq 1, \Omega \equiv \frac{\rho}{\rho_{\text{cr}}} = \frac{8\pi G\rho}{3H^2}$ where ρ is the present mass density of the Universe.] Consequently, the requirement that $\frac{\delta\rho}{\rho} > 1$ today – which is necessary for the formation of gravitationally bound systems such as galaxies – leads to the primordial amplitude: $\frac{\delta\rho}{\rho} \approx 10^{-3}$ at recombination. If the primordial perturbations are adiabatic (*ie* fluctuations for which the specific entropy per baryon is spatially constant), then at recombination^{4,8}

$$\left(\frac{\delta\rho}{\rho}\right)_B = \frac{3}{4}\left(\frac{\delta\rho}{\rho}\right)_\gamma = 3\frac{\Delta T}{T} \quad (1)$$

so that a fluctuation amplitude $\sim 10^{-3}$ in the baryon component would invariably result in an anisotropy $\sim 10^{-4}$ in the CMBR temperature on arc minute scales which has not been observed. On the other hand, perturbations in a nonbaryonic component, can begin growing soon after matter - radiation equality has been achieved (*ie* by $z \sim 10^4$ in the case of an $\Omega = 1$ Universe⁹), with the result that large final perturbations can develop from small initial values without violating the CMBR constraints on small scales. Fig. 3 shows how perturbations in baryons and in a nonbaryonic component grow after matter-radiation equality; it is clear that whereas perturbations in the non-baryonic component can grow soon after matter dominance, the growth in the baryonic component is suppressed until after recombination. (After recombination, baryons catch up with the perturbations in the non-baryonic component, by falling into the potential wells created by the non-baryonic perturbations.) Consequently, since the CMBR anisotropy in (1) is determined by the density fluctuation in the baryonic component at recombination, it is an order of magnitude smaller than it would have been in a purely baryonic scenario, and therefore does not come into conflict with observations.

Historically, the suggestion that a significant portion of the matter in the Universe may exist in the form of weakly interacting massive particles (WIMP's) such as massive neutrino's, was made by Marx and Szalay¹⁰ as well as by Cowsik and McClelland¹¹. Galaxy formation with massive neutrino's began to be studied in earnest after Lubimov claimed to have detected a mass of ~ 30 eV for the electron neutrino⁹. (The results of this work have however, since been called into question). Perturbations in a medium

made up of light collisionless particles such as massive neutrino's, are subject to collisionless phase mixing, (*ie.* weakly interacting particles free stream relativistically from regions of high density into low density regions), which effectively wipes out perturbations on scales smaller than $\lambda_{fs} \sim 40 \text{ Mpc} \frac{30 \text{ eV}}{m_\nu}$ – corresponding to scales of clusters of galaxies¹². *. Consequently the perturbation spectrum in a massive neutrino model (conveniently called the Hot Dark Matter model – to highlight the relativistic nature of its particle species), displays a sharp cutoff at the free - streaming scale λ_{fs} , which is shown in figure 4. The presence of a cutoff on scales $\sim 40 \text{ Mpc}$ ensures that the first objects to undergo gravitational collapse in this model will be of cluster scales, with smaller-scale objects forming out of the subsequent fragmentation of cluster-sized pancakes. Such a scenario for galaxy formation was originally suggested by Zeldovich in the 70's in connection with baryonic models of galaxy formation, and is popularly known as the *pancake* or *top-down* model of galaxy formation¹³. Some characteristic features of an HDM Universe which emerge from N-body simulations^{14,15}, include the presence of a cell-like structure with sharply demarkated filaments and voids on scales of roughly $\sim \lambda_{fs} \text{ Mpc}$.

Although the gross features of the HDM model do seem to reproduce several aspects of the observed large scale structure in the Universe – such as the superclusters and voids seen in redshift surveys, the model when tested quantitatively fails to agree with observations. N-body computer simulations of Hot Dark Matter models^{14,15} for instance, show that in order to agree with such statistical indicators of galaxy clustering as the observed two point galaxy-galaxy correlation function at the present epoch, galaxy formation in this model would have to have taken place very recently ($z \leq 1$), which clearly runs counter to observations of galaxies and quasars at redshifts $z \geq 3$. Although some attempts are still on for a revival of this model¹⁶, most theorists have since the mid 80's, begun to favour a different non-baryonic model for galaxy formation – the Cold Dark Matter (CDM) model.

The non-baryonic particle candidates in CDM models are assumed to be weakly interacting and nonrelativistic (hence "cold"). Particle candidates which fit this prescription are known to arise naturally in particle physics models incorporating supersymmetry, such as the supersymmetric partners of the photon and graviton – the photino and gravitino, respectively. Others, such as the axion, are required to provide consistency to low energy theories such as QCD¹⁷.

* λ_{fs} is the distance traversed by a relativistic massive neutrino until its momentum becomes non-relativistic

Due to the absence of free streaming, the fluctuation spectrum for CDM - δ_k^2 - ** does not show a cutoff on small scales^{18,19}, instead δ_k^2 falls off monotonically, approaching the asymptotic form $\delta_k^2 \propto k^{-3} \log^2 k$ for $k \gg k_{eq}$ (see (2) below) ($k = \frac{2\pi a(t)}{\lambda}$ is the comoving wavenumber).

The CDM spectrum is well described by the the semi-analytic approximation obtained by Starobinsky and Sahni^{19,21}

$$\delta_k^2 = \frac{1}{A^4 k^3} \frac{\ln^2(1+Bk)}{1 + \frac{\ln(1+Bk)}{(Ak)^2}} \quad (2)$$

where $A = 3.08\sqrt{\kappa}h^{-2}Mpc$; $B = 1.83\sqrt{\kappa}h^{-2}Mpc$; $\kappa = \frac{\Omega_{rad}}{\Omega_m}$. (See also ref.[44]).

The characteristic bend in the CDM spectrum occurs at length scales $\sim \lambda_{eq}$ ($= \frac{13}{\Omega h^2}$ Mpc), corresponding to the Horizon scale at matter - radiation equality. (This is the only length scale that enters into the spectrum). This bend arises because density perturbations on scales much smaller than λ_{eq} enter the horizon when the Universe was still radiation dominated. Since perturbations in radiation get ironed out on scales smaller than the Hubble radius (due to the free streaming of photons), matter perturbations have nothing to gravitate towards, and so grow at the exceedingly slow rate : $\delta \propto 1 + \frac{3}{2} \frac{a(t)}{a_{eq}}$ until matter dominance ($a = a_{eq}$). On the other hand, larger than horizon size perturbations continue to grow as $a^2(t)$, while the Universe is radiation dominated , with the result that modes entering the horizon at later times come in with correspondingly larger amplitudes, resulting in a bend in the shape of δ_k^2 . Perturbations on scales $> \lambda_{eq}$, reenter the horizon after matter dominance and so are left effectively untouched by the radiation era. As a result, the fluctuation spectrum preserves its primordial form $\delta_k^2 \propto k$, on scales $> \lambda_{eq}$.

A measure of the density contrast over a given scale is provided by $\frac{\delta\rho}{\rho} \sim (k^3 |\delta_k^2|)^{\frac{1}{2}}$, which grows as $\ln^2 k$ on small scales^{19,21} (see fig. 4). Consequently, smaller scales are the first to go non-linear , with larger scales following suit. As a result gravitational clustering takes place hierarchically with large units such as clusters and superclusters , forming out of the merger of smaller - galaxy and globular cluster-sized units. This hierarchical process of building larger structures out of smaller ones, is often referred to as the *bottom-up* scenario of galaxy formation, and is complementary to the *top-down* scenario which is associated with the HDM model. The *rms* fluctuation in the mass

** $\left(\frac{\delta\rho}{\rho}\right) = \frac{1}{(2\pi)^3} \int \delta_k \exp(k^\mu x_\mu) d^3 k$

found within a randomly placed sphere of radius $R : \frac{\Delta M}{M}$ is plotted in fig. 5 for the CDM spectrum given in (2). From fig. 5 and the relation $\frac{\Delta M}{M}(0,R) \simeq 1+z$, it is easy to show that objects of mass $\sim 10^{11} - 10^{12} M_{\odot}$ can begin forming by redshifts $\sim 3 - 4$, in the standard (unbiased), Cold Dark Matter model ($1+z = \frac{a_0}{a}$).

Due to the fact that non-baryonic matter is dissipationless and does not shine, CDM models can successfully account for the existence of spherically symmetric dark halo's surrounding galaxies, and also for the substantial quantity of non-luminous matter in clusters whose existence can be inferred from arguments based on the virial theorem. Other successes of the CDM model – demonstrated using N-body simulations – include its ability to account for the masses, sizes, angular momenta and abundance of galaxies²².

The CMBR anisotropies expected in a CDM Universe are⁵: $\frac{\Delta T}{T}(7.15') \simeq \frac{10^{-5}}{b}$ and $\frac{\Delta T}{T}(7^{\circ}) \simeq \frac{10^{-5}}{b}$, which are smaller than the observational upperlimits on these scales (b is the biasing factor; $h = 0.5$ is assumed for the Hubble parameter). For an unbiased CDM model ($b = 1$), the predictions for $\frac{\Delta T}{T}$ on large scales are close to being either confirmed or ruled out by COBE.*

The varied diversity of its successes made the CDM model very popular in the aftermath of the demise of the HDM scenario. However, recent observations of galaxy clustering on large scales^{1,23} (50 – 100 Mpc) – fig. 1, as well as measurements of the peculiar velocities of galaxies²⁴ on scales $\sim 50h^{-1}$ Mpc, have posed a serious observational challenge to the CDM scenario and have caused considerable debate in the cosmology community as to the viability of this model. Some of these observational tests will be discussed in the following two sections of this paper.

IV. Clustering of Galaxies

Galaxy clustering is most evident in wide angle survey's of the sky both in the optical as well as in the infrared and 21cm wave bands (see fig 6). Optical redshift

* Just as this paper was nearing completion, a press release announced the detection of an anisotropy in the microwave background measured by COBE⁴⁶: $\frac{\Delta T}{T} \simeq 1.1 \times 10^{-5}$, on scales $> 7^{\circ}$, which is consistent with the predictions made by the standard (unbiased) CDM model.

surveys have reconfirmed this result, discovering in addition, large-scale coherent structures such as the Perseus-Pisces supercluster chain, and the great void in Bootes^{25,26} (fig 7).

The clustering of galaxies can be characterised by several statistical indicators, the best known and most comprehensively studied being the two point galaxy - galaxy correlation function - $\xi(r)$, defined as the probability in excess of random, of finding a galaxy at a distance r from another, randomly picked galaxy .

On small scales ($r < 10h^{-1}$ Mpc) the correlation function has the well known form

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-1.8} \quad (3a)$$

where $r_0 = 5h^{-1}$ Mpc, is the clustering scale.

Since $\xi(r)$ is the fourier transform of the power spectrum δ_k^2

$$\xi(r) = \frac{1}{(2\pi)^3} \int |\delta_k^2| \exp(k^\mu x_\mu) d^3 k, \quad (3b)$$

it carries direct information of the form and amplitude of the density fluctuation spectrum. On large scales ($> \lambda_{eq} \approx \frac{13}{\Omega h^2}$ Mpc), δ_k^2 is not yet distorted by non-linear gravitational clustering, so that for primordial spectra of the form $\delta_k \propto k^n$, $\xi(r) \approx -\sin(\frac{\pi n}{2}) \times r^{-(3+n)}$; therefore, knowing the sign of $\xi(r)$ and its slope on linear scales, one can directly infer the primordial form of δ_k . Recent estimates of the angular galaxy - galaxy correlation function - $w(\theta)$, * made using the Automatic Plate Measuring Machine (APM) survey, and covering over 2 million galaxies, indicate that galaxies continue to be positively clustered upto scales $\sim (50 - 100)h^{-1}$ Mpc (see fig 1). Comparison of $w(\theta)$ with predictions of the standard CDM model, shows that galaxies appear to be more strongly clustered on large scales than can be accounted for in a $\Omega = 1$ CDM model with a conventional Harrison - Zeldovich spectrum: $\delta_k^2 \propto k$ on large scales. (The observational curve for $w(\theta)$ is consistent with a primordial spectral index $-1 \leq n \leq 0$, but inconsistent with $n = 1$.)

Other indications of the existence of greater power on large scales comes from recent redshift surveys of infrared galaxies taken with the help of the IRAS satellite²³. These surveys show an *rms.* fluctuation of ~ 0.3 in the number of galaxies on scales $\sim 20h^{-1}$

* $w(\theta)$ and $\xi(r)$ can be related via the Limber equation⁴, so that for power law spectra $\xi(r) \propto r^{-\gamma}$, $w(\theta) \propto \theta^{1-\gamma}$.

Mpc, which is about 2 - 3 times the predicted amplitude on these scales in a biased CDM scenario. **

There are several ways of circumventing the above problems with CDM. One is to drop the assumption that the primordial perturbation spectrum is of the scale invariant Harrison - Zeldovich type. Although a scale invariant spectrum arises naturally in most models of inflation, models do exist^{31,32} in which the inflationary expansion of the Universe is of the power - law kind: $a(t) \propto t^p, p > 1$, which leads to a more general spectrum for density fluctuations^{33,34}: $\delta_k^2 \propto k^n$ where $n = \frac{3-p}{1-p}$. Such models predict greater power on large scales for δ_k^2 , and consequently also for $\xi(r)$ and $w(\theta)$.

Another possibility for reconciling the observed evidence for large scale clustering, is by working with low density CDM models ($\Omega \approx 0.1$ being preferred from estimates of the mass to light ratio in clusters of galaxies). In such models the turnaround point in the CDM power spectrum (2), occurs at $\lambda_{eq} \sim \frac{13}{\Omega h^2}$ Mpc ~ 130 Mpc, so that on scales ~ 50 Mpc, the slope of the perturbation spectrum $\delta_k^2 \propto k^n$ is effectively $n \approx -1$ which is consistent with observations of $w(\theta)$ on these scales. However low density CDM Universes predict large distortions in the CMBR and are ruled out by observations⁵. One can get around this difficulty by introducing an effective cosmological constant into the model,* $\Lambda = 3H_0^2(1 - \Omega_m)$, so that $\Omega_\Lambda + \Omega_m = 1$. Such a cosmological model is consistent with the observed isotropy of the microwave background, predicts the correct slope and amplitude for $w(\theta)$ on large scales, and in addition, is old enough to explain the existence of the oldest observed star clusters in our galaxy³⁵ (whose ages lie in the range $15 \leq T \leq 18$ billion years, and are difficult to accommodate within the framework of a matter dominated $\Omega = 1$ cosmogony³⁶.)

V. Large-Scale Peculiar Velocities of Galaxies

The cosmic microwave background radiation defines for all practical purposes, an absolute frame of reference against which departures from a smooth Hubble flow may be

** Light does not trace mass in biased galaxy formation scenario's, consequently density fluctuations in these models, are smaller by a factor $b = \frac{\frac{\delta N}{N}}{\frac{\delta M}{M}}$, than in an unbiased scenario. ($\frac{\delta N}{N}$ is the variance in the number density of bright galaxies on a given scale, $\frac{\delta M}{M}$ is the variance in the mass, evaluated on the same scale.)

* Although a small value of the cosmological constant does not conflict with observations, its introduction necessitates some fine tuning which most cosmologists find unattractive.

measured**. An indication that our galaxy is not at rest with respect to this reference frame, comes from the presence of a dipole anisotropy in the CMBR having the form:

$$T(\theta) = T_0(1 + \frac{v}{c} \cos \theta) \quad (4)$$

where v is the velocity of the observer, and θ is the angle at which the microwave temperature is being measured with respect to the motion of the observer. The *dipole anisotropy* – indicating that the sky appears hotter in one direction and colder in the opposite one – has the well established value $\frac{\delta T}{T} = 1.2 \times 10^{-3}$, and implies (after correcting for the motion of the Earth) a velocity for the sun of $V_{CM}^{sun} = 360 \pm 25$ km/s. Since the Sun's motion in the Milky Way (~ 250 km/s), is in a direction roughly opposite to that of its motion with respect to the CMBR, we get $V_{MB}^{gal} = 540 \pm 50$ km/s, for the motion of our galaxy relative to the microwave background. Taking into account the relative motions of galaxies within the local group, (which constitutes ~ 20 members, the largest being the Milky Way and M31) we finally obtain $V_{MB}^{LG} = 610 \pm 50$ km/s, for the velocity of the barycentre of the local group with respect to the CMBR.

The nearest large concentration of mass in the vicinity of the local group is the Virgo cluster, located at a distance of $\sim 13h^{-1}$ Mpc. The relative peculiar velocity of the local group with respect to the Virgo cluster (commonly known as *infall towards Virgo*) is $V_{Virgo}^{LG} \approx 250$ km/s., indicating that the Virgo cluster contributes only *partially* to our overall peculiar motion. Furthermore, the direction of motion of the Local Group relative to the CMBR, is roughly at an angle of 45° *away* from the direction of the Virgo cluster, indicating that the Virgo cluster – with the Local Group at its periphery * – is moving at an overall velocity of ~ 400 km/s., in the direction of the Hydra - Centaurus supercluster. For sometime it was felt that perhaps the gravitational attraction towards Hydra - Centaurus might wholly account for the large peculiar velocity of Virgo and the Local Group (see fig 8). However work on ~ 400 elliptical galaxies by Lynden-Bell and others²⁴, revealed that large peculiar motions were not confined to the Local Group alone, but were shared by galaxies occupying large volumes of space $\sim 50h^{-1}$ Mpc., and probably extending to regions well beyond the Hydra - Centaurus supercluster.

Simple models have attributed most of the bulk motions (600 ± 100) km/s, to the presence of a large mass concentration ($\sim 10^{16}M_\odot$) located at a distance $\sim 42h^{-1}$ Mpc.

** If \mathbf{V} is the observed velocity of a galaxy located at a distance \mathbf{r} , then its peculiar velocity is $\mathbf{v} = \mathbf{V} - H \mathbf{r}$.

* constituting a patch $\sim 13h^{-1}$ Mpc. in scale

away from us and behind the Hydra - Centaurus supercluster, which has appropriately been dubbed - The Great Attractor. As of today, attempts to find the great attractor are on, and it is hoped that conclusive evidence of its existence - such as a reversal of galaxy infall on the far side of the Great Attractor⁴⁵ - will soon be forthcoming.

The existence of large scale peculiar motions of $\sim 600 \pm 100$ km/s on scales $\sim 50h^{-1}$ Mpc. has provided a stiff challenge to theories of galaxy formation, especially the Cold Dark Matter model. One can estimate the predicted value of bulk motions in a hierarchical theory of structure formation such as CDM, by noting that since most of the *action* is on large scales, it is safe to use linear theory to provide an estimate of the peculiar velocity of galaxies on such scales. Following this procedure, we note that a peculiar acceleration \mathbf{a}_p acting for a time τ , will induce a peculiar velocity $\mathbf{v} = \mathbf{a}_p \times \tau$, where \mathbf{a}_p is related to the over all mass enhancement on scales r : $\mathbf{a}_p = \frac{G\delta M r}{r^3} = \frac{4}{3}\pi G\delta\rho r = \frac{\Omega H}{2}\mathbf{v}_H \left(\frac{\delta\rho}{\rho}\right)$, where $\mathbf{v}_H = Hr$ is the Hubble velocity. Since⁴ $\frac{3}{2}H\Omega\tau \simeq \Omega^{0.6}$ we obtain:

$$\frac{|\mathbf{v}|}{v_H} \lambda = \frac{\Omega^{0.6}}{3} \left(\frac{\delta\rho}{\rho}\right)_\lambda \quad (5)$$

for the relative peculiar velocity on a scale λ .

Eq. (5) enables us to relate the peculiar velocity on a given scale with the corresponding value of the density enhancement on that scale. Thus on the scale of the Great Attractor ($\lambda \sim 50h^{-1}$ Mpc), $\frac{v}{v_H} \sim 0.1$ which gives $\left(\frac{\delta\rho}{\rho}\right)_{50} \simeq 0.3$, for an $\Omega = 1$ Universe. If we assume that the density perturbation spectrum on a given scale can be characterised by a power law: $\delta_k^2 \propto k^n$, then the density contrast on that scale is given by $\left(\frac{\delta\rho}{\rho}\right)_\lambda \sim (k^3 |\delta_k^2|)^{\frac{1}{2}} \sim \lambda^{-\frac{3+n}{2}}$. In order to yield sensible information, the above expression has first to be normalised to agree with the observed variance in the number counts of bright galaxies on a given scale. Requiring⁸ $\left(\frac{\delta\rho}{\rho}\right)_8 = \frac{\delta N}{N} (8h^{-1} Mpc) = 1$ gives

$$\left(\frac{\delta\rho}{\rho}\right)_\lambda = \left(\frac{\lambda}{8h^{-1}}\right)^{-\frac{3+n}{2}} \quad (6)$$

so that on scales of the Great Attractor, $\left(\frac{\delta\rho}{\rho}\right)_{50} = \left(\frac{8}{50}\right)^{\frac{3+n}{2}}$.

In an $\Omega = 1$ CDM model, the spectrum δ_k acquires its primordial Harrison - Zeldovich form $\delta_k^2 \propto k$ on scales ≥ 50 Mpc, so that substituting $n = 1$ in (6) we find $\left(\frac{\delta\rho}{\rho}\right)_{50} \simeq 0.03$, which is an order of magnitude smaller than the value inferred from bulk

flows on these scales. Since $\frac{\delta\rho}{\rho}$ is a Gaussian on large scales, the probability of finding a density enhancement $\sim \delta_{GA}$ on scales of $\sim 50h^{-1}$ Mpc is given by

$$P(\delta \geq \delta_{GA}) = \frac{1}{\sqrt{2\pi}\delta_{exp}} \int_{\delta_{GA}}^{\infty} \exp\left(-\left(\frac{\delta}{2\delta_{exp}}\right)^2\right) d\delta \quad (7)$$

where δ_{exp} is the expected value of $\frac{\delta\rho}{\rho}$ on a given scale. Substituting $\delta_{exp} \simeq 0.03$ and $\delta_{GA} \simeq 0.3$ in (7) we get $P(\delta \geq \delta_{GA}) \simeq 5 \times 10^{-24}$ – ie the probability of finding a region like the great attractor in a standard CDM Universe is extremely small^{41,42}.

In our discussion of galaxy clustering in the previous section, we mentioned that the CDM model could be reconciled with the observed large amplitude of the angular correlation function $w(\theta)$ if the spectral index on scales 50 – 100 Mpc. was less than unity – the preferred value being $n \simeq -1$. (This possibility arises for flat CDM models dominated by a cosmological constant.) Substituting $n = -1$ in (6) we find $(\frac{\delta\rho}{\rho})_{50} \simeq 0.16$ and the corresponding probability of finding a region like the great attractor is now $P(\delta \geq \delta_{GA}) \simeq 0.04$, which although still small, is not at all unlikely.

One can redo the above calculation for determining the velocity perturbation in an expanding Universe more accurately, by using the linearised continuity equation:

$$\dot{\delta} = \frac{1}{a} \nabla \vec{v} \quad (8)$$

Fourier transforming (8) we get: $\dot{\delta}_k = \frac{1}{a} i \vec{k} \vec{v}_k$; since $\frac{\dot{\delta}_k}{\delta_k} = \frac{d \log(\delta(t))}{d \log(a(t))} H(t) \simeq \Omega^{0.6} H(t)$ we finally obtain $\vec{v}_k = -i H \Omega^{0.6} \frac{\delta_k \vec{k}}{k^2}$. The rms value of the bulk velocity on a scale R can now be calculated by choosing a suitable window (filtering) function $W(kR)$, to filter out the small scale contribution, so that finally

$$\langle v^2 \rangle = \frac{1}{(2\pi)^3} \int d^3 k |v_k|^2 W(kR) = \frac{(H \Omega^{0.6})^2}{2\pi^2} \int |\delta_k^2| \exp\left(-\frac{(kR)^2}{2}\right) dk \quad (9)$$

(where we have used a Gaussian filter: $W(kR) = \exp\left(-\frac{(kR)^2}{2}\right)$).

The probability of measuring a peculiar velocity having an rms value – v_{rms} in the interval $v_1 \leq v \leq v_2$ is given by⁸

$$P(v) = \frac{1}{\pi} \int_{v_1}^{v_2} \left(\frac{v}{v_{rms}}\right)^2 \exp\left(-\frac{3}{2} \left(\frac{v}{v_{rms}}\right)^2\right) \frac{dv}{v_{rms}} \quad (11)$$

From (11) it follows that the probability of measuring a velocity v in the range $\frac{v_{rms}}{3} < v < 1.6 \times v_{rms}$ is $\sim 90\%$. Applying (9) and (11) to the CDM model with δ_k^2 given by (2) we obtain: $v_{rms}(r \simeq 50h^{-1}Mpc) \simeq 83kms^{-1}h^{-0.92}$, so that, at the 90% confidence level $30km/s < v(r = 50h^{-1})h^{-0.92} < 135km/s$, which once again demonstrates that high peculiar velocities on scales $\sim 50h^{-1}Mpc$ are extremely difficult to accommodate within the framework of an $\Omega = 1$ CDM model.

Assuming $\delta_k^2 \propto k^n$ in (9) we obtain : $v(r) \propto r^{-\frac{n+1}{2}}$. For an $\Omega = 1$ CDM model $\delta_k^2 \propto k$ on scales $\sim 50h^{-1}Mpc$, giving $v(r) \propto r^{-1}$ on large scales. On the other hand for low Ω CDM models, $n \simeq -1$ on scales $\sim 50h^{-1}Mpc$, which implies $v \approx constant$.

A comparison of $v(r) \propto r^{-\frac{n+1}{2}}$ with $\frac{\delta\rho}{\rho}(r) \propto r^{-\frac{3+n}{2}}$, shows that large scales are weighed much more heavily in $v(r)$ than in $\frac{\delta\rho}{\rho}(r)$. A quantity having still greater power on large scales than either $v(r)$ or $\frac{\delta\rho}{\rho}$, is the peculiar gravitational potential, which is related to $\frac{\delta\rho}{\rho}$ via the Poisson equation: $\Delta\phi = 4\pi G a^2 \delta\rho$. For a spherically symmetric distribution of mass –

$$\phi(r) \simeq \frac{G\delta M}{r} \simeq \frac{4}{3}\pi G\rho(a(t)r)^2 \frac{\delta\rho}{\rho} \quad (10)$$

where $a(t)r = R$, is the physical length scale, and ρ – the background density of the Universe. Since $\rho \propto a^{-3}(t)$, and $\frac{\delta\rho}{\rho} \propto a(t)$, we find that fluctuations in the linear gravitational potential are independent of time in an Einstein - de Sitter Universe. From (10) we also find that $\phi(r) \propto r^{-\frac{n-1}{2}}$, which demonstrates that $\phi(r)$ is *scale invariant* for a primordial Harrison - Zeldovich spectrum ($n = 1$).

VI. Loitering Cosmological Models

In the preceding two sections we have been mainly dealing with models in which non-barionic dark matter plays the key role in determining how and where structure forms in the Universe. The reason for excluding baryonic models from our consideration was two-fold. Firstly (as discussed in §2), baryonic models suffer from the *growth problem*: too little growth in density perturbations occurs in baryonic models, to account for the presence of galaxies by the present epoch. Secondly, in order to be consistent with inflation, one requires that $\Omega = 1$, whereas primordial nucleosynthesis strongly suggests¹⁷ $\Omega_b \leq 0.04h^{-2}$.

One would also like to point out, that in addition to these problems faced primarily by baryonic models of structure formation, all matter dominated cosmological models

with $\Omega = 1$, suffer from the so-called *age problem*, *ie.* the Universe turns out to be too young to accommodate a population of "old" objects, such as globular clusters³⁶.

In the remainder of this paper I will discuss a scenario for galaxy formation, in which only baryons cluster, and in which both the growth, as well as the age problems are absent. According to this scenario – dubbed *loitering Universe*, the Universe underwent a recent phase of very slow expansion (or loitering), during which its scale factor was either a constant, or grew *very slowly* with time³⁹. The advantages of such an expansion regime are two-fold: firstly, a Universe which loitered in the past can be much older than a matter dominated Einstein - de Sitter Universe whose age $T \simeq \frac{2}{3H} \leq 13$ billion years for values of the Hubble parameter $100 \geq H \geq 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ indicated by observations. (This is illustrated in fig(9).)

Secondly, since inhomogeneities grow quasi-exponentially during *loitering* (see fig(10)), galaxies can form by the present epoch out very small initial fluctuations. Therefore a low density of matter which clusters is not a hindrance to the growth of perturbations. Furthermore, any feature in the density power spectrum associated with the transition from radiation- domination to matter-domination (such as λ_{eq} in §3) is on a larger scale than in an Einstein- deSitter Universe, which might be the explanation for the extra power in $w(\theta)$ which is observed .

These features of a loitering Universe have been discussed in detail by Sahni, Feldman and Stebbins³⁹, who also show that if one limits the density in baryons to $\Omega_b \simeq 0.2$ (in keeping with virial estimates of dark matter in clusters of galaxies), then loitering could only have occurred in a fairly narrow redshift interval $2 \leq z \leq 7.2$. Thus it may be possible to observationally rule out (or confirm) loitering, by studying the number count of galaxies as a function of their redshift, as well as by observations relating to the number density of absorption lines in the spectra of distant QSO's. Some work in this direction has been reported in this conference by Patrick Das Gupta.

Discussion

In this brief review I have attempted to confront realistic models of structure formation with observational data describing the properties of our Universe on scales $\geq 20h^{-1}$ Mpc. On such scales density fluctuations are still linear and it is possible to make a systematic check of theoretical predictions of cosmological models *vs.* observations, without resorting to detailed nonlinear calculations such as N-body simulations.

We have seen that although the CDM model is very successful in explaining the small scale texture of galaxy clustering, it seems to lack sufficient large scale power to explain either the clustering of galaxies on scales $\sim 50 - 100$ Mpc., or to successfully account for the large observed bulk motions of galaxies on scales $\sim 50h^{-1}$ Mpc. As alternatives to the *standard* matter dominated CDM model, both a CDM model with a cosmological constant, as well as the loitering Universe model of §6, seem to have greater success in predicting more power on larger scales, as required by observations.

Note. Just as this paper was nearing completion, a press release announced the detection of an anisotropy in the microwave background measured by COBE⁴⁶: $\frac{\Delta T}{T} \simeq 1.1 \times 10^{-5}$, on scales $> 7^\circ$ (the associated quadrupole being $Q \simeq 5 \times 10^{-6}$). The COBE results indicate a spectrum $\delta_k^2 \propto k^n$, with $n = 1.1 \pm 0.6$ on scales $> 700h^{-1}$ Mpc. ($n = 1$ is the scale-invariant Harrison - Zeldovich spectrum.) The anisotropies in the CMBR measured by COBE, are consistent with the unbiased, matter dominated, CDM model, as well as with a CDM model with a non-zero cosmological constant. Most other models of structure formation however (with the exception of scenarios involving cosmic strings or a loitering epoch) seem to be ruled out.

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Figure Captions

Fig. 1 The angular correlation function $w(\theta)$ for galaxies in the Automatic Plate Measuring Machine (APM) galaxy survey¹ – dots represent galaxies. The CDM prediction is shown by the smooth curve.

Fig. 2a Spectrum of the CMBR (taken with the FIRAS instrument on COBE²) compared to a blackbody with a temperature of $2.735 \pm 0.06^\circ K$. Boxes are centered on measured points and show a size of assumed error $\sim 1\%$. The units for the vertical axis are $10^{-4} \text{ergs s}^{-1} \text{cm}^{-2} \text{sr}^{-1} \text{cm}$.

Fig 2b. Anisotropy limits^{2,3,43} on $\frac{\Delta T}{T}$. The solid line at $\theta > 7^\circ$ corresponds to the recent detection of a CMBR anisotropy: $\frac{\Delta T}{T} \simeq 1.1 \times 10^{-5}$ on scales $\theta > 7^\circ$ by COBE⁴⁶.

Fig. 3 Growth of linear density perturbations⁹. The wavelength of the perturbation is taken to be $\simeq \lambda_{eq}$ - the comoving horizon scale at matter - radiation equality ($\lambda_{eq} \simeq \frac{13}{\Omega h^2} Mpc$). The solid lines correspond to perturbations in baryons - $(\frac{\delta \rho}{\rho})_B$, and the dashed line to perturbations in a non - baryonic component - $(\frac{\delta \rho}{\rho})_X$, such as Hot or Cold Dark Matter. The dotted vertical lines z_{eq} and z_{rec} correspond to epochs of matter - radiation equality, and recombination, respectively.

Fig. 4 The density fluctuation spectrum $\overline{\rho} k^3 \times \delta_k^2$ is plotted for: The Hot Dark Matter Model¹² and the Cold Dark Matter Model^{19,21,44}. An arbitrary normalisation is assumed.

Fig. 5 The *rms* mass fluctuation on a scale R , $\frac{\Delta M}{M}(R)$, is plotted for an unbiased CDM Universe with $\Omega = 1$ and $h = 0.5$. $\frac{\Delta M}{M}$ is normalised to agree with the observed excess in the number density of galaxies at $8h^{-1} Mpc$. ($\frac{\Delta M}{M}^2(R, t_0) = \frac{1}{2\pi^2} \int_0^R k^2 dk |\delta_k^2| W^2(kR)$, $W(kR) = 3(\frac{\sin(kR)}{(kR)^3} - \frac{\cos(kR)}{(kR)^2})$, is the *top hat* window function¹⁷.)

Fig. 6 Equal area projections of the a) North, and b) South, galactic hemispheres, taken with the IRAS satellite⁴⁰. The galactic plane lies along the equator. The numbers along the circumference indicate longitude. The major clusters in the Northern hemisphere include - Virgo (centre) and Hydra - Centaurus ($270^\circ - 310^\circ$). In the Southern hemisphere: Persius - Pisces ($130^\circ - 150^\circ$), Pavo - Indus ($315^\circ - 345^\circ$) and Fornax.

Fig. 7 A CfA redshift slice³⁰ containing ~ 1067 galaxies. The survey strip is centered at $\delta = 29.5^\circ$ and is 6° in declination. The observed velocity *vs.* right ascension is shown.

Fig. 8 Velocities for the Local Group and the Virgo cluster are shown in a rough pen sketch (not drawn to scale!).

LG denotes the Local group, HC - the Hydra - Centaurus supercluster, and GA - the region of the Great Attractor.

Fig. 9 The scale factor in the loitering Universe scenario³⁹. The early evolution is matter dominated $a \propto t^{\frac{2}{3}}$ followed by a loitering phase $a \simeq constant$. As can be seen, the Universe is *very* old, *ie* $T_0 \gg H_0^{-1}$.

Fig. 10 Growth of linear inhomogeneities $\frac{\delta(t)}{\delta_1}$ superimposed over the scale factor $a(t)$ evolution for a loitering Universe. In this figure $\frac{\delta}{\delta_1}$ grows by a factor of 10^5 . (Scales for the two curves are different.)

