Betti numbers as a new probe of non-Gaussianity in the CMB

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Outline of talk

- Primordial non-Gaussianity of the CMB: simulations
- Minkowski Functionals and non-Gaussian deviations
- Betti numbers and non-Gaussian deviations

Temperature fluctuations of the CMB

$$\frac{\Delta T(\hat{n})}{T_o} = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$$

$$a_{\ell m} \propto \int d^3k \, \Phi(\vec{k}, t_i) \, \Delta_\ell(k, t_0) \, Y^*_{\ell m}(\hat{k})$$

Primordial gravitational potential

Transfer function

Primordial non-Gaussianity

Consider expansion to cubic order as:

$$\Phi(\vec{x}) = \Phi^G(\vec{x}) + f_{NL}\left((\Phi^G(\vec{x}))^2 - \langle(\Phi^G)^2\rangle\right) + g_{NL}(\Phi^G(\vec{x}))^3 + \ldots$$

- Characterized by non-linearity parameters f_{NL} and g_{NL}.
- *Local* since the non-linear contributions depend only on same spatial point.

Simulation of non-Gaussian CMB maps

Liguori, Mattarese & Moscardini (2003), Salmon (1996), Chingangbam & Park (2009)

Method:

Rewrite $a_{\ell m}$ as real space integral

$$a_{\ell m} = \int dr\, r^2 \Phi_{\ell m}(r) \Delta_\ell(r)$$

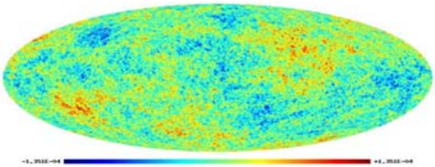
$$\Phi_{\ell m}(r) \equiv \Phi^G_{\ell m}(r) + f_{NL} \Phi^{NG}_{\ell m}(r) + g_{NL} \Phi^{NNG}_{\ell m}(r)$$

Non-Gaussian maps

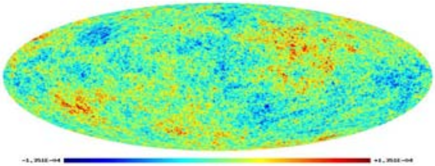
Chingangbam & Park (2009)

Resolution = 30 arcmin:

Gaussian $\longrightarrow$

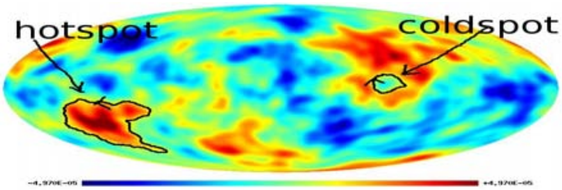


$g_{NL} = 5 \times 10^6 \longrightarrow$



Minkowski Functionals

Gott, Park et al (1990), Tomita (1986)



Threshold $\longrightarrow \nu \equiv \frac{\Delta T / T_0}{\sigma_0}$

Excursion set: consider set of all pixels above a chosen ν.

Consider iso-temperature contours for each threshold ν.

1. V_0 : Area fraction of the excursion set
2. V_1 : Contour length
3. V_2 : Genus = number of hot spots - number of cold spots

Minkowski Functionals for Gaussian fields

Gott, Park et al (1990), Tomita (1986)

Gaussian formula

$$V_0 = A_0 \, \mathrm{erfc}\,(\nu/\sqrt{2})$$

$$V_1 = A_1 \; e^{-\nu^2/2}, \qquad A_1 \propto \frac{\sigma_1}{\sigma_0}$$

$$V_2 = A_2 \; \nu \; e^{-\nu^2/2}, \qquad A_2 \propto \left(\frac{\sigma_1}{\sigma_0}\right)^2$$

Non-Gaussian deviations

$$\Delta V_i \equiv \left(V_i^{NG} - V_i^{G}\right) / V_i^{G,max}$$

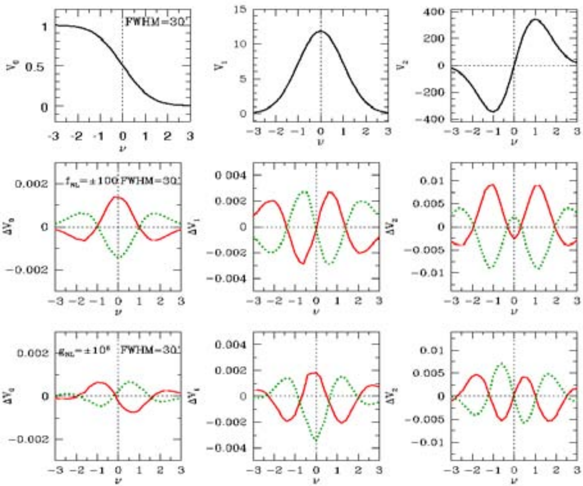
Minkowski Functionals and non-Gaussian deviations

Chingangbam & Park (2009), Hikage et al (2008), Schmalzing & Gorski (1997), Matsubara (2003), Hikage et al (2006), Pogosyan et al (2009), Matsubara (2010)

Gaussian $\longrightarrow$

$f_{\rm NL}$ $\longrightarrow$

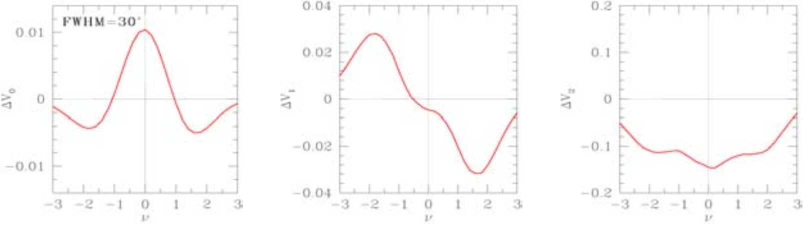
$g_{\rm NL}$ $\longrightarrow$



Minkowski Functionals and non-Gaussian deviations

Rocha et. al. (2005)

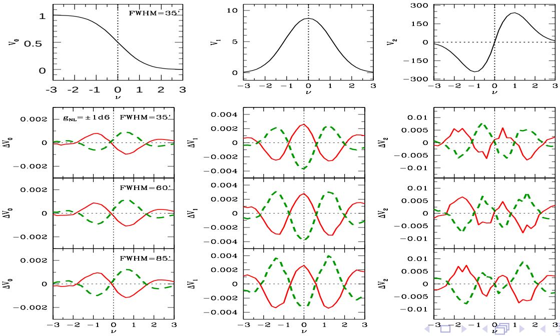
Non-Gaussian maps generated using HEALPIX:



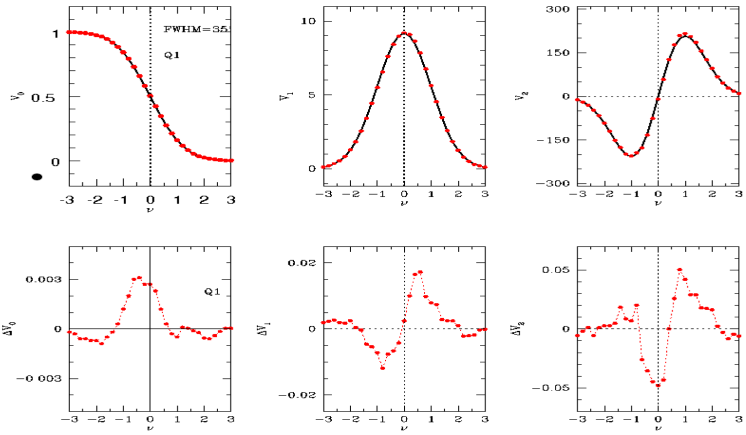
Comparision with WMAP data

Include observational effects in the simulations: beam size and shape, instrumental noise, pixellization effect, mask for Galaxy and point sources..

The characteristic non-Gaussian deviation shapes of the MFs are maintained.



Minkowski Functionals measured from WMAP data



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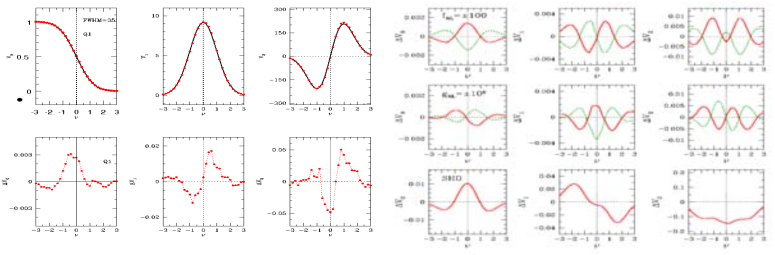


The non-Gaussian deviation shapes seen in the WMAP data are not ALL consistent with theoretical predictions of f_{NL} and g_{NL}.

Constraints on $f_{\rm NL}$ and $g_{\rm NL}$

Constraints on f_{NL} using Minkowski Functionals:

- $f_{\rm NL}$ from WMAP 5 yr data, Komatsu et al (2008) :

$$f_{NL} = -57 \; \pm 60 (68\% CL)$$

Constraints on f_{NL} and g_{NL} from bispectrum and trispectrum:

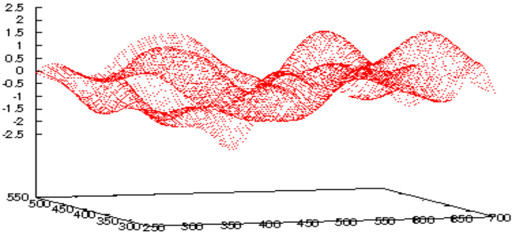
- $f_{\rm NL}$ from WMAP 7 yr data, Komatsu et al (2010) :

$$f_{\rm NL} = 32 \; \pm 21 (68\% CL)$$

- g_{NL} from WMAP 5 year data, Smidt et al (2010) :

$$-7.4 \times 10^5 < g_{\rm NL} < 8.2 \times 10^5 \quad (95\% CL)$$

Betti numbers: new statistics



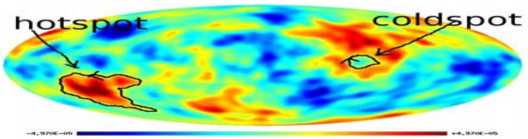
At each ν, if we consider the pixels above ν the temperature field "manifold" breaks up into connected components and holes.

The connected components are just the hot spots while the holes are the cold spots.

Betti numbers : new statistics

Definition of betti numbers:

- β_0 = number of connected components.
- β_1 = number of holes.



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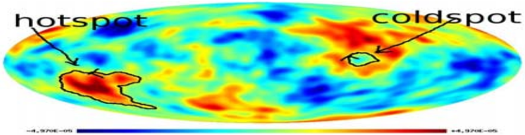

What is known analytically for Gaussian fluctuation field :

$$g(\nu) = \beta_0(\nu) - \beta_1(\nu) = A\,\nu\,e^{-\nu^2/2}. \qquad \text{Tomita (1986)}$$

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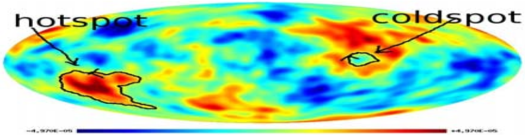

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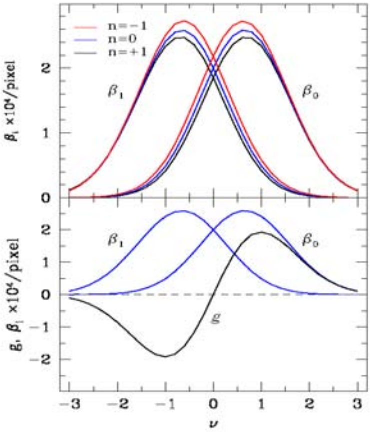
Individually, β_0 and β_1 not known analytically for Gaussian field.

Numerical computation of β_0 and β_1

Changbom Park, P. Chingangbam, R. Weygaert, W. Hellwing, J. Hidding, P. Pranab, H. Edelsbrunner



Input power spectrum:
$P(k) \propto k^n$.

Average over 1000 Gaussian simulations on a square grid.

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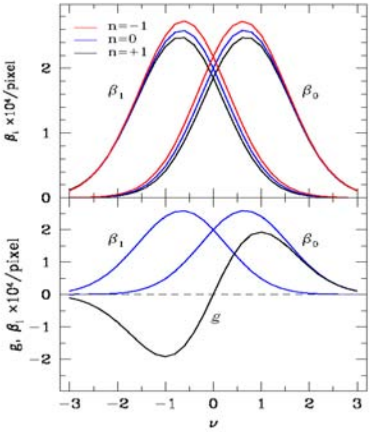
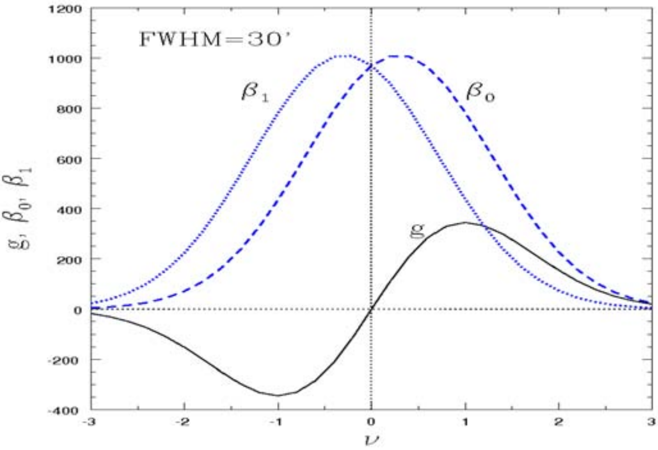


Average over 1000 Gaussian simulations on a square grid.

Input power spectrum: $P(k) \propto k^n$.

- Scaling with smoothing scale : $\propto \; R_s^2$.
- Scaling of the amplitude with n : $\beta_0 \, R_s^2 \;\propto\; n + 2 + 2/3$.
- Peak location : shifts towards zero as n becomes more negative.

Betti numbers for CMB maps

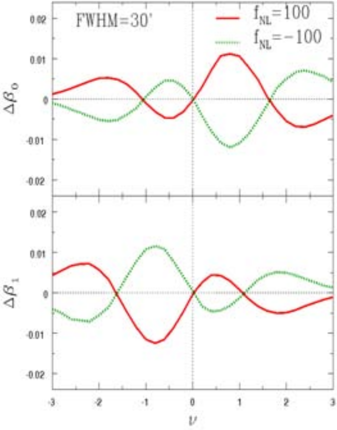


Average over 200 simulation maps.

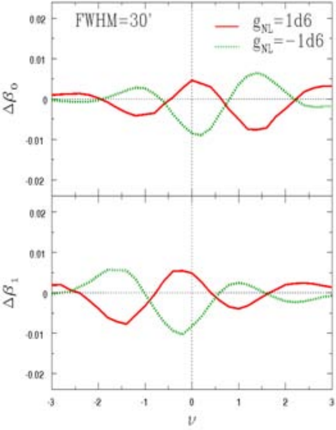
Non-Gaussian deviations of Betti numbers

Chingangbam & Park, in preparation

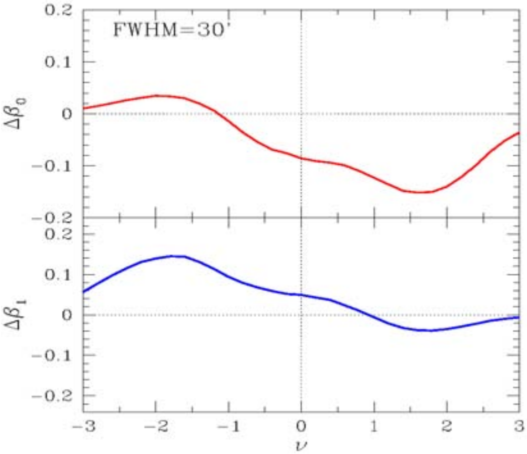
f_{NL}



g_{NL}



Non-Gaussian deviations of Betti numbers

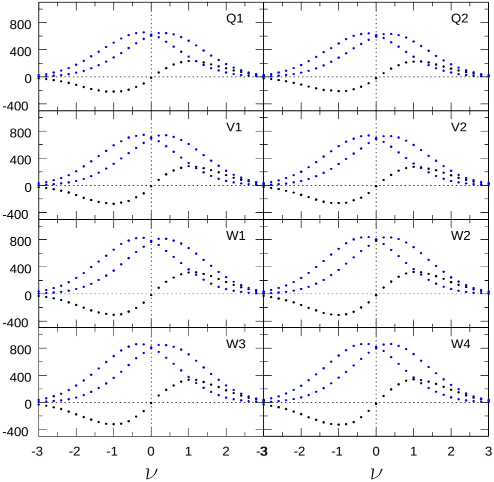


Summary

• Predictions of non-Gaussian deviations of the MFs for f_{NL} and g_{NL} due to f_{NL} and g_{NL} do not all agree with the deviations seen in the WMAP data.

• New statistics : Betti numbers or counts of hot and cold regions in the CMB. The hope is they will provide more information about non-Gaussian deviations, in addition to the MFs.

• Need to know the analytic formula for the Betti numbers for a Gaussian random field to be able to estimate the Gaussian component in the observational data.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Q2
V1
V2
W1
W2
W3
W4
800
400
0
-400
-3
-2
-1
0
1
2
3
ν