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What can Classical Gravity tell us about Quantum Structure of Spacetime?

T. Padmanabhan

IUCAA, Pune University Campus, Ganeshkhind, Pune 411007, INDIA.

E-mail: `paddy@iucaa.ernet.in`

Abstract. Several features of classical gravity, combined with the existence of Davies-Unruh temperature of horizons, support the following paradigm: Gravitational field equations in a wide class of theories, including Einstein's theory, should be viewed as describing the thermodynamic limit of the statistical mechanics of (as yet unknown) atoms of spacetime. I present the conceptual evidence for this emergent paradigm and discuss several facets of this approach.

1. Introduction

Classical gravity has no obvious $\hbar$ dependence while quantum theory has and hence — if we take the question in the title literally — classical gravity cannot tell us anything about the quantum structure of spacetime! But there is one effect, viz., the thermodynamics of spacetime horizons [1] which brings together the principles of quantum theory and gravity. This will be the main anchor in the attempt to learn about the quantum structure of spacetime using 'classical' gravity.

The entire paradigm [2] can be summarized in one sentence: Gravity is an emergent phenomenon like gas dynamics or elasticity with the gravitational field equations having the same status as, say, the equations of fluid dynamics/elasticity. Historically, this paradigm originated with Sakharov [3] and were implemented in different ways by Jacobson [4], Volovik [5], Bei-Lok Hu [6] and others. I will now elaborate on this theme drawing mainly from the work I was involved in.

2. Can such a top-down view help?

This programme is a "top-down" approach to quantum spacetime in the sense of zooming in from the top to smaller and smaller spatial scales. (Some people use the word "top-down" to mean exactly the opposite; I will use "top-down" the way I have defined it.) One may wonder whether such an approach will be really useful because one is attempting to determine the features of the microscopic theory from knowing its properties at the macroscopic scales. To reassure you, let me describe at least two other examples in which the deeper theory leaves a signature on the 'top layer'.

(i) *Boltzmann's conjecture of atoms*: Classical thermodynamics of a gas/fluid uses variables like density, pressure etc. in the continuum description. But the fact that such a fluid can store and exchange heat energy *cannot* be understood within the continuum theory. Boltzmann had the insight to suggest that thermal phenomena *demand* the existence of microscopic degrees of freedom in matter. In fact, the law of equipartition, expressed as $E/[(1/2)k_BT] = N$ relates two thermodynamic variables E, T (which are well-defined for a continuum fluid) to N, the number density of microscopic degrees of freedom, which cannot be interpreted in the continuum limit. The Avogadro's number, closely related to N, was determined even before we understood what exactly it counts and without any direct evidence for the molecular structure of matter. This is splendid example of our being able to say something about microscopic structure from the features of macroscopic theory.

(ii) *Equality of inertial and gravitational masses*: A more dramatic example is provided by Einstein's use of principle of equivalence which, of course, was known for centuries. Einstein realized that $m_i = m_g$ is not a trivial algebraic accident which should be taken for granted, as others before him have done, but requires an explanation. This led him to the description of gravity in terms of the geometry of spacetime. The relation $m_i = m_g$ was a signature of the deeper theory discernible in the approximate [top-layer] description.

These two examples show that for a top down approach to be useful, you need to ask the right questions! One way is to pick up features of the theory that are usually taken for granted ('algebraic accidents') — or not even noticed — and demand deeper explanations for them. This is the procedure I will follow in this programme to probe the quantum structure of spacetime from known aspects of classical gravity.

3. The conceptual road map

In this approach, it is necessary to make a clear distinction between quantum description of spacetime structure and a theory of quantum gravity.

Classical field equations of gravity *also* describe the classical dynamics of the spacetime because of the geometrical interpretation. In the emergent gravity paradigm, these field equations have a status similar to the equations of fluid mechanics or elasticity. So, if this paradigm is correct, one should *not* expect quantizing a classical theory of gravity to lead us to the quantum structure of spacetime any more than quantizing the equations of elasticity or hydrodynamics will lead us to atomic structure of matter! Quantizing the elastic vibrations of a solid will lead only to phonon physics [6] just as quantizing a classical theory of gravity will lead to graviton physics. The latter could be as different from a description of quantum structure of spacetime just as phonon physics is from that of the atom.

Combining the principles of GR and quantum theory is probably not just a technical problem that could be solved by sufficiently powerful mathematics. It is a more of a conceptual issue and the decades of failure of sophisticated mathematics in delivering quantum gravity indicates that we should try a different approach. This is very much in tune with item (ii) mentioned in Sec. 2. Einstein did not create a sophisticated mathematical model for m_i and m_g and try to interpret $m_i = m_g$. He used thought experiments to arrive at a conceptual basis in which $m_i = m_g$ can be embedded naturally so that $m_i = m_g$ will cease to be an algebraic accident. Once this is done, physics itself led him to the maths that was needed.

Of course, the key issue is what could play the role of a guiding principle similar to principle of equivalence in the present context. *For this, my bet will be on the thermodynamics of horizons.*[2, 7] A successful model will have the connection between horizon thermodynamics and gravitational dynamics *at its foundation* rather than this feature appearing as a result derived in the context of certain specific solutions to the field equations. There are three technical points closely related to this conjecture which needs to be recognized if this approach has to succeed:

- One must concentrate on the general context of observer dependent, local, thermodynamics associated with the local horizons, going beyond the *black hole* thermodynamics. Black hole horizons in the classical theory are far too special, on-shell, constructs to provide a sufficiently general back-drop to understand the quantum structure of spacetime. The

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preoccupation with the black hole horizons loses sight of the conceptual fact that all horizons are endowed with temperature as perceived by the appropriate class of observers. Observer dependence [8] of thermal phenomena is a feature and not a bug!

- One should also think beyond Einstein's theory and use the structure of, say, Lanczos-Lovelock models of gravity [9] in exploring the microstructure of spacetime. Previous work (starting from [10]) has shown that the interpretation of gravity as an emergent phenomenon transcends Einstein's theory and remains applicable to (at least) all Lanczos-Lovelock models. Exploiting this connection will allow us to discriminate between results of general validity from those which are special to Einstein's theory in $D = 4$. Irrespective of whether Lanczos-Lovelock models are relevant to real world, they provide a good test-bed to see which concepts/results are robust and general.
- A corollary is that one should not think of entropy of horizons as being proportional to their area. This result, which is true in Einstein's theory, fails for all higher order Lanczos-Lovelock models [11]. But all the general thermodynamic features still continue to remain valid. Because area brings in several other closely related geometrical notions, restricting oneself to Einstein's theory leads to an incorrect view of what entropy and quantum microstructure of spacetime could be.

4. Thermodynamics of horizons

1. Temperature of horizons

One can associate a temperature with any null surface that can act as horizon for a class of observers, in any spacetime (including flat spacetime). This temperature is determined by the behaviour of the metric close to the horizon and has *nothing to do with the field equations* (if any) which are obeyed by the metric.

The simplest situation is that of Rindler observers in flat spacetime with acceleration κ who will attribute a temperature $k_B T = (\hbar/c)(\kappa/2\pi)$ to the Rindler horizon — which is just a $X = T$ surface in the flat spacetime having no special significance to the inertial observers. While these results are usually proved for an eternally accelerating observer, they also hold in the appropriately approximate sense for an observer with variable acceleration [12]. In general, this result can be used to compare the vacuum state in a freely falling frame with that in a locally accelerated frame in an approximate manner.

An immediate consequence, not often emphasized, is that *all* thermodynamic variables must become observer dependent. If one considers a "normal" gaseous system with "normal" thermodynamic variables $(T, S, F$ etc.....$)$ as a highly excited state of the inertial vacuum, it is obvious that a Rindler observer will attribute to this gas different thermodynamic variables compared to what an inertial observer will attribute. Thus thermal effects in the accelerated frame brings in [8, 13] a new level of observer dependence even to *normal* thermodynamics.

More generally, whenever a bifurcation horizon divides the spacetime into two causally disconnected regions R and L, the global vacuum state $|\text{vac}\rangle$ of a quantum field theory, described by a functional $\langle\text{vac}|\phi_L, \phi_R\rangle$ in terms of the field modes in R and L, can be expressed in the form $\langle\phi_L|e^{-(\pi/\kappa)H_R}|\phi_R\rangle$ where H_R is the Hamiltonian describing the dynamics in one of the wedges [14] and κ is the suitably defined acceleration. Tracing out the modes ϕ_L beyond the horizon will lead to a thermal density matrix for the observables in the right wedge $\rho(\phi'_R, \phi_R) \propto \langle\phi'_R|e^{-(2\pi/\kappa)H_R}|\phi_R\rangle$ corresponding to the horizon temperature $T = \kappa/2\pi$. This result only depends on the near horizon geometry having the approximate form of a Rindler metric and is independent of the field equations of the theory.

2. Entropy of horizons

It seems reasonable to assume that *all* horizons should also possess entropy which is again, by necessity, observer dependent. One would have expected that if integrating out certain field modes leads to a thermal density matrix ρ, then the entropy of the system should be related to lack of information about the *same* field modes and should be given by $S = -\text{Tr}\ \rho \ln \rho$. This entropy (called entanglement entropy) (i) is proportional to area of the horizon and (ii) is divergent without a cut-off [15]. Such a divergence makes the result meaningless and thus we fail to attribute a unique entropy to horizon using just QFT in a background metric. That is, while the temperature of the horizon can be obtained through the study of QFT in an external geometry, one cannot understand the entropy of the horizon by the same procedure.

This is because, in sharp contrast to temperature, the *entropy* associated with a horizon in the theory depends on the field equations of the theory, which we will now briefly review. Given the principle of equivalence (interpreted as gravity being spacetime geometry) and principle of general covariance, one could still construct a wide class of theories of gravity. For example, if we take the action functional

$$A = \int d^D x \sqrt{-g}\left[L(R^{ab}_{cd}, g^{ab}) + L_{\text{matt}}(g^{ab}, q_A)\right] \tag{1}$$

(where L_{matt} is the matter Lagrangian for some matter variables denoted symbolically as q_A) and vary the metric with appropriate boundary conditions, we will get the field equations (see e.g., chapter 15 of [16]):

$$\mathcal{G}_{ab} = P_a^{\ cde} R_{bcde} - 2\nabla^c\nabla^d P_{acdb} - \frac{1}{2}L g_{ab} \equiv \mathcal{R}_{ab} - \frac{1}{2}L g_{ab} = \frac{1}{2}T_{ab} \tag{2}$$

where $P^{abcd} \equiv (\partial L/\partial R_{abcd})$. A nice subclass of theories in which the field equations remain second order in the metric is obtained if we choose L such that $\nabla_a P^{abcd} = 0$. The most general scalar functionals $L(R^{ab}_{cd}, g^{ij})$ satisfying this condition are specific polynomials in curvature tensor which lead to the Lanczos-Lovelock models [9] with the field equations:

$$P^{de}_{ac} R^{bc}_{de} - \frac{1}{2}L\delta^b_a \equiv \mathcal{R}^b_a - \frac{1}{2}L\delta^b_a = \frac{1}{2}T^b_a; \quad \mathcal{R}^b_a \equiv P^{de}_{ac}R^{bc}_{de} \tag{3}$$

and the structure of the theory is essentially determined by the tensor P^{ab}_{cd} which has the algebraic symmetries of curvature tensor and is divergence-free in all indices.

These field equations, in general, have black hole solutions with horizons in asymptotically flat spacetime. Studying the physical processes occurring in such spacetimes, one can obtain an expression for the entropy of the horizon (called Wald entropy [11]) in such theories. The resulting form of the entropy turns out to be closely related to the Noether current J^a which is conserved due to the diffeomorphism invariance of these theories under $x^a \to x^a + q^a(x)$. Computing the Noether current for Lanczos-Lovelock models, one can show that the entropy is given by:

$$S_{\text{Noether}} = \beta \int d^{D-1}\Sigma_a\ J^a \equiv \frac{\beta}{2}\int d^{D-2}\Sigma_{ab}\ J^{ab} = \frac{2\pi}{\kappa}\oint_{\mathcal{H}} P^{abcd}\epsilon_{ab}\epsilon_{cd} d\sigma \tag{4}$$

where $\beta^{-1} = \kappa/2\pi$ is the horizon temperature and $J^a \equiv \nabla_b J^{ab}$ with $J^{ab} = -J^{ba}$ ensuring $\nabla_i J^i = 0$. In the final expression the $(D-2)$-dimensional integral is on a space-like bifurcation surface with ϵ_{ab} denoting the bivector normal to the bifurcation surface. Thus horizon entropy is given by an integral over the horizon surface of the P^{abcd}, which we may call the *entropy tensor* of the theory.

In the simplest context of $L \propto R$, leading to $P^{ab}_{cd} \propto (\delta^a_c\delta^b_d - \delta^a_d\delta^b_c)$, we have $\mathcal{R}^a_b \propto R^a_b, \mathcal{G}^a_b \propto G^a_b$ and one recovers Einstein's equations and the entropy will be proportional to the area of the horizon. But in general, the entropy of the horizon is *not* proportional to the area and depends on the theory. This feature makes it impossible to identify the entanglement entropy as the entropy of the Lanczos-Lovelock models. (For the possible approach to tackle this issue within the emergent paradigm, which I will not discuss here, see ref. [17])

This dichotomous situation as regards temperature versus entropy is the first indication that the thermodynamics of the horizon, probed by QFT in a external gravitational field, is just the tip of an iceberg. As we will see the emergent paradigm provides a better understanding of these features.

5. Thermodynamic interpretation of field equations and action functionals

I stressed in Sec. 2 that for the top-down approach to be of some use, we need to identify the 'algebraic accidents' in the top level description which are usually taken for granted without a demand for explanation. I will briefly summarize four such issues in classical gravity, which can give us a clue about the microscopic theory.

(1) Gravitational field equations become a thermodynamic identity:

It can be shown that [18] the field equations in any Lanczos-Lovelock model, when evaluated on a static solution of the theory which has a horizon, can be expressed in the form of a thermodynamic identity $TdS = dE_g + PdV$. Here S is the correct Wald entropy of the horizon in the theory, E_g is a geometric expression involving an integral of the scalar curvature of the sub-manifold of the horizon and PdV represents the work function of the matter source. The differentials dS, dE_g etc. should be thought of as indicating the difference in S, E_g etc between two solutions in which the location of the horizon is infinitesimally displaced.

This equality between field equations on the horizon and the thermodynamic identity — originally obtained [19] for spherical horizons in Einstein's theory where $S = (1/4)4\pi a^2, T = \kappa/2\pi, E_g = a/2$ and P is the pressure of the source — has now been demonstrated for an impressively wide class of models [20] like stationary axisymmetric horizons and evolving spherically symmetric horizons in Einstein gravity, static spherically symmetric horizons and dynamical apparent horizons in Lanczos-Lovelock gravity, generic, static horizon in Lanczos-Lovelock gravity, three dimensional BTZ black hole horizons, FRW cosmological models in various gravity theories and even in the case Horava-Lifshitz gravity.

Incidentally, while Davies-Unruh temperature scales as $\hbar$ the entropy scales as $1/\hbar$ (coming from inverse Planck area), thereby making TdS independent of $\hbar$! This is reminiscent of the fact that in normal thermodynamics $T \propto 1/k_B, S \propto k_B$ making TdS independent of k_B. In both cases, the effects due to discrete microstructure (indicated by non-zero $\hbar$ or k_B) disappears in the continuum limit thermodynamics. Thermal phenomena requires microstructure but thermodynamical laws are independent of it!

There are significant differences between this identity $TdS = dE_g + PdV$ to which field equations reduce to and the so called Clausius relation $TdS = dE_m$ (used, for example, by Jacobson [4]) which are not often appreciated.

- In addition to the obvious existence of the work term PdV it should be stressed that E_m used in the Clausius relation is related to *matter stress tensor* while E_g in the first law is a purely geometrical construct built out of the metric. The origin of these differences can be traced to two different kinds of virtual displacements of the horizons considered in these two approaches to define the infinitesimal differences [21].
- More importantly, *while $TdS = dE_g + PdV$ holds in widely different contexts,* it has been found to be *impossible* to generalize $TdS = dE_m$ beyond Einstein's theory without introducing additional assumptions (like dissipation), the physical meaning of which remains unclear.

(2) Gravitational field equations and local entropy balance:

The above result has a simple generalisation in terms of local Rindler horizons. Let us choose any event $\mathcal{P}$ and introduce a local inertial frame (LIF) around it with Riemann normal coordinates $X^a = (T, \mathbf{X})$. Let k^a be a future directed null vector at $\mathcal{P}$ and we align the coordinates of LIF such that it lies in the $X - T$ plane at $\mathcal{P}$ and transform from the LIF to a local Rindler frame (LRF) with acceleration κ along the X axis. Let ξ^a be the approximate Killing vector corresponding to translation in the Rindler time. Usually, we shall do all the computation on a time-like surface (called a "stretched horizon") with $N^2 = -\xi_a\xi^a =$ constant, infinitesimally away from the Rindler horizon $\mathcal{H}$ which corresponds to $N^2 = 0$. An infinitesimal displacement of a local patch of the stretched horizon in the direction of its normal r_a, by an infinitesimal proper distance ϵ, will change the proper volume by $dV_{prop} = \epsilon\sqrt{\sigma}d^{D-2}x$ where σ_{ab} is the metric in the transverse space. The flux of energy through the surface will be $T^a_b\xi^b r_a$ and the corresponding entropy flux can be obtained by multiplying the energy flux by $\beta_{\rm loc} = N\beta$. Hence the 'loss' of matter entropy to the outside observer because the virtual displacement of the horizon has engulfed some matter is $\delta S_m = \beta_{\rm loc}\delta E = \beta_{\rm loc}T^{aj}\xi_a r_j dV_{prop}$. Recalling from Eq. (4) that $\beta_{loc}J^a$ gives the gravitational entropy current, the change in the gravitational entropy is given by $\delta S_{\rm grav} \equiv \beta_{loc} r_a J^a dV_{prop}$ where J^a is the Noether current corresponding to the local Killing vector ξ^a given by $J^a = 2\mathcal{G}^a_b\xi^b + L\xi^a$. As the stretched horizon approaches the true horizon, $Nr^a \to \xi^a$ and $\beta\xi^a\xi_a L \to 0$. Hence we get, in this limit: $\delta S_{\rm grav} \equiv \beta\xi_a J^a dV_{prop} = \beta 2\mathcal{G}^{aj}\xi_a\xi_j dV_{prop}$. Comparing $\delta S_{\rm grav}$ and δS_m we see that the field equations $2\mathcal{G}^a_b = T^a_b$ can be interpreted as the entropy balance condition $\delta S_{grav} = \delta S_{matt}$ thereby providing direct thermodynamic interpretation of field equations as local entropy balance in local Rindler frame.

In the conventional approach, there is no explanation for the fact that the entropy of the horizon is related to a Noether current arising from the diffeomorphism $x^a \to x^a + \xi^a(x)$. In the emergent paradigm, the spacetime is analogous to a solid made of atoms and $x^a \to x^a + \xi^a(x)$ is analogous to the deformation of an elastic solid. When such a deformation leads to changes in accessible information — like when one considers the virtual displacements of horizons — it is understandable that the gravitational entropy changes (See Sec. 7).

(3) Holography of gravitational action functionals:

If the gravitational dynamics and horizon thermodynamics are so closely related, with field equations becoming thermodynamic identities on the horizon, then the action functionals of the theory (from which we obtain the field equations) must contain information about this connection. This clue comes in the form of another unexplained algebraic accident related to the structure of the action functional.

Gravity is the only theory known to us for which the natural action functional preserving symmetries of the theory contain second derivatives of the dynamical variables but still leads to second order differential equations. (As we see from Eq. (3), this feature is shared by all the Lanczos-Lovelock action functionals.) It can be shown that this result will be true for actions that can be separated into a surface term and a bulk term with the surface term being an integral over $\partial_a(q^A\pi^a_A)$ where q^A are the dynamical variables and π^a_A are the canonical momentum. All Lanczos-Lovelock action functional have this form [10, 22]. This structure allows one to interpret all these action functionals, including Einstein-Hilbert action, as providing the momentum space description (see p. 292 of [16]) of the theory.

In the conventional approach this is hardly taken note of as something worthy of explanation and, in fact, there is no explanation possible within the standard framework. In contrast, this result has direct links with the thermodynamic interpretation. To see this, note that: (a) The field equations can be obtained from varying the bulk term after ignoring (or by canceling with a counter-term) the surface term. (b) But if we evaluate the surface term on the horizon of any solution to the field equations of the theory, one obtains the entropy of the horizon! How does the surface term, which was discarded before the field equations were obtained know about the entropy associated with a solution to those field equations?!

The answer lies in the fact mentioned before, viz. that the surface and bulk term of the Lagrangian are related in a specific manner

$$\sqrt{-g}L_{\rm sur} = -\partial_a \left(g_{ij}\frac{\delta\sqrt{-g}L_{\rm bulk}}{\delta(\partial_a g_{ij})}\right) \tag{5}$$

thereby duplicating the information about the horizon entropy [22]. This duplication of information allows one (i) to construct [24] a suitable variational principle based purely on the surface term and (ii) to obtain the full action [7] from the surface term alone using the entropic interpretation. (Incidentally, in the Riemann normal coordinates around any event $\mathcal{P}$ the gravitational action reduces to a pure surface term, again showing that the dynamical content is actually stored on the boundary rather than in the bulk.)

4. Gravitational action as free energy of spacetime

The reason gravitational actions have a surface and bulk terms is because they are related to the entropy and energy of a static spacetimes with horizons, adding up to make the action the free energy of the spacetime [23]. This is most easily seem for any Lanczos-Lovelock model by writing the time component of the Noether current in the form:

$$L = \frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}\,J^{0\alpha}\right) - 2\mathcal{G}^0_0 \tag{6}$$

Only spatial derivatives contribute in the first term on the right hand side when the spacetime is static. Integrating this to obtain the action it is is easy to see (using Eq. (4)) that the first term gives the entropy and the second term can be interpreted as energy.

Finally, I stress that the real importance of these results arise from the fact that they hold for all Lanczos-Lovelock models in an identical manner.

6. The Avogadro number of the spacetime

The results described in the previous sections suggest that there is a deep connection between horizon thermodynamics and the gravitational dynamics. Because the spacetime can be heated up just like a body of gas, the Boltzmann paradigm ("If you can heat it, it has microstructure") suggests a reinterpretation of gravity as an emergent phenomenon. This motivates the study of the microscopic degrees of freedom of the spacetime exactly the way people studied gas dynamics *before* they understood the atomic structure of matter.

There exists, fortunately, an acid test of this paradigm. Boltzmann's conjecture led to the equipartition law $\Delta E = (1/2)k_B T\Delta N$ relating the number density ΔN of microscopic degrees of freedom required to store an energy ΔE at temperature T and to the determination of Avogadro number of a gas. If our ideas are correct, we should be able to relate the E and T of a given spacetime to determine the number density of microscopic degrees of freedom of the spacetime. Remarkably enough, this can be done directly from the field equations [25]. In a hot spacetime, Einstein's equations *imply* the equipartition law

$$E = \frac{1}{2}k_B\int_{\partial\mathcal{V}}\frac{\sqrt{\sigma}\,d^2x}{L_P^2}\ \left\{\frac{Na^\mu n_\mu}{2\pi}\right\} \equiv \frac{1}{2}k_B\int_{\partial\mathcal{V}} dn\,T_{\rm loc} \tag{7}$$

(where $T_{\rm loc} = (Na^\mu n_\mu/2\pi)$ is the local acceleration temperature and $\Delta n = \sqrt{\sigma}\,d^2x/L_P^2$) thereby allowing us to read off the number density of microscopic degrees of freedom. We again see that gravity is holographic in the sense that the number density Δn scales as the proper area $\sqrt{\sigma}\,d^2x$ of the boundary of the region rather than the volume. (In the case of a gas, we would have got an integral over the volume of the form $dV(dn/dV)$ rather than the area integral.) We also notice that, in Einstein's theory, the number density (dn/dA) is a constant with every Planck area contributing a single degree of freedom.

The true value of this result again rests on the fact that it holds true for all Lanczos-Lovelock models! For an Lanczos-Lovelock model with an entropy tensor P^{ab}_{cd} one gets the result

$$E = \frac{1}{2}k_B \int_{\partial\mathcal{V}} dn T_{loc}; \qquad \frac{dn}{dA} = \frac{dn}{\sqrt{\sigma}d^{D-2}x} = 32\pi P^{ab}_{cd}\epsilon_{ab}\epsilon^{cd} \tag{8}$$

where ϵ_{ab} is the binormal on the codimension-2 cross-section. All these gravitational theories are holographic and the density of microscopic degrees of freedom encodes information about the theory through the entropy tensor. *I consider these results as the most direct evidence for the emergent paradigm of gravity.*

The holographic nature of gravity which I have alluded to several times shows that area elements play a significant role in the microscopic description of the theory. This is directly related to the fact that the basic unit of the theory is the Planck *area* $\mathcal{A}_P \equiv (G\hbar/c^3)$. (Only by taking a square root, rather artificially, one obtains the Planck *length*; also see, [26]) Classical gravity, in fact, should be described using $\mathcal{A}_P$ rather than using G with Newton's law of gravity written in the form $F = (\mathcal{A}_P c^3/\hbar)(m_1 m_2/r^2)$. This has the crucial consequence that one cannot really take $\hbar \to 0$ limit at fixed $\mathcal{A}_P$ and call it classical gravity. Gravity is intrinsically quantum mechanical at all scales [27] because of the microstructure of spacetime just as normal matter is intrinsically quantum mechanical due to the atomic structure. One cannot study classical elasticity, say, by taking the $\hbar \to 0$ limit in a crystal lattice. This is because such a limit (which can give elasticity from lattice dynamics) will also make all the electrons in the atom collapse!

7. Entropy density of spacetime an its extremum

So far we have been faithfully following the 'top-down' philosophy of starting from known results in classical gravity and obtaining consequences which suggests an alternative paradigm. For example, the results in the last section were obtained by starting from the field equations of the theory, rewriting them in the form of law of equipartition and thus determining the density of microscopic degrees of freedom.

Ultimately, however, we have to start from a microscopic theory and obtain the classical results as a consequence. As a first step in this direction, I will now describe a rather attractive procedure of inverting the arguments of the last section and obtaining the field equations from the density of microscopic degrees of freedom.

We know that the thermodynamical behaviour of any system can be described by an extremum principle for a suitable potential (entropy, free energy ...) treated as a functional of appropriate variables (volume, temperature ,....). If our ideas related to gravitational theories are correct, it must be possible to obtain the field equations by extremising a suitably defined entropy functional. The fact that null surfaces block information suggests that the entropy functional should be closely related to null surfaces in the spacetime. This expectation turns out to be correct. The Lanczos-Lovelock field equations can indeed be obtained [28] by extremising an entropy functional associated with the null vectors in the spacetime.

The mathematics involves associating with every null vector in the spacetime an entropy functional S_{grav} and demanding $\delta[S_{grav} + S_{matter}] = 0$ for *all* null vectors in the spacetime where S_{matter} is the relevant matter entropy. Recall that 'how gravity tell matter to move' can be determined by demanding the validity of special relativistic laws for all locally inertial observers. Similarly, 'how matter curves spacetime' can be determined by demanding that the entropy maximization should hold for *all* local Rindler observers. The entropy functional we will use can be interpreted as the entropy of the null surface (which these observers perceive as horizon) plus matter entropy. The form of S_{matter} relevant for this purpose turns out to be

$$S_{\rm matt} = \int_{\mathcal{V}} d^k x \sqrt{-g}\, T_{ab} n^a n^b \tag{9}$$

where n^a is a null vector field and T_{ab} is the matter energy-momentum tensor. This expression can be interpreted as a limiting case of the entropy flux associated with the energy flux into the local Rindler horizon as viewed by the Rindler observers. The integration could be over this spacetime volume ($k = D$) or even over a nontrivial sub-manifold ($k < D$), say, a set of null surfaces. (This does not affect the variational principle or the resulting equations but the interpretation could be that of minimizing the rate of entropy production.) The simplest choice for S_{grav} is a quadratic expression in the derivatives of the null vector:

$$S_{grav} = -4 \int_{\mathcal{V}} d^k x \sqrt{-g}\, P_{ab}^{cd} \nabla_c n^a \nabla_d n^b \tag{10}$$

where P_{ab}^{cd} is a tensor having the symmetries of curvature tensor and is divergence-free in all its indices. So the P^{abcd} in Eq. (10) can be expressed as $P^{abcd} = \partial L/\partial R_{abcd}$ where L is the Lanczos-Lovelock Lagrangian and R_{abcd} is the curvature tensor [2]. This choice will also ensure that the equations resulting from the entropy extremisation do not contain any derivative of the metric which is of higher order than second. (More general possibilities exist which I will not discuss here.) Hence the expression for the total entropy becomes:

$$S[n^a] = -\int_{\mathcal{V}} d^k x \sqrt{-g} \left(4 P_{ab}^{cd} \nabla_c n^a \nabla_d n^b - T_{ab} n^a n^b \right) , \tag{11}$$

We now vary the vector field n^a in Eq. (11) after adding a Lagrange multiplier function $\lambda(x)$ for imposing the condition $n_a \delta n^a = 0$. Straight forward algebra (see e.g., Section 7.1 of ref. [2]) now shows that the condition $\delta[S_{grav} + S_{matter}] = 0$ leads to the equations

$$(2\mathcal{R}^a_b - T^a_b + \lambda \delta^a_b)\, n_a = 0 , \tag{12}$$

with $\mathcal{R}^a_b \equiv P^{jk}_{bi} R^{ai}_{jk}$ which we demand should hold for all null vector fields n^a. Using the generalized Bianchi identity and the condition $\nabla_a T^a_b = 0$ we obtain [2, 28] from Eq. (12) the equations

$$\mathcal{G}^a_b = \mathcal{R}^a_b - \frac{1}{2}\delta^a_b L = \frac{1}{2} T^a_b + \Lambda \delta^a_b \tag{13}$$

where Λ is a constant. These are precisely the field equations for gravity in a theory with Lanczos-Lovelock Lagrangian L.

In addition to providing a purely thermodynamic extremum principle for the field equations of gravity, the above approach also has the following attractive features.

- The extremum value of the entropy functional, when computed on-shell for a solution, leads to the Wald entropy. This is a non-trivial consistency check on the approach because it was not designed to reproduce the Wald entropy. It also shows that when the field equations hold, the total entropy of a region $\mathcal{V}$ resides on its boundary $\partial\mathcal{V}$ which is yet another illustration of the holographic nature of gravity.
- In the semi-classical limit, one can show [29] that the gravitational (Wald) entropy is quantized with $S_{\rm grav}$ [on-shell] $= 2\pi n$. In the lowest order Lanczos-Lovelock theory, the entropy is proportional to area and this result leads to area quantization. More generally, it is the gravitational entropy that is quantized. The law of equipartition for the surface degrees of freedom is closely related to this entropy quantization because both arise from the existence of discrete structures on the surfaces in question.

- The entropy functional in Eq. (11) is invariant under the shift $T_{ab} \to T_{ab} + \rho_0 g_{ab}$ which shifts the zero of the energy density. This symmetry allows any low energy cosmological constant, appearing as a parameter in the variational principle, to be gauged away thereby alleviating the cosmological constant problem to a great extent [30]. I will not discuss this issue here.

There is another way of interpreting Eq. (12) which is more in tune with the emergent perspective of gravity [31]. Note that, while Eq. (12) holds for any vector field once the normalization condition is imposed through the Lagrange multiplier, the entropy was originally attributed to *null* vectors and hence it is natural to study Eq. (12) when $n^a = \ell^a$, the null normal of a null surface $\mathcal{S}$ in the spacetime and project Eq. (12) onto the null surface. If $\boldsymbol{\ell}$ is the normal to $\mathcal{S}$, then such a projection leads to the equations:

$$R_{mn}\ell^m q^n_a = 8\pi T_{mn}\ell^m q^n_a; \quad R_{mn}\ell^m\ell^n = 8\pi T_{mn}\ell^m\ell^n \tag{14}$$

where $q_{ab} = g_{ab} + \ell_a k_b + \ell_b k_a$ with k^a being another auxiliary null vector satisfying $\boldsymbol{\ell}\cdot\boldsymbol{k} = -1$. The q_{ab} with $q_{ab}\ell^b = 0 = q_{ab}k^b$ acts as a projector to $\mathcal{S}$ (see ref. [31] for details). It is possible to rewrite the first equation in Eq. (14) in the form of a Navier-Stokes equation thereby providing a hydrodynamic analogy for gravity. This equation is known in the literature and is called Damour-Navier-Stokes (DNS) equation [32] and is usually derived by rewriting the field equations. Our analysis [31] provides an entropy extremisation principle for the DNS equation which makes the hydrodynamic analogy natural and direct.

It may also be noted that the gravitational entropy density — which is the integrand $\Im_{grav} \propto (-P^{cd}_{ab}\nabla_c\ell^a\nabla_d\ell^b)$ in Eq. (10) — obeys the relation:

$$\frac{\partial\Im_{\text{grav}}}{\partial(\nabla_c\ell^a)} \propto (-P^{cd}_{ab}\nabla_d\ell^b) \propto (\nabla_a\ell^c - \delta^c_a\nabla_i\ell^i) \tag{15}$$

where the second relation is for Einstein's theory. This term is analogous to the more familiar object $t^c_a = K^c_a - \delta^c_a K$ (where K_{ab} is the extrinsic curvature) that arises in the (1+3) separation of Einstein's equations. (More precisely, the projection to 3-space leads to t^c_a.) This combination can be interpreted as a surface energy momentum tensor in the context of membrane paradigm [33] because t_{ab} couples to δh^{ab} on the boundary surface when we vary the gravitational action (see, e.g., eq.(12.109) of [16]). Equation (15) shows that the entropy density of spacetime is directly related to t^c_a and its counterpart in the case of null surface. This term also has the interpretation as the canonical momentum conjugate to the spatial metric in (1+3) context and Eq. (15) shows that the entropy density leads to a similar structure.

Further, the *functional* derivative of the gravitational entropy in Eq. (10) has the form, in any Lanczos-Lovelock model:

$$\frac{\delta S_{\text{grav}}}{\delta\ell^a} \propto \mathcal{R}_{ab}\ell^b \propto J_a \tag{16}$$

Previous work [2, 25, 8] has shown that the current $J_a = 2\mathcal{R}_{ab}\ell^b$ plays a crucial role in interpreting gravitational field equations as entropy balance equations. In the context of local Rindler frames, when ℓ^a arises as a limit of the time-like Killing vector in the local Rindler frame, J_a can be interpreted as the Noether (entropy) current associated with the null surface. In that case, the generalization of the two projected equations in Eq. (14) to Lanczos-Lovelock model will read as

$$J_a\ell^a = \frac{1}{2}T_{ab}\ell^a\ell^b; \quad J_a q^a_m = \frac{1}{2}T_{ab}\ell^a q^b_m \tag{17}$$

which relate the gravitational entropy density and flux to matter energy density and momentum flux. (The second equation in the above set becomes the DNS equation in the context of Einstein's theory.) All these results, including the DNS equation, will have direct generalization to Lanczos-Lovelock models which can be structured using the above concepts. We again see that all these ideas find a natural home in the emergent paradigm.

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References

[1] Bekenstein J D 1972 *Nuovo Cim. Lett.* **4** 737–740; Hawking S W 1975 *Commun. Math. Phys.* **43** 199–220.; Davies P C W (1975) *J. Phys.* A **8** 609–616; Unruh W G (1976) *Phys. Rev.* D **14** 870.
[2] Padmanabhan T., *Rep. Prog. Phys.*, **73** (2010) 046901, [arXiv:0911.5004].
[3] Sakharov A D 1968 *Sov. Phys. Dokl.* **12** 1040.
[4] Jacobson T 1995 *Phys. Rev. Lett.* **75** 1260.
[5] Volovik G E 2003 *The universe in a helium droplet*, (Oxford University Press).
[6] B. L. Hu, (2010)*Gravity and Nonequilibrium Thermodynamics of Classical Matter* [arXiv:1010.5837]; *General Relativity as Geometro-Hydrodynamics* [arXiv:gr-qc/9607070].
[7] T. Padmanabhan, *Phys. Reports*, **406**, 49 (2005) [gr-qc/0311036].
[8] Padmanabhan T 2009 *A Dialogue on the Nature of Gravity* [arXiv:0910.0839]
[9] Lanczos C 1932 *Z. Phys.* **73** 147; Lovelock D 1971 *J. Math. Phys.* **12** 498.
[10] T. Padmanabhan, *Dark Energy: Mystery of the Millennium*, Albert Einstein Century International Conference, Paris, July 2005, *AIP Conference Proceedings* **861**, 858, [astro-ph/0603114]
[11] Wald R. M., *Phys. Rev. D* (1993), **48** 3427, [gr-qc/9307038]; Iyer V. and R. M. Wald, (1995), *Phys. Rev. D* **52** 4430, [gr-qc/9503052].
[12] D. Kothawala, T. Padmanabhan , *Phys.Letts.*, **B 690** , 201-206 (2010) [arXiv:0911.1017]; A. Raval, B.L. Hu, Don Koks, *Phys.Rev*, **D55**, 4795 (1997) [gr-qc/9606074].
[13] Marolf D, Minic D and Ross S 2004 *Phys.Rev.* **D69** 064006.
[14] Lee T D 1986 *Nucl. Phys.* B **264** 437; Unruh W G and Weiss N 1984 *Phys. Rev.* D **29** 1656.
[15] See, for e.g., L. Bombelli et al., *Phys.Rev.*, **D34**, 373 (1986); M. Srednicki, *Phys.Rev.*, **D71**, 66 (1993); T. Nishioka *et al. J.Phys.,* **A42**, 504008 (2009) [arXiv:0905.0932].
[16] T.Padmanabhan (2010) *Gravitation: Foundations and Frontiers*, Cambridge University Press, UK.
[17] Padmanabhan T. *Phys. Rev. D* **82**, 124025 (2010). [arXiv:1007.5066]
[18] Kothawala D and Padmanabhan T 2009 *Phys. Rev.* D **79** 104020 [arXiv:0904.0215].
[19] Padmanabhan T 2002 *Class. Quan. Grav.* **19** 5387 [gr-qc/0204019].
[20] There is large literature on this subject; see e.g., ref.110-124 of [2].
[21] D. Kothawala, *The thermodynamic structure of Einstein tensor*, [arXiv:1010.2207].
[22] A. Mukhopadhyay, T. Padmanabhan, *Phys.Rev.*, **D 74**, 124023 (2006) [hep-th/0608120].
[23] Sanved Kolekar, T. Padmanabhan , *Phys.Rev.*, **D 82**, 024036 (2010) [arXiv:1005.0619].
[24] T. Padmanabhan , *Gen.Rel.Grav.*, **38**, 1547-1552 (2006); *Int.J.Mod.Phys.*, **D 15**, 2029 (2006) [gr-qc/0609012]; *Adv. Sci. Lett.* , **2**, , 174 (2009) [arXiv:0807.2356].
[25] This was done in 2004 in the form of a relation $E = 2TS$ in Einstein's theory and is now generalized to all Lanczos-Lovelock models. T. Padmanabhan, *Class.Quan.Grav.*, **21**, 4485 (2004) [gr-qc/0308070]; *Mod. Phys. Lett.* **A 25**, 1129 (2010), [arXiv:0912.3165]; *Phys. Rev.*, **D 81**, 124040 (2010), [1003.5665].
[26] J.Makela, *On distance and area,* [arXiv:1011.2052].
[27] T. Padmanabhan , *Mod.Phys.Letts. A* , **17**, 1147 (2002). [hep-th/0205278]; *Gen.Rel.Grav.*, **34**, 2029-2035 (2002) [gr-qc/0205090].
[28] Padmanabhan T., (2008), *Gen.Rel.Grav.*, **40**, 529-564 [arXiv:0705.2533]; Padmanabhan T. and Paranjape A., (2007), *Phys.Rev.* D, **75**, 064004 [gr-qc/0701003].
[29] D. Kothawala, T. Padmanabhan, S. Sarkar, *Phys.Rev.*, **D78**, 104018 (2008) [arXiv:0807.1481]
[30] T. Padmanabhan, *Gen.Rel.Grav.*, **40**, 529 (2008) [arXiv:0705.2533]; *Adv. Sci. Lett.*, **2,** 174 (2009) [arXiv:0807.2356].
[31] Padmanabhan T., *Entropy density of spacetime and the Navier-Stokes fluid dynamics of null surfaces*, [arXiv:1012.0119].
[32] T. Damour, (1979), Thèse de doctorat d'État, Université Paris (available at http://www.ihes.fr/∼damour/Articles/).
[33] R.H. Price and K.S. Thorne, (1986), *Phys. Rev.* **D 33**, 915.