# THE ROLE OF GENERAL RELATIVITY IN ASTROPHYSICS

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# THE ROLE OF GENERAL RELATIVITY IN ASTROPHYSICS

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It gives me great pleasure to attend this Seminar dedicated to the Diamond Jubilee celebrations of my *alma mater*. I have happy recollections of attending lectures in this very lecture theatre as a student in 1955-57. While talking about general relativity may I also point out that this subject is about as old as this University. The first paper on this subject by Einstein(1) appeared in November 1915.

1. **Introduction.** Unlike its predecessor, the special theory of relativity, the general theory has taken a long time to be assimilated in the rest of physics. Even today it stands somewhat aloof from the mainstream of theoretical physics.

A look at the nature of the theory will give an indication as to the reason why. The theory deals basically with the law of gravitation and therefore his applications in the laboratory physics. It is worth recalling the ratio

$$\frac{G\,m_e\,m_p}{e^2} \cong 10^{-10} \qquad (1{\cdot}1)$$

expressing the smallness of the gravitational attraction between an electron and a proton when compared to their electorostatic attraction. It is because gravitation is such a weak interaction that it finds so few applications in the atomic, nuclear or particle physics.

It is when dealing with large masses that gravitation comes into its own. And such masses are only to be found in astronomy. Even Newton's law was first applied in the domain of astronomy, to the motion of the heavenly bodies in our solar system. Later fruitful application of this law were made to stellar structure, stellar dynamics, cosmology etc. The trajectories of man made spacecrafts are computed with this same law today.

General relativity was conceived by Einstein with several aims. There are conceptual problems with the formulation of Newton's laws of motion and gravitation which, though not of serious practical consequence, are indicators of the incompleteness of the Newtonian framework One example is the concept of instantaneous action at a distance. Einstein wanted to remove such defects. Further he was impressed by the so called Mach's principle(2). He was also guided by the desire to unite the whole of physics under one banner : the unified field theory.

It is well known that general relativity represents only a partial fulfilment of these ideals. Yet in the last analysis, its success or failure depends on the bread and butter question which is faced by any physical theory : "How far does it succeed in explaining natural phenomena in preference to other theories ?" I will be concerned here with this problem. Granted that we have a much simpler theory of Newton to work with, where should we look for applications of general relativity ? In what areas can we use this theory with greater confidence than we can use Newtonian gravitation ? These are the questions I hope to answer briefly today.

## 2. Strong and weak gravitational fields

As a general rule we can say that general relativity is to be preferred to Newtonian gravity in the situations where strong gravitational fields prevail. How to decide whether the field is weak or strong ? I will illustrate with an example.

Consider a spherical object of gravitational mass $M$ and surface area $4\pi R^2$. Construct the parameter

$$\alpha=\frac{2GM}{c^2R}\,, \tag{2.1}$$

where $c$=velocity of light. The gravitational field in the neighbourhood of this object can be considered weak if $\alpha << 1$. For example, for the sun $\alpha \cong 10^{-6} << 1$. In this case the equations of general relativity may be approximated by Newtonian equations and it would be permissible to use the latter in most cases. I have said 'most', not 'all'; because there are experimental tests which distinguish between the two theories even in the gravitational field of the Sun. These are the so called classical tests involving (i) the bending of light, (ii) the motion of the perihelion of Mercury (iii), the gravitational redshift, (iv) the precession of gyroscopes etc.

Great precision of measurement is required to verify the small differences involved in these tests. I shall not go into details here beyond stating that present measurements favour the theory of relativity.

However, these differences are small enough to justify the use of Newtonian gravitation in most cases. It is when one comes to strong gravitational fields ($\alpha \sim 1$) that the two theories differ considerably and the use of general relativity becomes more desirable. One such situation occurs in the study of cosmogical models. Although Newtonian cosmology has been studied(3,4), the real progress has come from applying the relativity theory to construct models of the universe, starting with the work of Friedmann(5). I will not discuss cosmology in this lecture but will confine myself to the application of general relativity to compact objects.

### 3. Supermassive objects

In 1962-63 the combined effort of radio and optical astronomers led to the discovery of the quasistellar objects (QSOs in brief). These objects are very bright and compact and the first two QSOs to be discovered, 3C 48 and 3C 273 were mistaken for stars in the Galaxy. Spectral measurements, however, revealed the remarkable fact that these objects possess large redshifts. How could these redshifts arise ?

There are three possible explanations. The redshift could be due to the expansion of the universe. Or it could be Doppler redshift arising from the fact that the QSOs have been ejected from our Galaxy or a nearby galaxy in an explosion so that they are receding from us. A third explanation is that the redshift is gravitational. Even today the real reason for QSO-redshift is not known, although a majority of astronomers would be inclined to put their money on the first, cosmological explanation.

Nevertheless, the brightness of QSOs implied that they were considerably more massive than ordinary stars and this led to the construction of models of supermassive stars with masses $M \gtrsim 10^6 M_\odot$. It is easy to see from (2.1) that the average density of matter is given by

$$\bar{\rho} \simeq 10^{16} \left(\frac{M_\odot}{M}\right)^2 \alpha^3. \tag{3.1}$$

For $M \sim 10^6 M_\odot$, $\alpha \simeq 10^{-2}$, we have $\bar{\rho} \simeq 10^{-2}\, g\, cm^{-3}$. Thus even for fairly low densites general relativity becomes significant. Work by Fowler(6) and others(7) has shown how even for $\alpha$ as small as $10^{-2}$, stellar equilibrium considerations are seriously modified by general relativity. For example, for such supermassive stars nuclear reactions are not triggered off in time to prevent the gravitational contraction. Such stars therefore undergo a gravitational collapse-i. e., continued unimpeded gravitational contraction.

Although there had been earlier instances of important work on gravitational collapse(8) the subject as such received a boost only after the discovery of QSOs. The collapse of a spherical distribution is easier to handle in general relativity. Once spherical symmetry is abandoned, the problem becomes much more difficult to handle and exact solutions are few. I shall return to this question later.

It is however, possible to have equilibrium solutions of spherical distributions with cold matter with arbitrarily high masses and redshifts. P. K. Das(9) has discussed such objects recently with a view to constructing models of QSOs. Within fairly realistic limits on the equations of state he is able to obtain stable models with redshifts $z \lesssim 2{\cdot}5$. These are purely gravitational redshifts with the light coming from the centre rather than the surface of the object. The idea of central gravitational redshifts was first proposed by Hoyle and Fowler(10).

## 4. Black holes

If a spherical object undergoes gravitational collapse, it crashes through the barrier set by $\alpha = 1$. A signal sent out from the surface of the object will never reach an external observer if $\alpha \geqslant 1$. The object therefore becomes totally invisible and has been given the name 'black hole'. The spherical black hole is often called the Schwarzschild black hole. The existence of such a black hole can only be discerned through its gravitational field.

A sysem of two stars going round each other provides a clue to the detection of a black hole if one of the members is a black hole. In a typical scenario the black hole pulls matter from its companion which spirals into it and in the process radiates $X$-rays. The recent detection of $X$-rays from a binary system associated with Cygnus $X-1$ is often quoted as an example of black hole detection(11). This system consists of a supergiant star with an invisible companion. Is the companion, which is a compact object, a black hole ? The answer depends on two factors, both theoretical in nature.

The first involves tho determination of the mass of the compact object from the double star dynamics. The answer is around 5-6 $M_\odot$. How does this compare with the maximum mass sustainable as a neutron star by a star at the end of its evolution ? This brings in the second question : 'What is the equation of state of highly dense matter ?" Investigations proceeding at present suggest limits around $\sim 2-3\, M_\odot$ although some physicists quote somewhat higher limits(12). If the compact object is higher in mass then these limits then the conclusion is that it must be a black hole, i. e. a star undergoing gravitational collapse. At present the inference that 'the compact object is a black hole' cannot be considered as established beyond doubt.

However, the observational uncertainty has not prevented theoreticians from investigating the important aspects of black holes. By considering certain aspects of non spherical gravitational collapse, theoreticians conclude that within the frame work of gravitation and electromagnetism the most general black hole is characterized by mass, charge and angular momentum. This is the Kerr-Newman black hole(14). The solution with mass and angular momentum was earlier obtained by Kerr(15). Theoreticians(16) have also derived the laws of black hole dynamics which have a curious resemblance to the laws of thermodynamics.

A couple of years ago Hawking(17) has raised the interesting possibility that black holes are not really black when viewed quantum mechanically. Like the tunnel effect of quantum theory it is possible to associate a temperature with the black hole depending on its surface gravity. The temperature of the Schwarzschild black hole of mass $M$ is given by

$$T=5\times10^{-8}\left(\frac{M_\odot}{M}\right)^{\circ}K. \tag{4.1}$$

Thus a typical star black hole has very low temperature and it hardly radiates. The effect is higher as $M$ decreases. Hawking and his collaborators argue that for $M \lesssim 10^{15} 9$ the radiation is large enough to evaporate the entire black hole :

## 5. Conclusions

These are some aspects of relativistic astrophysics. Limitations of time prevent me from talking about other interesting topics like white holes, gravitational radiation, etc. The interesting aspect is that general relativity has found increasing applications as a result of its interaction with astrophysics.

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