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Problems of Astro-Particle Physics in the Quasi-Steady State Cosmology

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Abstract

This work highlights the conceptual and theoretical issues underlying the quasi-steady state cosmology which was proposed by the F. Hoyle, G. Burbidge and the author as an alternative to the standard big bang cosmology.

To begin with it is shown with the help of a toy model how the problems of spacetime singularity and violation of the energy momentum conservation law that are present in the standard cosmology can be avoided by introducing a scalar field minimally coupled to gravity and having its sources in events where matter is created.

It is then shown that matter creation preferentially occurs near collapsed massive objects and the scalar field created at such mini-creation events has a feedback on spacetime geometry causing the universe to have a steady expansion as in the de Sitter model but with periodic phases of expansion and contraction superposed on it. The parameters of the toy model can be empirically fixed in relation to the cosmological observations thus providing tests of the theory.

Next it is argued that the toy model arises from a deeper theory which is Machian in origin with the inertia of a particle determined by the rest of the particles in the universe in a long range conformally invariant scalar interaction. The characteristic mass of a particle created is then the Planck mass. The Planck particle decays quickly to baryons. It is shown that the inertial effects produced by the Planck particles during their brief existence generate the scalar field of the toy model while the inertial effects of the stable baryonic particles give the more familiar Einstein equations of relativity.

The baryons into which the Planck particle decays form an SU3 octet which, in the high density – high energy environment of a mini-creation event finally forms the nuclei of hydrogen, helium and other elements of low atomic masses. These predicted abundances match those actually found.

Finally it is shown that extending the theory to the most general conformally invariant form automatically leads to the cosmological constant whose sign and magnitude are of the right cosmological order.

1. Introduction

We begin with the tentative definition that cosmology refers to a study of those aspects of the universe for which spatial isotropy and homogeneity can be used, with the spacetime metric taking the form

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \quad (1)$$

in terms of coordinates t, r, θ, ϕ with $r = 0$ at the observer. The topological constant k in this so-called Robertson–Walker form can be shown to be 0 or ± 1. The "particles" to which (1) applies are thought of as galaxies or clusters of galaxies, each "particle" having spatial coordinates r, θ, ϕ independent of the universal time t. They form what is often referred to as the Hubble flow.

Big-Bang cosmology in all its forms is obtained from the equations of general relativity,

$$R_{ik} - \frac{1}{2}g_{ik}R + \lambda g_{ik} = -8\pi G T_{ik}, \quad (2)$$

which follow from the variation of an action formula

$$A = \frac{1}{16\pi G}\int_V (R + 2\lambda)\sqrt{-g}d^4x + \int_V \mathcal{L}_{phys}(X)\sqrt{-g}d^4x \quad (3)$$

with respect to a general Riemannian metric

$$ds^2 = g_{ik}dx^i dx^k \quad (4)$$

within a general spacetime volume V. The physical Lagrang an $\mathcal{L}_{phys}(X)$ generates the energy-momentum tensor T_{ik} in this variation of g_{ik}. In the standard big-bang cosmology the physical lagrangian includes only particles and the electromagnetic field, whereas in inflationary forms of big-bang cosmology a scalar field is also considered to be added to $\mathcal{L}_{phys}$. This is done in various ways, being severally advocated by different authors (see Narlikar and Padmanabhan 1991 for a review).

The initial conditions assumed in the standard model are :

(i) The universe was sufficiently homogeneous and isotropic at the outset for the metric (1) to be used immediately over a range of the r-coordinate of relevance to presentday observation,

(ii) $k = 0$,

(iii) $\lambda = 0$,

(iv) The initial balance of radiation and baryonic matter was such that the light elements D, ^{3}He, ^{4}He, ^{7}Li were synthesised in the early universe in the following relative abundances to hydrogen

$$\frac{D}{H} \simeq \frac{^3He}{H} \simeq 2\times 10^{-5}, \frac{^4He}{H} \simeq 0.235, \frac{^7Li}{H} \simeq 10^{-10}.$$

From detailed calculations these abundances can be shown to require

$$\rho_{\rm baryon} \simeq 10^{-32}T^3 \text{ gcm}^{-3}, \quad (5)$$

the radiation temperature being in degrees kelvin. (Gamow 1946, Alpher, et al 1950, 1953, Hoyle and Tayler 1964).

There is a fundamental problem here. The instant t = 0 is the so-called spacetime singularity at which the field equations (2) break down. This is identified with the big bang epoch. All matter that we see in the universe (as well as radiation) is supposed to be given as an initial condition at $t = \epsilon > 0$. The initial instant ϵ can be taken arbitrarily close to $t = 0$ but not identified with it. Thus the action principle (3) itself gets restricted in validity since the singular epoch must be excluded from it too.

Conceptually this is an exceptional step to take. In theoretical physics the basic laws or principles like the action principle are considered superior to the specific solutions based on them. Yet here we seek to restrict the validity of (2) and (3) because the solution so warrants it! There is thus a clear indication here of an inconsistency of the overall framework.

The other problems of the standard big bang model often referred to as the horizon and flatness problems also relate to the above initial conditions assumed at $t = \epsilon > 0$. While the need for such far reaching assumptions as (i) to (iv) has always prompted a measure of unease they were widely accepted for a decade and a half, and are indeed still fully accepted by the more orthodox supporters of the standard model. Others, however, welcomed the inflationary idea of including a scalar field in the physical lagrangian that initially dominated both matter and other fields and which varied adiabatically in such a way as to give

$$\frac{\dot{S}^2}{S^2} = C, \quad S(t) = S(0)\exp\sqrt{C}t, \quad (6)$$

with C a constant. The solution (6) is considered to apply from $t = \epsilon > 0$, where $\epsilon \sim 10^{-36}$s to a value of t large compared to $1/\sqrt{C}$. It greatly reduces the range of the r-coordinate over which (i) is needed and it effectively removes the k-term from (1). It also removes any initial contributions from matter and radiation, but these are considered to be reasserted through a physical transition of the scalar field, which jumps the solution (6) to

$$\dot{S}^2 = \frac{A}{S}, S \simeq \left(\frac{9}{4}A\right)^{\frac{1}{3}} t^{\frac{2}{3}}, \quad (7)$$

which is the so-called closure model with matter just having sufficient expansion to reach a state of infinite dispersal, a condition that is considered most favourable for the eventual formation of stars and galaxies.

A major problem associated with inflation is how to effectively eliminate the cosmological constant. The value of this constant which gave the exponential solution (6) above must reduce to zero or, if the cosmological observations so demand, become as small as 10^{-108} of its initial value. Any theoretical trick invoked to achieve this has a contrived appearance (Weinberg 1989).

Last year F. Hoyle, G. Burbidge and I (1993, 1994 a,b,c) proposed an alternative scenario for cosmology called the quasi-steady state cosmology (QSSC) that gets round these conceptual difficulties as well as provides an adequate explanation of all the crucial cosmological observations. A review of the latter will be found in our previous papers (Hoyle et al op. cit.). The ideas discussed here are brief description of the theoretical aspects of our recent work.

It is commonly assumed by particle physicists that the very high energy regime that brings about a grand unification is obtainable only in the very early moments of the big bang cosmology. We show here that similar regimes exist near a typical mini-creation event of the QSSC. Moreover, unlike the 'once only never again' phase of very high energy of the big bang cosmology, here we have such phases occuring again and again, thus lending physical testability to the predictions made therein.

2. Creation of Matter: A Toy Model

The action principle (3) has a second term which is supposed to include physical contributions other than gravity. A close parallel exists between the scalar field used for inflation and the scalar field used earlier by Hoyle and Narlikar (1963) for obtaining the steady state model from Einstein's field equations. To begin with we will use the 1963 formalism as a "toy model" for describing creation of matter without violating the law of conservation of energy-momentum and without encountering spacetime singularity.

Thus the classical Hilbert action leading to the Einstein equations is modified by the inclusion of a scalar field C whose derivatives with respect to the spacetime coordinates x^i are denoted by C_i. The action is given by

$$\mathcal{A} = -\sum_a \int_{\Gamma_a} m_a ds_a + \int_V \frac{1}{16\pi G} R\sqrt{-g} d^4x - \frac{1}{2} f \int_V C_i C^i \sqrt{-g} d^4x + \sum_a \int_{\Gamma_a} C_i da^i \quad (8)$$

where C is a scalar field and $C_i = \partial C/\partial x^i$. f is a coupling constant. The last term of (8) is manifestly path-independent and so, at first sight it appears to contribute no new physics. The first impression, however, turns out to be false if we admit the existence of broken worldlines. For, if particles $a, b, \ldots$ are created at world points $A_0, B_0, \ldots$ respectively, then the last term of (8) contributes a non-trivial sum

$$-\{C(A_0) + C(B_0) + \ldots\}$$

to $\mathcal{A}$.

Thus, if the worldline of particle a begins at point A_0, then the variation of $\mathcal{A}$ with respect to that worldline gives

$$m_a \frac{da^i}{ds_a} = g^{ik} C_k \quad (9)$$

at A_0. In other words, the C-field balances the energy-momentum of the created particle.

The field equations likewise get modified to

$$R_{ik} - \frac{1}{2} g_{ik} R = -8\pi G \left[\underset{m}{T_{ik}} + \underset{c}{T_{ik}} \right] \quad (10)$$

where

$$\underset{c}{T_{ik}} = -f \left\{ C_i C_k - \frac{1}{2} g_{ik} C^l C_l \right\}. \quad (11)$$

Thus the energy conservation law is

$$\underset{m}{T^{ik}}{}_{;k} = -\underset{c}{T^{ik}}{}_{;k} = f C^i C^k{}_{;k}. \quad (12)$$

That is, matter creation via a nonzero left hand side of (12) is possible while conserving the overall energy and momentum. The C-field tensor has negative stresses which lead to the expansion of spacetime, as in the case of inflation.

From (9) we therefore get a necessary condition for creation as

$$C_i C^i = m_a^2; \qquad (13)$$

This is the 'creation threshold' which must be crossed for particle creation. How this can happen near a massive object, can be seen from the following simple example.

The Schwarzschild solution for a massive object M of radius $R > 2GM/c^2$ is

$$ds^2 = dt^2\left(1 - \frac{2GM}{r}\right) - \frac{dr^2}{1 - \frac{2GM}{r}} - r^2(d\theta^2 + \sin^2\theta d\phi^2), \qquad (14)$$

for $r \geq R$. Now if the C-field does not seriously change the geometry, we would have at $r \gg R$,

$$\dot{C} \approx m, \qquad C' \equiv \frac{\partial C}{\partial r} \cong 0. \qquad (15)$$

If we continue this solution closer to $r \approx R$, we find that

$$C^i C_i \equiv \left(1 - \frac{2GM}{r}\right)^{-1} m^2. \qquad (16)$$

In other words $C_i C^i$ increases towards the object and can become arbitrarily large if $r \approx 2GM$. So it is possible for the creation threshold to be reached *near* a massive collapsed object even if $C_i C^i$ is *below* the threshold far away from the object. In this way massive collapsed objects can provide new sites for matter creation. Further, because of the negative stresses the created matter is expelled outwards from the site while the C-field quanta escape with the speed of light. Thus, instead of a single big bang event of creation, we have mini-creation events near collapsed massive objects.

Since the C-field is a global cosmological field, we expect the creation phenomenon to be globally cophased and to have cosmological consequences. Thus, there will be phases when the creation activity is large, leading to the generation of the C-field strength in large quantities. However, the C-field growth because of its large negative stresses leads to a rapid expansion of the universe and a consequent drop in its background strength. When that happens creation is reduced and takes place only near the most collapsed massive objects thus leading to a drop in the intensity of the C-field. The reduction in C-field slows down the expansion, even leading to local contraction and so to a build-up of the C-field strength. And so on!

I emphasise a point that might be missed by particle physicists accustomed to working in a flat spacetime. When attempt is made to quantize a negative energy field, the negativity of energy leads to a run-away cascading, thus making quantization impossible. In curved spacetime, with a dynamical feedback of the field energy momentum tensor on spacetime geometry via Einsetin's equations the negative energy and stresses cause expansion of space, thus providing a control on the run-away situation but also provides a feed back on the quantization process. The detailed working of the latter is still to be carried out whereas the former part is reasonably worked out.

We can describe this up and down type of activity of the mini-creation events and its impact on cosmology as an oscillatory solution superposed on a steadily expanding de Sitter type solution of the field equations as follows. For the Robertson–Walker line element the equations (10)–(12) give

$$3\frac{\dot{S}^2 + kc^2}{S^2} = 8\pi G(\rho - \frac{1}{2} f\dot{C}^2), \qquad (17)$$

$$2\frac{\ddot{S}^2}{S} + \frac{\dot{S}^2 + kc^2}{S^2} = 4\pi G f\dot{C}^2, \qquad (18)$$

where $S(t)$ is the scale factor and k the curvature parameter ($= 0, \pm 1$). The cosmic time is given by t. These equations have a deSitter type solution given by

$$S \propto \exp(t/P), k = 0, \qquad \dot{C} = \text{constant}, \qquad \rho = \text{constant} \qquad (19)$$

The oscillatory solution is given by

$$k = +1, \qquad \dot{C} \propto 1/S^3, \qquad \rho \propto 1/S^3. \qquad (20)$$

Thus (17) becomes, in the latter case

$$\dot{S}^2 = -c^2 + \frac{A}{S} - \frac{B}{S^4}, \quad A, B = \text{constant}. \qquad (21)$$

Here the oscillatory cycle will typically have a period $Q \ll P$.

Although the exact solution of (21) will be difficult to obtain, we can use the following approximate solution of (19) and (20) to describe the short-term and long-term cosmological behaviour :

$$S(t) = \exp\left(\frac{t}{P}\right)\left\{1 + \alpha\cos\frac{2\pi t}{Q}\right\}. \qquad (22)$$

Note that the universe has a long term secular expanding trend, but because $|\alpha| < 1$, it also executes non-singular oscillations around it. For this reason this model has been called "quasi-steady state cosmology". We can determine α and our present epoch $t = t_0$ by the observations of the present state of the universe. Thus an acceptable set of parameters is

$$\alpha = 0.75, \qquad t_0 = 0.85Q, \qquad Q = 4 \times 10^{10} yr., \qquad P = 20Q. \qquad (23)$$

Although the set is not unique and there will be a *range* of acceptable values, we have worked with this set to illustrate the performance of the model (Hoyle et al 1994 a,b) vis-a-vis observations of the universe.

What is the nature of the created particle? A deeper theory which is outlined in §4 tells us that the particle in question has Planck mass. We will assume this result to begin with and explore its consequences for astroparticle physics.

3. Problems of high energy physics

The Planck particle is unstable and within a timescale of $\sim 10^{-43}$s, it decays into a large number of secondaries. The process involves a release of high energy since it begins with energy source of $\sim 10^{19}$ GeV which gets distributed over particles and radiation, the ultimate decay products being baryons, leptons and photons etc. We may see here an analogy with the descending energy ladder in the big bang cosmology, from $\sim 10^{19}$ GeV, through the GUT energy of $\sim 10^{16}$ GeV, down to the electroweak unification energy of $\sim 10^2$ GeV, to ~ 1 GeV for baryons. Instead of a single big bang, however, we now have numerous mini-creation events involving 'Planck fireballs' centred on all decaying Planck particles.

This transition from 10^{19} GeV to 10^2 GeV has the same range of interesting physics that particle physicists like to study in the context of the big bang cosmology. The advantage with the QSSC is that the Planck fireballs are physical objects that can be studied just like any other repetative physical phenomena. (In the 'early universe' of big bang cosmology the events are non-repetative). Moreover, many mini-creation events occur at modest redshifts $\lesssim 5$) and so are, in principle directly accessible to extragalactic astronomy, which is not the case for the early universe of big bang cosmology. As an example Narlikar and DasGupta (1993) have shown that the mini-creation events can be detected by gravity-wave detectors being planned now.

One interesting issue that is handled differently by the QSSC is the observed lack of balance between matter and antimatter. In the big bang cosmology the symmetry between matter and antimatter is normally sought to be broken during the GUTs era. Somewhat contrived scenarios are needed to understand the observed photon to baryon ratio. In the QSSC the problem is posed differently. Since the universe 'renews itself' over a few oscillations, we have to understand *why*, given a matter dominated phase now, it will persist even with the renewed phase. Since the C-field is a globally interacting field the imbalance in the current phase is expected to be propagated into the next. How exactly the propagation of broken symmetry takes place is still to be worked out.

Finally, let us see what happens at the limit of the decay process, when from the Planck particle we end up with a group of baryons and radiation. At temperature $\gg 1$ GeV we expect an equipartition of all eight particles of the baryon octet. Eventually, however, all except the more long-lived neutron and proton decay to proton and end up as hydrogen nuclei. The neutron and proton combine to form the helium nuclei. A simple counting thus tells us that with two out of eight particles forming helium we expect the helium abundance to be ~ 0.25 by mass.

A more detailed calculation has been given by Hoyle et al (1993) and it leads to values of not only the helium abundance but also of D, Li, Be, B including their isotopes that agree with observations. The important difference is that instead of the assumption (iv) and equations (5) of §1, we have here a different set of values for ρ and T with the result that the deuterium abundance does not constrain the cosmological baryonic matter density. In other words, dark matter can be baryonic. This issue therefore has implications not only for astrophysics and cosmology (Hoyle et al a,b) but also for particle physics.

4. Scale-Invariant Gravity

An important property of physical theories is scale invariance or conformal invariance. Maxwell's equations and the Dirac equation for massless particle are conformally invariant but general relativity is not. If, however, the inertial mass transforms inversely as the length scale in conformal transformation then the Dirac equation for a massive fermion as well as classical and quantum electrodynamics will become conformally invariant. Can general relativity be suitably reformulated to be conformally invariant? We indicate the steps towards this goal since they naturally lead to a comprehensive theory of matter creation that encompasses the toy model described above.

It is necessary to begin by finding an action $\mathcal{A}$ that is unaffected in its value by a scale transformation. The second term on the right-hand side of (3) can be made to satisfy this requirement. For a set of particles $a, b, \ldots$ of masses $m_a, m_b, \ldots$ the form of $\mathcal{L}_{\rm phys}$ usually considered in gravitational theory is

$$\sum_{a,b,\ldots} \int \frac{\delta_4(X,A)}{\sqrt{-g(A)}} m_a(A) da. \tag{24}$$

where the possibility of the particle masses varying with the spacetime position requires the mass $m_a(A)$ of particle a to vary with the point A on its path, and similarly for the other particles. Hence the second term on the right-hand side of (3) is

$$-\sum_a \int m_a(A)da. \tag{25}$$

With $da^* = \Omega da$ and $m_a^* = \Omega^{-1} m_a$ it is clear that (25) is invariant with respect to a conformal (scale) transformation.

The possibility of particle masses varying with spacetime coordinates arises most naturally in a Machian approach. Here the property of inertia is not entirely intrinsic to a particle but is also related to its presence in a non-empty universe. A quantitative description of this idea that we will follow here is based on an early work by two of us (Hoyle and Narlikar 1964). In this inertia is expressed as a scalar conformally invariant long range interaction between particles.

To begin with choose a scalar mass field $M(X)$ to satisfy

$$\Box_X M(X) + \frac{1}{6} R M(X) = \sum_a \int \frac{\delta_4(X,A)}{\sqrt{-g(A)}} da. \tag{26}$$

Equation (26) has both advanced and retarded solutions. We particularize an advanced solution $M^{\text{adv}}(X)$ and a retarded solution $M^{\text{ret}}(X)$ in the following way. $M^{\text{ret}}(X)$ is to be the so-called fundamental solution in the flat spacetime limit (Courant and Hilbert, 1962). This removes for $M^{\text{ret}}(X)$ the ambiguity that would obviously arise from the homogeneous wave equation. The corresponding ambiguity for $M^{\text{adv}}(X)$ is removed by the physical requirement that fields without sources are to be zero. Since

$$\Box[M^{\text{adv}} - M^{\text{ret}}] + \frac{1}{6} R[M^{\text{adv}} - M^{\text{ret}}] = 0, \tag{27}$$

the immediate consequences of this boundary condition is that $M^{\text{adv}} - M^{\text{ret}}$, being without sources, must be zero, so that

$$M^{\text{adv}}(X) = M^{\text{ret}}(X) = M(X) \text{ say}. \tag{28}$$

The gravitational equations are now obtained by putting

$$m_a(A) = M(A), \quad m_b(B) = M(B), \cdots \tag{29}$$

It can also be shown that in a conformal transformation the mass field $M(X)$ transforms as

$$M^*(X) = \Omega^{-1}(X) M(X), \tag{30}$$

a result that follows from the form of the wave equation (10) (c.f. Hoyle and Narlikar, 1974, 111). The outcome (*loc. cit.*, 112 *et seq*) is

$$K(R_{ik} - \frac{1}{2} g_{ik} R) = -T_{ik} + M_i M_k - \frac{1}{2} g_{ik} g^{pq} M_p M_q + g_{ik} \Box K - K_{;ik}, \tag{31}$$

where

$$K = \frac{1}{6} M^2. \tag{32}$$

These gravitational equations are scale invariant. It may seem curious that from a similar beginning, (24) for the action rather than (3), the outcome is more complicated, but this seems to be a characteristic of the physical laws. As the laws are improved they become simpler and more elegant in their initial statement but more complicated in their consequences.

Now make the scale change

$$\Omega(X) = M(X)/\bar{m}_0, \tag{33}$$

where $\bar{m}_0$ is a constant with the dimensionality of $M(X)$. After the scale change the particle masses simply become $\bar{m}_0$ everywhere and in terms of transformed masses the derivative terms drop out of the gravitational equations. And defining the gravitational constant G by

$$G = \frac{3}{4\pi \bar{m}_0^2}, \tag{34}$$

the equations (31) take the form of general relativity

$$R_{ik} - \frac{1}{2} g_{ik} R = -8\pi G T_{ik}. \tag{35}$$

It now becomes clear why the equations of general relativity are not scale invariant. These are the special form to which the scale invariant equations (31) reduce with respect to a particular scale, namely that in which particle masses are everywhere the same.

It is also clear that the transition from (31) to (35) is justified provided $\Omega(X) \neq 0$ or $\Omega(X) \neq \infty$. For example, if $M(X) = 0$ on a spacelike hypersurface the above conformal transformation breaks down. It is because of the existence of such time sections that the use of (35) leads to the (unphysical) conclusion of a spacetime singularity. It has been shown [Hoyle and Narlikar 1974, Kembhavi 1979] that the various spacetime singularities like that in the big bang or in a black hole collapse arise because of the illegitimate use of (35) in place of (31).

It is easily seen from the wave equation (26) that $M(X)$ has dimensionality $(\text{length})^{-1}$, and so has $\bar{m}_0$. Units are frequently used in particle physics for which both the speed of light c and Planck's constant $\hbar$ are unity and in these units mass has dimensionality (length)$^{-1}$. If we suppose these units apply to the above discussion then from (34)

$$\bar{m}_0 = (3/4\pi G)^{1/2}, \tag{36}$$

which with $c = \hbar = 1$ is the mass of the Planck particle. This suggests that in a gravitational theory without other physical interactions the particles must be of mass (36), which in ordinary practical units is about 10^{-5} gram, the empirically determined value of G being used. This conclusion can be supported at greater length [See Hoyle, et al 1944c]. We will, however, next consider what happens when the Planck mass decays into the much more stable baryons.

5. The creation of matter : A Machian description

A typical Planck particle a exists from A_0 to $A_0 + \delta A_0$, in the neighbourhood of which it decays into n stable baryonic secondaries, $n \simeq 6.10^{18}$, denoted by $a_1, a_2, \ldots a_n$. Each such secondary contributes a mass field $m^{(a_r)}(X)$, say, which is the fundamental solution of the wave equation

$$\Box m^{(a_r)} + \frac{1}{6} R m^{(a_r)} = \frac{1}{n} \int_{\sim A_0+\delta A_0} \frac{\delta_4(X,A)}{\sqrt{-g(A)}} da, \tag{37}$$

while the brief existence of a contributes $c^{(a)}(X)$, say, which satisfies

$$\Box c^{(a)} + \frac{1}{6} R c^{(a)} = \int_{A_0}^{A_0+\delta A_0} \frac{\delta_4(X,A)}{\sqrt{-g(A)}} da. \tag{38}$$

Summing $c^{(a)}$ with respect to $a, b, \ldots$ gives

$$c(X) = \sum_a c^{(a)}(X), \tag{39}$$

the contribution to the total mass $M(X)$ from the Planck particles during their brief existence, while

$$\sum_a \sum_{r=1}^{n} m^{(a_r)}(X) = m(X) \tag{40}$$

gives the contribution of the stable particles.

Although $c(X)$ makes a contribution to the total mass function

$$M(X) = c(X) + m(X) \tag{41}$$

that is generally small compared to $M(X)$, there is the difference that, whereas $m(X)$ is an essentially smooth field, $c(X)$ contains small exceedingly rapid fluctuations and so can contribute significantly to the derivatives of $c(X)$. The contribution to $c(X)$ from Planck particles a, for example, is largely contained between two light cones, one from A_0, the other from $A_0 + \delta A_0$. Along a timelike line cutting these two cones the contribution to $c(X)$ rises from zero as the line crosses the light cone from A_0, attains some maximum value and then falls back effectively to zero as the line crosses the second light cone from $A_0 + \delta A_0$. The time derivative of $c^{(a)}(X)$ therefore involves the reciprocal of the time difference between the two light cones. This reciprocal cancels the short duration of the source term on the right-hand side of (40). The factor in question is of the order of the decay time τ of the Planck particles, $\sim 10^{-43}$ seconds. No matter how small τ may be the reduction in the source strength of $c^{(a)}(X)$ is recovered in the derivatives of $c^{(a)}(X)$, which therefore cannot be omitted from the gravitational equations.

The derivatives of $c^{(a)}(X), c^{(b)}(X), \ldots$ can as well be negative as positive, so that in averaging many Planck particles, linear terms in the derivatives do disappear. It is therefore not hard to show that after such an averaging the gravitational equations become

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{6}{m^2} \Big[- T_{ik} + \frac{1}{6} (g_{ik} \Box m^2 - m^2_{;ik}) + (m_i m_k - \frac{1}{2} g_{ik} m_l m^l) + \frac{2}{3} (c_i c_k - \frac{1}{4} g_{ik} c_l c^l) \Big]. \tag{42}$$

Since the same wave equation is being used for $c(X)$ as for $m(X)$, the theory remains scale invariant. A scale change can therefore be introduced that reduces $M(X) = m(X) + c(X)$ to a constant, or one that reduces $m(X)$ to a constant. Only that which reduces $m(X)$ to a constant, viz

$$\Omega = \frac{m(X)}{m_0} \tag{43}$$

has the virtue of not introducing small very rapidly varying ripples into the metric tensor. Although small in amplitude such ripples produce non-negligible contributions to the derivatives of the metric tensor, causing difficulties in the evaluation of the Riemann tensor, and so are better avoided. Simplifying with (43) does not bring in this difficulty, which is why separating of the main smooth part of $M(X)$ in (41) now proves an advantage, with the gravitational equations simplifying to

$$8\pi G = \frac{6}{m_0^2}, \quad m_0 \text{ a constant}, \tag{44}$$

$$R_{ik} - \frac{1}{2}g_{ik}R = -8\pi G[T_{ik} - \frac{2}{3}(c_i c_k - \frac{1}{4}g_{ik}c_l c^l)]. \tag{45}$$

Using the metric (1) with $k = 0$ the dynamical equations for the scale factor $S(t)$ are

$$\frac{2\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} = \frac{4\pi}{3}G\bar{\dot{c}^2}, \tag{46}$$

$$\frac{3\dot{S}^2}{S^2} = 8\pi G\left(\bar{\rho} - \frac{1}{2}\bar{\dot{c}^2}\right), \tag{47}$$

with $\bar{\rho}$ the average particle mass density and $\bar{\dot{c}^2}$ being the average value of $\dot{c}^2$, the average value of terms linear in c and of $\dot{c}$ being zero. It is easily shown from (46) and (47) that

$$\frac{\partial\bar{\rho}}{\partial t} + \frac{3\dot{S}}{S}\bar{\rho} = \frac{1}{2}\left(\frac{\partial\bar{\dot{c}^2}}{\partial t} + \frac{4\dot{S}}{S}\bar{\dot{c}^2}\right). \tag{48}$$

If at a particular time there is no creation of matter then at that time the left-hand side of (48) is zero with $\bar{\rho} \propto S^{-3}$. And with the right-hand side also zero at that time $\bar{\dot{c}^2} \propto S^{-4}$. The sign of the $\bar{\dot{c}^2}$ term in (46) is that of a negative pressure, a characteristic of the fields introduced into inflationary cosmological models. The concept of Planck particles forces the appearance of a negative pressure. In effect the positive energy of created particles is compensated by the sign of the $\dot{c}^2$ terms, which in (46) increases $\ddot{S}/S$ and so causes the universe to expand. One can say that the universe expands because of the creation of matter. The two are connected because the divergence of the right-hand side of the gravitational equations (45) is zero.

As would be expected from this conservation property the sign of the $\bar{\dot{c}^2}$ term in (47) is that of a negative energy field. Such fields have generally been avoided in physics because in flat spacetime they would produce catastrophic instabilities – creation of matter with positive energy producing a negative energy $\bar{\dot{c}^2}$ term, producing more matter, producing a still larger $\bar{\dot{c}^2}$ term, and so on. Here the effect is to produce explosive outbursts from regions where any such instability takes hold, through the $\bar{\dot{c}^2}$ term in (46) generating a sharp increase of $\ddot{S}$. The sites of the creation of matter are thus potentially explosive. The explosive expansion of space serves to control the creation process and avoids the catastrophic cascading down the negative energy levels.

The requirement is in agreement with observational astrophysics which in respect of high energy activity is all of explosive outbursts, without evidence for the ingoing motions required by the popular accretion-disk theory for which there is no direct observational support. The profusion of sites where X-ray and γ-ray activity is occurring are on the present theory sites where the creation of matter is currently taking place.

A connection with the toy model can now be given. Writing

$$C(X) = \tau c(X), \tag{49}$$

where τ is the decay lifetime of the Planck particle, the action contributed by Planck particles $a, b, \ldots,$

$$-\sum_a \int_{A_0}^{A_0+\delta A_0} c(A)da \tag{50}$$

can be approximated as

$$-C(A_0) - C(B_0) - \ldots, \tag{51}$$

which form was used in the toy model. And the wave-equation for $C(X)$, using the same approximation, is

$$\Box C + \frac{1}{6}RC = \tau^{-2}\sum_a \frac{\delta_4(X, A_0)}{\sqrt{-g(A_0)}}, \tag{52}$$

which was also used in the toy model, except that the $1/6RC$ term is included in the wave equation and previously an unknown constant f appeared in place of τ^2.

6. A derivation of the cosmological constant

Writing $M^{(a)}(X), M^{(b)}(X), \ldots$ as the mass fields produced by the individual Planck particles $a, b, \ldots,$ the total mass field

$$M(X) = \sum_a M^{(a)}(X) \tag{53}$$

satisfies the wave equation (26) when $M^{(a)}, M^{(b)}, \ldots$ satisfy

$$\Box M^{(a)} + \frac{1}{6}RM^{(a)} = \int \frac{\delta_4(X,A)}{\sqrt{-g(A)}}da, \ldots \tag{54}$$

Scale invariance throughout requires all the mass fields to transform as

$$M^{*(a)} = M^{(a)}\Omega^{-1} \tag{55}$$

with respect to the scale change Ω, when both the left and right hand sides of every wave equation transform to its starred form multiplied by Ω^{-3}, i.e. the left hand side of (54) goes to $(\Box M^{*(a)} + \frac{1}{6}R^* M^{*(a)})\Omega^{-3}$ and the right hand side to

$$\Omega^{-3}\int \frac{\delta_4(X,A)}{\sqrt{-g^*(A)}}da^*. \tag{56}$$

Then the factor Ω^{-3} cancels to give the appropriate invariant equation. This cancellation is evidently unaffected if, instead of (54) for the wave equation satisfied by M^a, we have

$$\Box M^{(a)} + \frac{1}{6}RM^{(a)} + M^{(a)^3} = \int \frac{\delta_4(X,A)}{\sqrt{-g(A)}}da. \tag{57}$$

Since the cube term transforms to $M^{*(a)^3}\Omega^{-3}$ with respect to Ω changing (54) to (57) preserves scale invariance in what appears to be its widest form. Since in other respects the laws of physics always seem to take on the widest ranging properties that are consistent with the relevant forms of invariance we might think it should also be here, in which case (57) rather than (54) is the correct wave equation for $M^{(a)}$, and similarly for $M^{(b)}, \ldots$, the mass fields of the other Planck particles.

But this departure from linearity in the wave equations for the individual particles prevents a similar equation being obtained for $M = \sum_a M^{(a)}$. Nevertheless, the addition of the individual equations can be considered in a homogeneous universe to lead to an approximate wave equation for M of the form

$$\Box M + \frac{1}{6}RM + \Lambda M^3 = \sum_a \int \frac{\delta_4(X,A)}{\sqrt{-g(A)}}da, \tag{58}$$

$$\Lambda = N^{-2}, \tag{59}$$

where N is the effective number of particles contributing to the sum $\sum_a M^{(a)}$. The latter can be considered to be determined by an Olbers-like cut-off, contributed by the portion of the universe surrounding the point X in $M(X)$ to a redshift of order unity. In the observed universe this total mass $\sim 10^{22}M_\odot$, sufficient for $\sim 2.10^{60}$ Planck particles. The actual particles are of course nucleons of which there are $\sim 10^{79}$ But if suitably aggregated they would give $\sim 2.10^{60}$ Planck particles and with this value for N

$$\Lambda \simeq 2.5 \times 10^{-121}. \tag{60}$$

The next step is to notice that the wave-equation (58) would be obtained in usual field theory from $\delta\mathcal{A} = 0$ for $M \to M + \delta M$ applied to

$$\mathcal{A} = -\frac{1}{2}\int (M_i M^i - \frac{1}{6}RM^2)\sqrt{-g}d^4x + \frac{1}{4}\Lambda\int M^4\sqrt{-g}d^4x - \sum_a \int \frac{\delta_4(X,A)}{\sqrt{-g(A)}}M(X)da. \tag{61}$$

In the scale in which M is m_0 everywhere the derivative term in (61) vanishes and since $G = 3/4\pi m_0^2$ the term in R is the same as in (3), as are also the line integrals, requiring the remaining term to be the same gives

$$\lambda = -3\Lambda m_0^2. \tag{62}$$

Thus we have obtained not only a cosmological constant but also its magnitude, something that lies beyond the scope of the usual theory. With 2.5×10^{-121} for Λ as in (60) and with m_0 the inverse of the Compton wavelength of the Planck particle, $\sim 3.10^{32}$ cm^{-1}, (62) gives

$$\lambda \simeq -2.10^{-56}\ \text{cm}^{-2}, \tag{63}$$

agreeing closely with the magnitude that has previously been assumed for λ. In the classical big bang cosmology there is no dynamical theory to relate the magnitude of λ to the density or other physical properties of matter. For observational consistency it is assumed that λ is of order (63). A dynamical derivation is possible if one goes into the very early inflationary epochs. However, the values of λ deduced from those calculations are embarrassingly large, being $10^{108} - 10^{120}$ times the value given by (63). The problem then is, how to reduce λ from such high values to the presently acceptable range (Weinberg 1989). By contrast, the present derivation leads to the acceptable range of values with very few theoretical assumptions.

7. Conclusions

The theory developed in this paper differs from big-bang cosmology in what we believe to be an important aspect, that the gravitational equations are scale invariant. The gravitational equations including both the creation terms and the cosmological constant then reduce in the constant mass frame to

$$R_{ik} - \frac{1}{2}g_{ik}R + \lambda g_{ik} = -8\pi G[T_{ik} - \frac{2}{3}(c_i c_k - g_{ik}\frac{1}{4}c_l c^l)]. \qquad (64)$$

The immediate successes of the theory are :

(i) The circumstance that G determined by (34) is necessarily positive requires gravitation to act as an attractive force. Aggregates of matter must tend to pull together. This is unlike general relativity where gravitation can as well be centrifugal, with aggregates of matter blowing always apart, as follows if G in the action (3) of general relativity is chosen to be negative.

(ii) In the cosmological case with homogeneity and isotropy the pressure contributed by the c-field term in the gravitational equations is negative, explaining the expansion of the universe.

(iii) Also in the cosmological case, the energy contribution of the c-field is negative, which ensures that when the creation conditions (9) are satisfied the creation process tends to cascade with explosive consequences.

(iv) The magnitude of the constant λ is shown to be of the order needed for cosmology. Unlike big-bang cosmology this is a deduction not an assumption.

There are also a few unsolved questions relating to particle physics which are briefly stated thus :

(i) How is the theory quantized in curved spacetime, including its dynamical feed-back on spacetime geometry?

(ii) How do we understand the particle-antiparticle asymmetry as an effect propagated (through a global interaction) from one generation of oscillation to the next?

(iii) Do we need non-baryonic dark matter at all?

(iv) Can we relate the parameters (23) to fundamental physical constants?

We hope that, given the successes achieved by the empirical model in explaining the cosmological observations, we can interest at least a few particle physicists in the attempts to come to gripps with the solution of the above problems.

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