

On the general correspondence between field theories and the theories of direct interparticle action†

BY J. V. NARLIKAR

Institute of Theoretical Astronomy, University of Cambridge

(Received 4 September 1967)

Abstract. It is well known that classical electrodynamics can be described both as a field theory and as a theory of direct interparticle action. In the present paper it is shown that, provided certain general conditions are satisfied, fields of arbitrary spin have their counterparts in 'direct particle fields'. This correspondence between the two formalisms is established in the Riemannian space-time used for general relativity.

1. *Introduction.* The two fundamental interactions of classical physics, electrodynamics and gravitation, have been described in two different ways. One makes use of the concept of direct interactions between pairs of particles, while the other involves interactions between particles and fields. Historically the former came first. Newton's law of gravitation was stated as a law of action at a distance between pairs of material particles. This was followed by a similar law in electrodynamics—the Coulomb law. However, Coulomb's law did not have the same success in electrodynamics as Newton's law had in gravitation. The law failed to give an adequate description of interaction between rapidly moving charges. The reason for failure lay in the concept of *instantaneous* action at a distance. As early as 1845, Gauss recognized this and went on to suggest that the law be modified to make the action travel at a finite speed such as *the speed of light*.

The impasse in electrodynamics was, however, resolved in an altogether different manner. In 1876 Maxwell proposed the theory of electromagnetic fields. According to this theory, charges interact with each other through an independent entity called the electromagnetic field. The disturbances propagate through such fields with a characteristic speed, the speed of light.

The success of Maxwell's theory established the field concept in physics. Furthermore, with the advent of special relativity, it became clear that the concept of instantaneous action at a distance is untenable. Even Newton's law of gravitation had to be modified. Einstein's efforts in this direction led him to the general theory of relativity. Although general relativity is an unusual theory in many respects, it is closer to the 'field' point of view than to the 'action at a distance' point of view.

Action at a distance was, however, revived by several theoretical physicists in this century. The work of Schwarzschild (11), Tetrode (12) and Fokker (2-4) gave a mathematical formulation of Gauss's idea of delayed action at a distance. Such a formulation

† Supported in part by NSG-695.

was necessarily time-symmetric and appeared, at first, inadequate to describe the time-asymmetric phenomena such as electromagnetic radiation. This difficulty was resolved by Wheeler and Feynman (13, 14), who pointed out that such phenomena can be accounted for in a 'perfectly absorbing' universe. The cosmological implications of the Wheeler-Feynman theory have also been subsequently investigated, with interesting results (cf. (5, 6)). These considerations have shown that the concept of direct interparticle action can be made to work in classical electrodynamics.

There is, however, nothing special about electrodynamics. If we want to reinstate the concept of direct interparticle action on an equal footing with field theory in classical physics, we should be able to generalize this result to other interactions. More specifically, we ask the following question: 'Given a field theoretic description for an interaction, can we formulate an analogous description in terms of direct interparticle action?' Here we confine ourselves to the formal aspects of this problem, and will not consider such things as the absorber theory of radiation. In the following section we illustrate these formal aspects with the familiar example of electrodynamics. In the subsequent section we will generalize the result to fields of arbitrary spin.

2. *Classical electrodynamics.* Classical electromagnetic field theory can be derived from an action principle. The action is given by

$$J_F = \frac{1}{16\pi G} \int R \sqrt{(-g)} d^4x - \sum_a \int m_a da - \frac{1}{16\pi} \int F_{ik} F^{ik} \sqrt{(-g)} d^4x - \sum_a e_a \int A_i da^i. \quad (1)$$

Here A_i is the 4-potential and F_{ik} the corresponding electromagnetic field. m_a and e_a are the mass and charge of a typical particle. The first term is purely gravitational and leads to the Einstein tensor of general relativity. (Throughout this discussion we shall use the Einstein description of gravitation.) G is the constant of gravitation. The velocity of light is taken to be unity. The second and fourth terms in (1) involve integrals over the world lines of particles. The second term arises from the inertia of the particle while the fourth term describes the interaction of the charge with the field. The third term contains no information about particles; it is purely a field term.

The formal aspects of the theory are all contained in the above action principle. The relevant information is obtained by requiring that the change in J_F be zero for small variations of various quantities. In effect there are the following variations.

(a) The variation of particle world lines leads to their equation of motion. The fourth term in (1) gives the Lorentz force formula.

(b) The variation of the A_i leads to the Maxwell equations of the electromagnetic field. The last two terms in (1) contribute in this variation.

(c) The variation of g_{ik} leads to the gravitational equations. The electromagnetic energy momentum tensor is contributed by the third term.

The corresponding direct interparticle action is given by

$$J = \frac{1}{16\pi G} \int R \sqrt{(-g)} d^4x - \sum_a \int m_a da - \sum_{a < b} \sum_b 4\pi e_a e_b \iint \bar{G}_{i_A i_B} da^{i_A} db^{i_B}. \quad (2)$$

The first two terms in (2) are the same as in (1). There is no term involving field quantities, however. Instead, the last term in (2) expresses the sum of interactions

between pairs of particles. Thus, $\bar{G}_{i_A i_B}$ is a propagator connecting a typical point A on the world line of particle \mathbf{a} with a typical point B on the world line of particle \mathbf{b} . $\bar{G}_{i_A i_B}$ is a two-point tensor with index i_A at A and i_B at B . It is symmetric, i.e.

$$\bar{G}_{i_A i_B} = \bar{G}_{i_B i_A} \tag{3}$$

and satisfies the wave equation

$$g^{i_A k_A} \bar{G}_{i_A i_B; i_A k_A} + R_{i_A}{}^{i_A} \bar{G}_{i_A i_B} = \delta_4(A, B) \bar{g}_{i_A i_B} / \sqrt{(-\bar{g}(A, B))}, \tag{4}$$

where $\bar{g}_{i_A i_B}$ is a parallel propagator (cf. Synge (10)) and $\bar{g}(A, B) = \det. \|\bar{g}_{i_A i_B}\|$. $\bar{G}_{i_A i_B}$ may be formally written (cf. DeWitt and Brehme (1))

$$\bar{G}_{i_A i_B} = \frac{1}{4\pi} [\Delta^{\frac{1}{2}} \bar{g}_{i_A i_B} \delta(s_{AB}^2) - \bar{v}_{i_A i_B} \theta(s_{AB}^2)] \tag{5}$$

where s_{AB}^2 is the square of the distance between A, B measured along the geodesic (assumed to be unique) joining them. Δ is given by

$$\Delta = -\det. [\frac{1}{2} s_{AB}^2; i_A i_B] / \bar{g}(A, B). \tag{6}$$

The first term in (5) contains the delta function and describes the part of the interaction which travels with the speed of light. The second term in (6) contains a Heaviside function and describes the part of the interaction scattered *inside* the light cone. In flat space $\bar{v}_{i_A i_B} = 0$, $\Delta = 1$ and we get only the first part of $\bar{G}_{i_A i_B}$.

Although we could describe all electromagnetic phenomena entirely through the action (2), it is convenient to use the so-called *direct particle fields*. These are entities defined in terms of particle world lines and the propagators. Thus, we define the 4-potential $A_{i_X}^{(a)}$ at X due to particle \mathbf{a} by

$$A_{i_X}^{(a)} = 4\pi e_a \int \bar{G}_{i_X i_A} da^{i_A}. \tag{7}$$

The corresponding direct particle field is given by

$$F_{i_X k_X}^{(a)} = A_{k_X; i_X}^{(a)} - A_{i_X; k_X}^{(a)}. \tag{8}$$

The formal variational problem in this theory is as follows:

(a) The equations of motion of particles are obtained by the variation of particle world lines. We get the analogue of Lorentz force from the third term of (2) with the difference that all fields acting on a typical particle \mathbf{a} are direct particle fields of particles *other than a*.

(b) There is no analogue of (b) in this case, as there are no independent entities called fields. The analogue of Maxwell equations is however contained in (4). The propagator is so defined that all direct particle fields satisfy the Maxwell equations identically. Also the gauge condition $A_{;i}^i = 0$ is satisfied so long as charge is conserved.

(c) Although there is no term in (2) analogous to the third term in (1), we still get a non-zero contribution to the Einstein equations from the electromagnetic term. This is because the variation of g_{ik} causes $\bar{G}_{i_A i_B}$ to change. This change can be calculated and the result expressed as an energy momentum tensor of the electromagnetic interaction. The tensor has the form

$$T_{[em]}^{ik} = \sum_{a < b} \sum \frac{1}{4\pi} [\frac{1}{2} g^{ik} F^{(a)lm} F_{lm}^{(b)} - F^{(a)i} F^{(b)k} - F^{(a)k} F^{(b)i}]. \tag{9}$$

This is analogous to the energy momentum tensor of field theory

$$T^{ik}_{[em]} = \frac{1}{4\pi} [\frac{1}{2}g^{ik}F^{lm}F_{lm} - F^i{}_l F^k{}_l]. \tag{10}$$

When this result was first obtained (7) it looked more like a coincidence. Similar results were obtained in the case of the *C*-field (7) for the Dirac field (9). However, the methods employed there were of the ‘slogging’ type and did not give insight into the close relationship between field theories and theories of direct interparticle action.

The purpose of this paper is to emphasize this close relationship. To this end we will proceed in the following way. First we will write down an action for an arbitrary field. Corresponding to this action we will construct a theory of direct interparticle action which resembles the original field theory as in the case of electromagnetism described above. For such a correspondence to exist the original field theory must satisfy certain linearity conditions which are generally satisfied by fields discussed in theoretical physics.

3. *Fields of arbitrary spin.* We will first consider tensor fields of arbitrary rank. Later we will show how the calculation can be extended easily to spinor fields.

Let ϕ be a tensor field of rank *N*, in interaction with particles. As in the electromagnetic case, we will assume that its properties can be derived from an action J_F of the form

$$J_F = \frac{1}{16\pi G} \int R \sqrt{(-g)} d^4x - \sum_a \int m_a da + \int L[\phi] \sqrt{(-g)} d^4x + \sum_a \int I[\phi, a] da. \tag{11}$$

Here the third term contains a Lagrangian of the field ϕ and the fourth term describes the interaction of ϕ with particles. We will now state the conditions $L[\phi]$ and $I[\phi, a]$ must satisfy to enable us to construct a direct particle interaction theory analogous to (11).

(i) $L[\phi]$ is a bilinear invariant composed of ϕ and its first derivatives. The coefficients appearing in the bilinear form must be functions of space-time geometry.

Thus, for a tensor field ϕ_{ik} , $L[\phi]$ would be a combination of the form

$$A^{kl}{}_{mn} \phi_{ik} \phi^{mn} + B^{ikl}{}_{nmp} \phi_{ik};{}_l \phi^{nm};{}_p + C^{ikl}{}_{mn} \phi_{ik};{}_l \phi^{mn}, \tag{12}$$

where *A*, *B*, *C* are tensors connected with space-time quantities only.

In the case of a tensor field of rank *N*, we can write down a similar expression. However, to avoid writing down too many suffixes explicitly we use the following convention. We will write $\phi^{\bar{l}}$ to denote $\phi^{i_1 i_2 \dots i_N}$ a typical component of ϕ . Thus $L[\phi]$ will be an expression of the form

$$A^{\bar{l}\bar{m}} \phi_{\bar{l}} \phi_{\bar{m}} + B^{\bar{l}\bar{m}ik} \phi_{\bar{l}};{}_i \phi_{\bar{m}};{}_k + C^{\bar{l}\bar{m}k} \phi_{\bar{l}};{}_k \phi_{\bar{m}}. \tag{13}$$

Summation convention over repeated indices is understood as usual. As in (12), *A*, *B*, *C* contain g_{ik} and their derivatives.

(ii) $I[\phi, a]$ is an expression of the form

$$D\phi_{\bar{m}} \xi^{(a)\bar{m}}, \tag{14}$$

where D is a coupling constant and $\xi^{(a)\bar{m}}$ is a tensor of rank N depending entirely on the world line of particle \mathbf{a} .

It is possible to generalize the conditions (i) and (ii) further and still maintain the linearity. However, the expressions (13) and (14) are sufficient for our present purpose. Most of the fields discussed in theoretical physics meet these requirements.

The expressions (13) and (14) are written in an invariant form. This is essential for any action principle. However, once they are written, we can regroup the different terms in a form more convenient for calculations. This will destroy the invariance of individual terms, but not of the sum of all terms. In the following calculation we will replace all covariant derivatives by ordinary derivatives, and will also include the factor $\sqrt{(-g)}$ with $L[\phi]$. Thus we write

$$\int L[\phi] \sqrt{(-g)} d^4x = \int \mathcal{L}[\phi] d^4x, \tag{15}$$

where
$$\mathcal{L}[\phi] = \alpha^{\bar{i}\bar{m}} \phi_{\bar{i}} \phi_{\bar{m}} + \beta^{\bar{i}\bar{m}ik} \phi_{\bar{i},i} \phi_{\bar{m},k} + \gamma^{\bar{i}\bar{m}k} \phi_{\bar{i},k} \phi_{\bar{m}}. \tag{16}$$

Note that we have replaced $\phi_{\bar{m};k}$ by $\phi_{\bar{m},k}$, etc., etc. α, β, γ are no longer tensors, but they still involve only the geometrical quantities.

As in the electromagnetic case we have three types of variation.

(a) The variation of particle world lines leads to the equation of motion. The second and fourth term of (11) contribute to this variation.

(b) The variation of ϕ leads to the field equation for ϕ . In the case of (16) we have, on variation of ϕ ,

$$\begin{aligned} & [(\beta^{\bar{i}\bar{m}ik} + \beta^{\bar{m}\bar{l}ki}) \phi_{\bar{i},i}]_{,k} - \gamma^{\bar{i}\bar{m}k} \phi_{\bar{i},k} + [\gamma^{\bar{m}\bar{l}k} \phi_{\bar{l}}]_{,k} - [\alpha^{\bar{i}\bar{m}} + \alpha^{\bar{m}\bar{l}}] \phi_{\bar{i}} \\ & = D \sum_a \int \delta_a(X, A) \bar{g}_{\bar{m}\bar{m}_A} \xi^{(a)\bar{m}_A} da, \end{aligned} \tag{17}$$

where the indices i, l, k, \dots refer to a general point X and \bar{m}_A to the point A on the world line of \mathbf{a} . $\bar{g}_{\bar{m}\bar{m}_A}$ is a parallel propagator of N components.

(c) The variation of g_{ik} leads to the Einstein field equations. The third term of (11) contributes an energy momentum tensor $T^{ik}_{[\phi]}$ of the ϕ -field. We get

$$\begin{aligned} - \int \frac{1}{2} T^{ik}_{[\phi]} \delta g_{ik} \sqrt{(-g)} d^4x &= \delta \int \mathcal{L} d^4x \\ &= \int [\delta \beta^{\bar{i}\bar{m}ik} \phi_{\bar{i},i} \phi_{\bar{m},k} + \delta \gamma^{\bar{i}\bar{m}k} \phi_{\bar{i},k} \phi_{\bar{m}} + \delta \alpha^{\bar{i}\bar{m}} \phi_{\bar{i}} \phi_{\bar{m}}] d^4x. \end{aligned} \tag{18}$$

In the above variation, only the coefficients α, β, γ are affected since $\phi_{\bar{i},i}$ and $\phi_{\bar{i}}$ are kept constant. Although the right-hand side of (18) does not appear invariant, we can always write T^{ik} as a tensor after $\delta\alpha, \delta\beta, \delta\gamma$ have been evaluated. This is because $\int \mathcal{L} d^4x$ is an invariant.

We now construct a direct particle theory analogous to the above field theory. As in the electromagnetic case, we look for a suitable propagator connecting a pair of particle world lines. Since the field ϕ has N indices, the propagator \bar{G} is a bitensor with N indices at each end. We shall denote the propagator between A and B by $\bar{G}_{\bar{m}_A \bar{m}_B}$. This is

a Green's function satisfying the equation

$$[(\beta^{i\bar{m}ik} + \beta^{\bar{m}iki}) \bar{G}_{i\bar{m}B, i}]_{,k} - \gamma^{i\bar{m}k} \bar{G}_{i\bar{m}B, k} + [\gamma^{\bar{m}ik} \bar{G}_{i\bar{m}B}]_{,k} - [\alpha^{i\bar{m}} + \alpha^{\bar{m}i}] \bar{G}_{i\bar{m}B} = \delta_4(X, A) \bar{g}^{\bar{m}}_{\bar{m}B}, \tag{19}$$

where the suffix A has been suppressed for convenience of writing. $\bar{G}_{\bar{m}A \bar{m}B}$ is symmetric with respect to A, B , i.e.

$$\bar{G}_{\bar{m}A \bar{m}B} = \bar{G}_{\bar{m}B \bar{m}A}. \tag{20}$$

We define the direct particle field $\phi_{\bar{m}X}^{(a)}$ at X due to particle a by

$$\phi_{\bar{m}X}^{(a)} = D \int \bar{G}_{\bar{m}X \bar{m}A} \xi^{(a)\bar{m}A} da. \tag{21}$$

The action describing the whole theory can be written down as

$$J = \frac{1}{16\pi G} \int R\sqrt{(-g)} d^4x - \sum_a \int m_a da + \sum_{a < b} \sum D^2 \iint \bar{G}_{\bar{m}A \bar{m}B} \xi^{(a)\bar{m}A} \xi^{(b)\bar{m}B} da db. \tag{22}$$

Writing

$$\phi_{a\bar{m}X} = \sum_{b \neq a} \phi_{\bar{m}X}^{(b)}, \tag{23}$$

we can rewrite (22) as

$$J = \frac{1}{16\pi G} \int R\sqrt{(-g)} d^4x - \sum_a \int m_a da + \frac{1}{2} \sum_a D \int \phi_{a\bar{m}X} \xi^{(a)\bar{m}A} da. \tag{24}$$

Thus the third term in (24) appears to be analogous to the fourth term of (11)—apart from the factor 1/2. This feature is common to all direct particle theories.

We now consider the analogues of the variations (a)–(c) in the present case.

(a) The variation of the world line of particle a gives the equation of motion of a . The 'force' contributed by the interaction in the present theory is the same as that from the field theory, with the difference that ϕ is replaced by ϕ_a . This is obvious from the fact that the force is derived from the fourth term in (11) and the third term in (24). (The factor 1/2 does not appear in the latter case.) Thus in the present theory self-action is excluded.

(b) We define the *total* direct particle field at x by

$$\phi_{\bar{m}X} = \sum_a \phi_{\bar{m}X}^{(a)}. \tag{25}$$

It is then easy to see that, by virtue of the definition (21), $\phi_{\bar{m}X}$ satisfies the equation (17) *identically*. As in the electromagnetic case, there are no 'fields' to vary, and this identity replaces the field equation (17).

(c) Finally we consider the variation of g_{ik} . This changes the $\bar{G}_{\bar{m}A \bar{m}B}$ and hence J . To evaluate δJ we first consider the variation of (19).

Writing the change in $\bar{G}_{\bar{m}X \bar{m}B}$ as $\delta \bar{G}_{\bar{m}X \bar{m}B}$, we get

$$\begin{aligned} & [(\beta^{i\bar{m}ik} + \beta^{\bar{m}iki}) \delta \bar{G}_{i\bar{m}B, i}]_{,k} - \gamma^{i\bar{m}k} \delta \bar{G}_{i\bar{m}B, k} + [\gamma^{\bar{m}ik} \delta \bar{G}_{i\bar{m}B}]_{,k} - [\alpha^{i\bar{m}} + \alpha^{\bar{m}i}] \delta \bar{G}_{i\bar{m}B} \\ & = - [(\delta \beta^{i\bar{m}ik} + \delta \beta^{\bar{m}iki}) \bar{G}_{i\bar{m}B, i}]_{,k} + \delta \gamma^{i\bar{m}k} \bar{G}_{i\bar{m}B, k} \\ & \quad - [\delta \gamma^{\bar{m}ik} \bar{G}_{i\bar{m}B}]_{,k} + [\delta \alpha^{i\bar{m}} + \delta \alpha^{\bar{m}i}] \bar{G}_{i\bar{m}B}. \end{aligned} \tag{26}$$

Again we have suppressed suffix X for convenience. We can use the original Green's function to write down the solution of (25) in an integral form. This is permitted provided the changes δg_{ik} are of first order and we neglect quantities of higher order of smallness. We therefore get $\delta \bar{G}_{\bar{m}_A \bar{m}_B}$ as

$$\begin{aligned} \delta \bar{G}_{\bar{m}_A \bar{m}_B} = & \int -\bar{G}_{\bar{m}_A \bar{m}} [(\delta \beta^{\bar{i} \bar{m} i k} + \delta \beta^{\bar{m} i k i}) \bar{G}_{\bar{i} \bar{m}_B, i}, k] d^4 x \\ & + \int \bar{G}_{\bar{m}_A \bar{m}} \delta \gamma^{\bar{i} \bar{m} k} \bar{G}_{\bar{i} \bar{m}_B, k} d^4 x - \int \bar{G}_{\bar{m}_A \bar{m}} [\delta \gamma^{\bar{m} i k} \bar{G}_{\bar{i} \bar{m}_B}, k] d^4 x \\ & + \int \bar{G}_{\bar{m}_A \bar{m}} [\delta \alpha^{\bar{i} \bar{m}} + \delta \alpha^{\bar{m} i}] \bar{G}_{\bar{i} \bar{m}_B} d^4 x. \end{aligned}$$

The first and third terms in the above can be rewritten by partial integration. Assuming that the variations vanish on the surface of the region of variation, we get

$$\begin{aligned} \delta \bar{G}_{\bar{m}_A \bar{m}_B} = & \int [(\delta \beta^{\bar{i} \bar{m} i k} + \delta \beta^{\bar{m} i k i}) \bar{G}_{\bar{m}_A \bar{m}, k} \bar{G}_{\bar{i} \bar{m}_B, i} \\ & + \delta \gamma^{\bar{i} \bar{m} k} \bar{G}_{\bar{m}_A \bar{m}} \bar{G}_{\bar{i} \bar{m}_B, k} + \delta \gamma^{\bar{m} i k} \bar{G}_{\bar{m}_A \bar{m}, k} \bar{G}_{\bar{i} \bar{m}_B} \\ & + (\delta \alpha^{\bar{i} \bar{m}} + \delta \alpha^{\bar{m} i}) \bar{G}_{\bar{m}_A \bar{m}} \bar{G}_{\bar{i} \bar{m}_B}] d^4 x. \end{aligned} \tag{27}$$

Using (21) we get

$$\begin{aligned} \sum_{a < b} D^2 \iint \delta \bar{G}_{\bar{m}_A \bar{m}_B} \zeta^{(a) \bar{m}_A} \zeta^{(b) \bar{m}_B} da db \\ = \int [(\delta \beta^{\bar{i} \bar{m} i k} + \delta \beta^{\bar{m} i k i}) \phi_{\bar{m}, k}^{(a)} \phi_{\bar{i}, i}^{(b)} + \delta \gamma^{\bar{i} \bar{m} k} \phi_{\bar{m}}^{(a)} \phi_{\bar{i}, k}^{(b)} \\ + \delta \gamma^{\bar{m} i k} \phi_{\bar{m}, k}^{(a)} \phi_{\bar{i}}^{(b)} + (\delta \alpha^{\bar{i} \bar{m}} + \delta \alpha^{\bar{m} i}) \phi_{\bar{m}}^{(a)} \phi_{\bar{i}}^{(b)}] d^4 x. \end{aligned} \tag{28}$$

Comparison of (28) and (18) shows the similarity of the energy momentum tensors in the two theories. The similarity noticed in the electromagnetic case was therefore no accident.

In general it is much simpler to calculate the energy momentum tensors in a field theory. The above result shows that in the corresponding direct particle theory it is not necessary to carry through the calculation of $\delta \bar{G}_{\bar{m}_A \bar{m}_B}$. We can arrive at the energy momentum tensor in this theory by using the following rule:

In the energy momentum tensor of the field theory substitute for $\phi_{\bar{m}}$ the sum (25). The tensor then becomes a double sum over particle pairs. From this sum delete all the 'degenerate' pairs, i.e. pairs of identical particles $[a, a]$. The remaining terms in the sum represent the energy momentum tensor of the analogous theory of direct interparticle action. The deleted terms correspond to the self action.

The above results can be easily extended to spinor fields. This is because no use was made of the general covariance of the fields. Thus spinor fields could be considered with the use of spinor indices in addition to the tensor ones. The only point of difference that arises in the spinor case is that the entity to be varied in (c) is not g_{ik} but $g_{i\alpha\beta}$. $g_{i\alpha\beta}$ is related to g_{ik} by

$$g_{i\alpha\beta} g_k^{\alpha\beta} = 2g_{ik}. \tag{29}$$

(In view of the variety of notation in use to describe spinors, it is best to state that

the notation used here is that of Hoyle and Narlikar (8.) Although $g_{i\alpha\beta}$ can be varied with 16 degrees of freedom, only 10 of these constitute a genuine geometrical variation. We can express this in the form

$$\delta g_{i\alpha\beta} = \frac{1}{2} \delta g_{ik} g^k_{\alpha\beta}. \quad (30)$$

The remaining 6 degrees of freedom correspond to Lorentz transformations, and are not of interest in the present variational problem.

Corresponding to (17) we have for spinor fields

$$\delta \int \mathcal{L} d^4x = - \int_{[\phi]} P^{i\alpha\beta} \delta g_{i\alpha\beta} \sqrt{(-g)} d^4x. \quad (31)$$

Using (30) we can construct a symmetric T^{ik} :

$$T^{ik} = \frac{1}{2} [g^i_{\alpha\beta} P^{k\alpha\beta} + g^k_{\alpha\beta} P^{i\alpha\beta}]. \quad (32)$$

The same result carries through for direct particle theories.

4. *Conclusion.* The work of the previous section shows that, provided the conditions (i), (ii) are satisfied, there exists a direct particle analogue of every field theory. That these conditions are necessary is seen from the fact that such a correspondence does not exist where L is not a bilinear of the form (16), or I is not of the form (14). A notable example of this is a Dirac field in interaction with a scalar field. The interaction term is $\sim \bar{\psi}\psi\phi$. This is linear in ϕ but not in ψ . We therefore cannot express ψ as a direct particle field. This difficulty was encountered by Islam (9) in his discussion of the Dirac field. The difficulty is removed if we regard ϕ alone as a direct particle field and treat ψ as an entity specifying the particle. Then $\bar{\psi}\psi$ can be treated as ξ^m in (14) (cf. Hoyle and Narlikar (8)).

The correspondence considered in this paper is confined entirely to classical physics. As yet no progress has been made towards understanding the quantum nature of direct interparticle action. The quantum theory of fields, on the other hand, has been studied extensively. Although it has produced several results in agreement with experiments, the quantum field theory cannot be regarded as a perfect theory. The difficulties of self-action and vacuum polarization are only too well known. Perhaps these difficulties will disappear when we have a proper understanding of the quantum theory of direct interparticle action.

This work was completed when the author was visiting the University of Maryland. He wishes to thank the Department of Physics and Astronomy for the hospitality extended to him.

REFERENCES

- (1) DEWITT, B. S. and BREHME, R. W. *Annals Phys.* **9** (1960), 220.
- (2) FOKKER, A. D. *Z. Phys.* **58** (1929a), 386.
- (3) FOKKER, A. D. *Physica* **9** (1929b), 33.
- (4) FOKKER, A. D. *Physica* **12** (1932), 145.
- (5) HOGARTH, J. E. *Proc. Roy. Soc. Ser. A* **267** (1962), 365.
- (6) HOYLE, F. and NARLIKAR, J. V. *Proc. Roy. Soc. Ser. A* **277** (1963), 1.

- (7) HOYLE, F. and NARLIKAR, J. V. *Proc. Roy. Soc. Ser. A* **282** (1964), 184.
- (8) HOYLE, F. and NARLIKAR, J. V. *Proc. Roy. Soc. Ser. A* **299** (1967), 188.
- (9) ISLAM, J. N. *Proc. Roy. Soc. Ser. A* **294** (1966), 437.
- (10) SYNGE, J. L. *Relativity: the general theory* (North Holland Publishing Co., 1960).
- (11) SCHWARZSCHILD, K. *Nachr. G. Wiss. Göttingen, Math.-Phys. Kl. II* **128** (1903), 132.
- (12) TETRODE, H. *Z. Phys.* **10** (1922), 317.
- (13) WHEELER, J. A. and FEYNMAN, R. P. *Rev. Modern Phys.* **17** (1945), 156.
- (14) WHEELER, J. A. and FEYNMAN, R. P. *Rev. Modern Phys.* **21** (1949), 424.