Cosmological solution of Machian gravity

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The standard model of cosmology predicts that more than 95% matter in the universe consists of dark components namely dark matter and dark energy. In spite of several attempts to measure these components, there is not a single direct observational evidence for these components till date. Hence, different alternate models of cosmology have been put forward by different authors. However, most of these models have their own problems. Therefore, in this paper, a new cosmological model has been proposed. This model is based on the Machian gravity model, which will be discussed in detail in a later paper. The model can provide an exactly similar cosmology as that of the standard cosmological model without demanding any ad-hoc dark matter or dark energy components. The paper shows that when the field equations from Machian gravity (a 5 dimensional model) are projected to the 4-dimensional space-time, some new mathematical terms arise in the equations that behave exactly like dark matter and dark energy. These mathematical terms come completely from the geometry of the universe and therefore these do not have any connection with the real matter. As the General theory of Relativity does not follow Mach's principle, the FLRW model that is based on GR, cannot provide the correct solution to the cosmological model and demands extra forms of matter and energy to give any predictions consistent with the observations.

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I. INTRODUCTION

At the present time, Einstein's General theory of Relativity is believed to be the most accurate theory of gravity. The standard model of cosmology is based on the General theory of Relativity and it can explain the observational data up to an extremely high accuracy just by using a handful set of parameters. This makes the standard model the most popular theory of cosmology. However, it demands that more than 95% of the universe may be made of dark matter and dark energy, a hypothetical form of matter and energy. If these unknown form of matter and energy are inserted into the cosmological model then all the observational data can be explained very precisely. Several models for dark matter such as hot dark matter, warm dark matter, cold dark matter and models for dark energy, such as Λ, Quintessence, k-essence, chaplain gas etc. have been proposed by researchers. All these hypothetical models have been tested from different indirect cosmological experiments. Several attempts have also been made for direct observation of these dark components of the universe. However, even after several attempts to search for dark matter in the past few decades, there is not a single direct evidence for dark matter till date. This shows that there might be some fundamental problem in the basic theory and that it might need some alternate theory to explain the entire phenomenon without demanding any dark component.

Attempts were made for alternate cosmological theories. However, most of these alternate theories, proposed in the recent past are designed to explain the cosmological data set and have no underlying physical meaning. MOND[1, 2] has been proposed to explain the galactic rotation curves. However, it violates the basic principles of physics like momentum conservation principle. Several attempts were made by researchers to make the MOND a physical theory. Theories like AQUAL[3, 4] and TeVeS[5] were proposed to develop MOND like field equations from the Lagrangian, which guarantee to obey all the conservation laws. A completely independent attempt was made by Moffat by proposing MoG/STVG[6–9] which can explain galactic velocity profiles, accelerated expansion of the universe, baryonic acoustic oscillations etc without requiring any dark matter or dark energy in the universe. However, all these theories require extra vector field and scalar fields and some random form of potentials to be added in the Lagrangian, just to match the observational data. There is no concrete arguments of logic behind taking these highly complicated Lagrangians in the theory. Therefore, these theories can only be taken as nice mathematical tricks to explain all the observational phenomenon, but they cannot be considered as physical theories unless logical explanations for the mathematics are provided. On the other hand theories like $f(R)$ gravity[10] though motivated from string theory, requires us to choose the function $f()$ in a random manner just to match the observational dataset. In addition, the theory demands extra dark matter to explain the cosmological phenomenon.

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In this paper, an alternate model of cosmology is proposed that can explain all the phenomenon of the standard cosmological model properly without demanding any ad-hoc dark matter or dark energy components. Mach's principle tells us that the inertial properties of matter depend on the background, where the object is located. In the next paper, it will be shown that to incorporate the Mach's principle, an extra dimension is needed to be added which somehow quantify the effects from the background. The cosmological model proposed in this paper is based on the Machian gravity model, which will be discussed in detail in a later paper. It will be shown that the line element in a flat space will take the form $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 + \epsilon(\hbar^2/4)d\zeta^2$. Here, ϵ is the signature of the background dimension ζ and $\hbar$ is the reduced Planck constant. This particular line element will come naturally from the philosophy and can explain the quantum mechanics in a completely classical way. Details of these calculations will be discussed in the next paper. In a later paper it will also be shown that the field equations for gravity will take the same form as that of Einstein's equation but in five dimension. An approximate solution for spherically symmetric static vacuum condition from this new theory will also be derived. It will be shown that the galactic velocity profiles can be explained from the theory without demanding any dark matter component. However, in this paper, we will only assume the field equation and the stress energy tensor from the theory, which happens to be same as that of the Einstein's field equation but in five dimension and show how the theory can explain all the cosmological phenomenon without going to the details of the theory. The goal of this paper is to describe the cosmological solutions of from the theory. In this paper, it is shown that if the field equations of five dimensional Machian gravity is projected in the 3+1 dimensional space-time then it will pick up some extra terms from geometry and these extra terms will have the properties as that of the Dark matter and the Dark energy. Therefore, the cosmology from the Machian gravity model behaves in exactly the same way as that of the standard cosmological model. The logic behind going through the top down approach of showing the results first and then discussing the actual theory in the later papers is that it will help the readers to understand the logic easily and they will be able to the connect the logic with the mathematics properly.

The mathematics developed in this paper is based on the calculations of Wesson's induced matter hypothesis[11–13], where it was shown that the matter can arise from the space-time curvature. Though the logic behind the Machian gravity and the field equations of the new theory are completely different from that of the induced gravity model, mathematical tricks similar to the induced gravity model can be applied here to show that the dark matter and the dark energy can emerge from the geometry completely mathematically.

The paper is organized as follows. The second section shows how a general five dimensional theory can be expanded in terms of a four dimensional theory. In the third part, the most general line element for the cosmological model from the Machian gravity is taken and then it is shown how the dark matter and the dark energy can emerge from the geometry. In the fourth part, the nature of the dark matter and dark energy in the FLRW universe is discussed. Finally, the last part gives the conclusion.

II. GENERAL PROJECTION OF 5 DIMENSIONAL GEOMETRY INTO 4 DIMENSION

It will be shown in the next paper that gravity is a five dimensional phenomenon. Therefore, to incorporate its effect, it is important to consider the curvature of the five dimensional space-time-background and not only the four dimensional space-time. However, GR describes gravity by the curvature of the four dimensional space-time. In this section we describe all the changes in the Einstein's field equation when a general five dimensional metric is projected in a four dimensional space.

Any generalized five dimensional metric can be written in terms of a four-dimensional metric along with a scalar field and a vector field. The metric used for our purpose is given below

$$\tilde{g}_{AB} = \begin{pmatrix} g_{\alpha\beta} + \kappa^2\phi^2 A_\alpha A_\beta & \kappa\phi^2 A_\alpha \\ \kappa\phi^2 A_\beta & \phi^2 \end{pmatrix} . \tag{1}$$

Here ϕ is a scalar field, A_α is a vector field, κ is a constant and $g_{\alpha\beta}$ is the metric in the four dimension. For simplicity the constant c and $\frac{\hbar}{2}$ are taken as 1 throughout the paper. Although this metric is the most general metric, the coordinate system can be chosen in such a way that the off-diagonal terms corresponds to the fifth dimension in the metric becomes 0, i.e. the vector field vanishes. This choice of coordinate system will highly simplify the calculations. Therefore, the metric used here is

$$\tilde{g}_{AB} = \begin{pmatrix} & & & & 0 \\ & & & & 0 \\ & g_{\alpha\beta} & & & 0 \\ & & & & 0 \\ 0 & 0 & 0 & 0 & \epsilon\phi^2 \end{pmatrix} . \tag{2}$$

Here ϵ is chosen as the signature of the background dimension of the metric. Also remember that, symbols $A, B, \ldots$ will vary from 0 to 4 and the symbols $\alpha, \beta, \ldots$ varis from 0 to 3. Throughout the paper, '$\tilde{()}$', i.e. 'tilde' has been used to say the the tensors are calculated in the five dimensional geometry.

The nonzero components of the Christoffel's symbols calculated from the metric after breaking it into the 4+1 dimensional format, i.e. the space-time and the background-separated format are

$$\Gamma^4_{\alpha\beta} = -\frac{1}{2}\frac{\epsilon}{\phi^2}\partial_4 g_{\alpha\beta}\,, \qquad \Gamma^A_{4A} = g^{AB}\partial_4 g_{AB}\,, \qquad \Gamma^\alpha_{\beta 4} = \frac{1}{2}g^{\alpha C}\partial_4 g_{\beta C}\,, \qquad \Gamma^A_{\alpha 4} = \epsilon g^{A4}\phi\partial_\alpha\phi + \frac{1}{2}g^{A\gamma}\partial_4 g_{\alpha\gamma}\,,$$

$$\Gamma^4_{\alpha A} = \frac{\epsilon}{2\phi^2}\left(\partial_\alpha g_{A4} - \partial_4 g_{\alpha A}\right)\,, \qquad \Gamma^\alpha_{44} = -\epsilon g^{\alpha\beta}\phi\partial_\beta\phi\,, \qquad \Gamma^\alpha_{4\alpha} = \frac{1}{2}g^{\alpha\beta}\partial_4 g_{\alpha\beta}\,, \qquad \Gamma^\alpha_{\beta\alpha} = \frac{1}{2}g^{\alpha\gamma}\partial_\beta g_{\alpha\gamma}\,,$$

$$\Gamma^4_{44} = \frac{\partial_4\phi}{\phi}\,. \tag{3}$$

The five dimensional Ricci tensor i.e. $\tilde{R}_{AB}$ can be written in terms of the Christoffel's symbol as follows

$$\tilde{R}_{AB} = \left(\Gamma^C_{AB}\right)_{,C} - \left(\Gamma^C_{AC}\right)_{,B} + \Gamma^C_{AB}\Gamma^D_{CD} - \Gamma^C_{AD}\Gamma^D_{CB}\,. \tag{4}$$

Now the indices (A, B, $\ldots$) can be separated out in terms of the four dimensional space-time and the separate background dimension. Therefore breaking the first 4×4 components of the Ricci tensor in that particular format we can get

$$\begin{aligned}\tilde{R}_{\alpha\beta} &= \left(\Gamma^\gamma_{\alpha\beta}\right)_{,\gamma} - \left(\Gamma^\gamma_{\alpha\gamma}\right)_{,\beta} + \Gamma^\gamma_{\alpha\beta}\Gamma^\delta_{\gamma\delta} - \Gamma^\gamma_{\alpha\delta}\Gamma^\delta_{\gamma\beta} + \left(\Gamma^4_{\alpha\beta}\right)_{,4} - \left(\Gamma^4_{\alpha 4}\right)_{,\beta} + \Gamma^4_{\alpha\beta}\Gamma^D_{4D} + \Gamma^\gamma_{\alpha\beta}\Gamma^4_{\gamma 4} - \Gamma^4_{\alpha D}\Gamma^D_{4\beta} - \Gamma^\gamma_{\alpha 4}\Gamma^4_{\gamma\beta} \\ &= R_{\alpha\beta} + \left(\Gamma^4_{\alpha\beta}\right)_{,4} - \left(\Gamma^4_{\alpha 4}\right)_{,\beta} + \Gamma^4_{\alpha\beta}\Gamma^D_{4D} + \Gamma^\gamma_{\alpha\beta}\Gamma^4_{\gamma 4} - \Gamma^4_{\alpha D}\Gamma^D_{4\beta} - \Gamma^\gamma_{\alpha 4}\Gamma^4_{\gamma\beta}\,.\end{aligned} \tag{5}$$

Therefore, it can be seen that the first 4×4 components of the five dimensional Ricci tensor can be written in terms of four dimensional Ricci tensor and some extra terms. Thus by replacing the values of the Christoffel's symbols from Eq.(3) into Eq.(4) and Eq.(5) we can get the Ricci tensor as

$$\tilde{R}_{\alpha\beta} = R_{\alpha\beta} - \frac{\phi_{\alpha;\beta}}{\phi} + \frac{\epsilon}{2\phi^2}\left(\frac{\partial_4\phi\partial_4 g_{\alpha\beta}}{\phi} - \partial_4\partial_4 g_{\alpha\beta} + g^{\mu\lambda}\partial_4 g_{\alpha\mu}\partial_4 g_{\beta\lambda} - \frac{1}{2}g^{\mu\nu}\partial_4 g_{\mu\nu}\partial_4 g_{\alpha\beta}\right)\,, \tag{6}$$

$$\tilde{R}_{44} = -\epsilon\phi g^{\mu\nu}\phi_{\mu;\nu} - \frac{1}{2}\partial_4 g^{\mu\nu}\partial_4 g_{\mu\nu} - \frac{1}{2}g^{\mu\nu}\partial_4\partial_4 g_{\mu\nu} + \frac{1}{2\phi}\partial_4\phi g^{\mu\nu}\partial_4 g_{\mu\nu} - \frac{1}{4}g^{\mu\nu}g^{\lambda\sigma}\partial_4 g_{\lambda\mu}\partial_4 g_{\sigma\nu}\,, \tag{7}$$

$$\tilde{R}_{4\alpha} = \phi P^\beta_{\alpha;\beta}\,. \tag{8}$$

In the above equations ';' represents the covariant derivative. In Eq.(6) and Eq.(7) $\phi_\alpha = \partial_\alpha\phi$ and $\phi_{\alpha;\beta} = \left(\phi_{\alpha,\beta} - \phi_\lambda\Gamma^\lambda_{\alpha\beta}\right)$. In the Eq.(8) P^β_α is a 2^{nd} rank tensor and given by $P^\beta_\alpha = \frac{1}{2\phi}\left(g^{\beta\lambda}\partial_4 g_{\lambda\alpha} - \delta^\beta_\alpha g^{\mu\nu}\partial_4 g_{\mu\nu}\right)$.

While calculating the cosmology for the FLRW universe, Einstein's four-dimensional General theory of Relativity is used. In General theory of Relativity we relate $R_{\alpha\beta}$ with the stress energy tensor i.e. $T_{\alpha\beta}$. However in Machian gravity it is important to relate $\tilde{R}_{AB}$ to $\tilde{T}_{AB}$. However, $\tilde{T}_{\alpha\beta} \approx T_{\alpha\beta}$, because the values of $\tilde{T}_{AB}$ involving the ζ dimension are very small as there are factors of $\hbar$ or $\hbar^2$ present in these forms. So what is essentially done here is just relating $\tilde{R}_{\alpha\beta}$ to $T_{\alpha\beta}$. The standard cosmology which is calculated by relating $R_{\alpha\beta}$ to $T_{\alpha\beta}$ needs Dark matter and Dark energy to match with the observational results. Also, from Eq.(6), it is known that $\tilde{R}_{\alpha\beta} = R_{\alpha\beta} + Q_{\alpha\beta}$ where $Q_{\alpha\beta}$ represents the residual terms on the right hand side of Eq.(6). Therefore, if it is shown that the terms $Q_{\alpha\beta}$ have the property same as that of dark matter and dark energy then our theory can predict everything in the same way as that of the standard cosmology without demanding any form of dark matter and dark energy.

In the above paragraph we have discussed the theory conceptually. However, when we come to the mathematics, $R_{\mu\nu}$ is not equalized to $T_{\mu\nu}$ due to some mathematical problem. Instead the Einstein's tensor is related to $T_{\mu\nu}$ where the Einstein's tensor is the just a linear function of $R_{\mu\nu}$. The Einstein's tensor in five dimension and four dimension are actually calculated from the five dimensional and the four dimensional Ricci tensors using the following formulae.

$$\tilde{G}_{AB} = \tilde{R}_{AB} - \frac{1}{2}\tilde{g}_{AB}\,\tilde{R}\,, \tag{9}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}\,R\,. \tag{10}$$

Also the Einstein's tensor in one covariant and one contra variant rank can be written as $\tilde{G}^A_B = \tilde{G}_{BC}\tilde{g}^{AC}$ and $G^\alpha_\beta = G_{\beta\gamma}g^{\alpha\gamma}$.

III. COSMOLOGICAL SOLUTION FROM A GENERALIZED METRIC

In the previous section, we have mentioned how a 5D metric can depict the effects of dark components of the universe. In this section, the most general line element for explaining some cosmological model is taken and the quantities, which give the effects of dark matter and the dark energy in cosmology, are calculated.

The line element taken here is

$$ds^2 = e^{\omega}dt^2 - e^{\kappa}dr^2 - R^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \epsilon e^{\mu}d\zeta^2\,. \tag{11}$$

The exponentials are just taken to make sure that these quantities can't be negative. The extra parameter ϵ determine the signature of the background dimension. Thus, for the line element the 5-dimensional metric can be written as $\tilde{g}_{AB} = \mathrm{diag}\left(\begin{array}{ccccc} e^{\omega} & -e^{\kappa} & -R^2 & -R^2\sin^2\theta & \epsilon e^{\mu}\end{array}\right)$ and the metric for the four dimensional space-time hyperspace can be written as $g_{\mu\nu} = \mathrm{diag}\left(\begin{array}{cccc} e^{\omega} & -e^{\kappa} & -R^2 & -R^2\sin^2\theta\end{array}\right)$. The nonzero Christoffel's symbols from this metric can be calculated as

$$\Gamma^0_{00} = \frac{\dot{\omega}}{2}\,, \qquad \Gamma^1_{00} = \frac{\omega'}{2}e^{\omega-\kappa}\,, \qquad \Gamma^4_{00} = -\frac{\epsilon}{2}\omega^* e^{\omega-\mu}\,, \qquad \Gamma^0_{01} = \frac{\omega'}{2}\,,$$

$$\Gamma^1_{01} = \frac{\omega}{2}\,, \qquad \Gamma^2_{02} = \frac{\dot{R}}{R}\,, \qquad \Gamma^3_{03} = \frac{\dot{R}}{R}\,, \qquad \Gamma^0_{04} = \frac{\omega^*}{2}\,, \qquad \Gamma^4_{04} = \frac{\dot{\mu}}{2}\,,$$

$$\Gamma^0_{11} = \frac{\kappa}{2}e^{\kappa-\omega}\,, \qquad \Gamma^1_{11} = \frac{\kappa'}{2}\,, \qquad \Gamma^4_{11} = \frac{\epsilon}{2}\kappa^* e^{\kappa-\mu}\,, \qquad \Gamma^2_{12} = \Gamma^3_{13} = \frac{R'}{R}\,,$$

$$\Gamma^1_{14} = \frac{\kappa^*}{2}\,, \qquad \Gamma^4_{41} = \frac{\mu'}{2}\,, \qquad \Gamma^0_{22} = R\dot{R}e^{-\omega}\,, \qquad \Gamma^1_{22} = -RR'e^{-\kappa}\,,$$

$$\Gamma^4_{22} = \epsilon RR^* e^{-\mu}\,, \qquad \Gamma^3_{23} = \cot\theta\,, \qquad \Gamma^2_{24} = \Gamma^3_{34} = \frac{R^*}{R}\,, \qquad \Gamma^0_{33} = R\dot{R}e^{-\omega}\sin^2\theta\,,$$

$$\Gamma^1_{33} = -R\dot{R}e^{-\kappa}\sin^2\theta\,, \qquad \Gamma^2_{33} = -\sin\theta\cos\theta\,, \qquad \Gamma^4_{33} = \epsilon RR^* e^{-\mu}\sin^2\theta\,,$$

$$\Gamma^0_{44} = -\frac{\epsilon}{2}\dot{\mu}e^{\mu-\omega}\,, \qquad \Gamma^1_{44} = \frac{\epsilon}{2}\mu' e^{\mu-\kappa}\,, \qquad \Gamma^4_{44} = \frac{\mu^*}{2}\,. \tag{12}$$

In these expressions $\dot{x}$ represents the derivative with respect to the normal time, x' represents the derivative with respect to the radius vector i.e. r and finally x^* represents the derivatives with respect to the background coordinate ζ. Now, once the Christoffel's symbols are calculated, the 5D Ricci tensor for this metric can be calculated using the formula given in Eq.(4). The nonzero components of the Ricci tensor are given by

$$\begin{aligned}\tilde{R}_{00} &= -\frac{\ddot{\kappa}}{2}-\frac{\ddot{\mu}}{2}-2\frac{\ddot{R}}{R}+\frac{\dot{\omega}\dot{\kappa}}{4}+\frac{\dot{\omega}\dot{\mu}}{4}+\frac{\dot{\omega}\dot{R}}{R}-\frac{\dot{\kappa}^2}{4}-\frac{\dot{\mu}^2}{4}+e^{\omega-\kappa}\left(\frac{\omega''}{2}+\frac{\omega'^2}{4}-\frac{\omega'\kappa'}{4}+\frac{\omega'\mu'}{4}+\frac{\omega'R'}{R}\right)\\&+\epsilon e^{\omega-\mu}\left(-\frac{\omega^{**}}{2}-\frac{\omega^{*2}}{4}+\frac{\omega^*\mu^*}{4}-\frac{\omega^*\kappa^*}{4}-\frac{\omega^*R^*}{R}\right), \qquad (13)\end{aligned}$$

$$\tilde{R}_{01} = -\frac{\dot{\mu}'}{2}-\frac{\dot{\mu}\mu'}{4}+\frac{\omega'\dot{\mu}}{4}+\frac{\dot{\kappa}\mu'}{4}+\frac{\dot{\kappa}R'}{R}+\frac{\omega'\dot{R}}{R}-\frac{2\dot{R}'}{R}, \qquad (14)$$

$$\tilde{R}_{04} = -\frac{\dot{\kappa}^*}{2}-\frac{\dot{\kappa}\kappa^*}{4}+\frac{\dot{\kappa}\omega^*}{4}+\frac{\kappa^*\dot{\mu}}{4}+\frac{\dot{\mu}R^*}{R}+\frac{\omega^*\dot{R}}{R}-\frac{2\dot{R}^*}{R}, \qquad (15)$$

$$\begin{aligned}\tilde{R}_{11} &= -\frac{\omega''}{2}-\frac{\mu''}{2}-\frac{{\omega'}^2}{4}-\frac{\mu'^2}{4}+\frac{\kappa'\omega'}{4}+\frac{\kappa'\mu'}{4}+\frac{\kappa'R'}{R}-\frac{2R''}{R}+e^{\kappa-\omega}\left(\frac{\ddot{\kappa}}{2}+\frac{\dot{\kappa}^2}{4}-\frac{\dot{\kappa}\dot{\omega}}{4}+\frac{\dot{\kappa}\dot{\mu}}{4}+\frac{\dot{\kappa}\dot{R}}{R}\right)\\&+\epsilon e^{\kappa-\mu}\left(\frac{\kappa^{**}}{2}+\frac{\kappa^{*2}}{4}+\frac{\kappa^*\omega^*}{4}-\frac{\kappa^*\mu^*}{4}+\frac{\kappa^*R^*}{R}\right), \qquad (16)\end{aligned}$$

$$\tilde{R}_{14} = -\frac{\omega'^*}{2}-\frac{\omega'\omega^*}{4}+\frac{\kappa^*\omega'}{4}+\frac{\mu'\omega^*}{4}+\frac{\kappa^*R'}{R}+\frac{\mu'R^*}{R}-\frac{2R'^*}{R}, \qquad (17)$$

$$\begin{aligned}\tilde{R}_{22} &= 1+R^2e^{-\omega}\left(\frac{\dot{R}^2}{R^2}+\frac{\ddot{R}}{R}-\frac{\dot{R}}{2R}(\dot{\omega}-\dot{\kappa}-\dot{\mu})\right)-R^2e^{-\kappa}\left(\frac{R'^2}{R^2}+\frac{R''}{R}+\frac{R'}{2R}(\omega'-\kappa'+\mu')\right)\\&\epsilon R^2e^{-\mu}\left(\frac{R^{*2}}{R^2}+\frac{R^{**}}{R}+\frac{R^*}{2R}(\omega^*+\kappa^*-\mu^*)\right), \qquad (18)\end{aligned}$$

$$\tilde{R}_{33} = \tilde{R}_{22}\sin^2\theta, \qquad (19)$$

$$\begin{aligned}\tilde{R}_{44} &= -\frac{\omega^{**}}{2}-\frac{\omega^{*2}}{4}-\frac{\kappa^{**}}{2}-\frac{\kappa^{*2}}{4}+\frac{\mu^*\omega^*}{4}+\frac{\mu^*\kappa^*}{4}+\frac{\mu^*R^*}{R}-\frac{2R^{**}}{R}-\epsilon e^{\mu-\omega}\left(\frac{\ddot{\mu}}{2}+\frac{\dot{\mu}^2}{4}-\frac{\dot{\mu}\dot{\omega}}{4}+\frac{\dot{\mu}\dot{\kappa}}{4}+\frac{\dot{\mu}\dot{R}}{R}\right)\\&+\epsilon e^{\mu-\kappa}\left(\frac{\mu''}{2}+\frac{\mu'^2}{4}+\frac{\mu'\omega'}{4}-\frac{\mu'\kappa'}{4}+\frac{\mu'R'}{R}\right). \qquad (20)\end{aligned}$$

Using the components of the 5D Ricci tensor the Ricci scalar can be calculated as

$$\begin{aligned}\tilde{R} &= -\frac{2}{R^2}-e^{-\omega}\left(\ddot{\kappa}+\frac{\dot{\kappa}^2}{2}+\ddot{\mu}+\frac{\dot{\mu}^2}{2}-\frac{\dot{\omega}\dot{\kappa}}{2}-\frac{\dot{\omega}\dot{\mu}}{2}-\frac{2\dot{R}}{R}(\dot{\omega}-\dot{\kappa}-\dot{\mu})+\frac{\dot{\mu}\dot{\kappa}}{2}+\frac{2\dot{R}^2}{R^2}+\frac{4\ddot{R}}{R}\right)\\&+e^{-\kappa}\left(\omega''+\frac{\omega'^2}{2}+\mu''+\frac{2R'}{R}(\omega'-\kappa'+\mu')+\frac{\mu'^2}{2}-\frac{\omega'\kappa'}{2}+\frac{\omega'\mu'}{2}-\frac{\mu'\kappa'}{2}+\frac{2R'^2}{R^2}+\frac{4R''}{R}\right)\\&-\epsilon e^{-\mu}\left(\omega^{**}+\frac{\omega^{*2}}{2}+\kappa^{**}+\frac{\kappa^{*2}}{2}+\frac{\kappa^*\omega^*}{2}-\frac{\kappa^*\mu^*}{2}+\frac{2R^*}{R}(\omega^*+\kappa^*-\mu^*)\right.\\&\left.-\frac{\mu^*\omega^*}{2}+\frac{2R^{*2}}{R^2}+\frac{4R^{**}}{R}\right). \qquad (21)\end{aligned}$$

The above expressions give the Ricci tensor and Ricci scalar in a five dimensional universe. Now if we do not consider the background dimension i.e. ζ as it done in General Relativity then the nonzero components of four dimensional Ricci tensor can be written as

$$R_{00} = -\frac{\ddot{\kappa}}{2} - 2\frac{\ddot{R}}{R} + \frac{\dot{\omega}\dot{\kappa}}{4} + \frac{\dot{\omega}\dot{R}}{R} - \frac{\dot{\kappa}^2}{4} + e^{\omega-\kappa}\left(\frac{\omega''}{2} + \frac{\omega'^2}{4} - \frac{\omega'\kappa'}{4} + \frac{\omega' R'}{R}\right), \tag{22}$$

$$R_{01} = \frac{\dot{\kappa}R'}{R} + \frac{\omega'\dot{R}}{R} - \frac{2\dot{R}'}{R}, \tag{23}$$

$$R_{11} = -\frac{\omega''}{2} - \frac{{\omega'}^2}{4} + \frac{\kappa'\omega'}{4} + \frac{\kappa' R'}{R} - \frac{2R''}{R} + e^{\kappa-\omega}\left(\frac{\ddot{\kappa}}{2} + \frac{\dot{\kappa}^2}{4} - \frac{\dot{\kappa}\dot{\omega}}{4} + \frac{\dot{\kappa}\dot{R}}{R}\right), \tag{24}$$

$$R_{22} = 1 + R^2 e^{-\omega}\left(\frac{\dot{R}^2}{R^2} + \frac{\ddot{R}}{R} - \frac{\dot{R}}{2R}(\dot{\omega} - \dot{\kappa})\right) - R^2 e^{-\kappa}\left(\frac{R'^2}{R^2} + \frac{R''}{R} + \frac{R'}{2R}(\omega' - \kappa')\right), \tag{25}$$

$$R_{33} = R_{22}\sin^2\theta\,. \tag{26}$$

From these four-dimensional Ricci tensor the Ricci scalar can be calculated as

$$\begin{aligned} R &= -\frac{2}{R^2} - e^{-\omega}\left(\ddot{\kappa} + \frac{\dot{\kappa}^2}{2} - \frac{\dot{\omega}\dot{\kappa}}{2} - \frac{2\dot{R}}{R}(\dot{\omega} - \dot{\kappa}) + \frac{2\dot{R}^2}{R^2} + \frac{4\ddot{R}}{R}\right) \\ &\quad + e^{-\kappa}\left(\omega'' + \frac{\omega'^2}{2} - \frac{\omega'\kappa'}{2} + \frac{2R'}{R}(\omega' - \kappa') + \frac{2R'^2}{R^2} + \frac{4R''}{R}\right). \end{aligned} \tag{27}$$

So using the above expressions (Eq.(13)-Eq.(27)), the nonzero components of 5D Einstein's tensor can be written in terms of the 4D Einstein's tensor as follows

$$\begin{aligned} \tilde{G}^0_0 &= G^0_0 + e^{-\omega}\left(\frac{\dot{\mu}\dot{\kappa}}{4} + \frac{\dot{\mu}\dot{R}}{R}\right) - e^{-\kappa}\left(\frac{R'\mu'}{R} - \frac{\kappa'\mu'}{4} + \frac{\mu''}{2} + \frac{\mu'^2}{2}\right) \\ &\quad + \epsilon e^{-\mu}\left(\frac{\kappa^{**}}{2} + \frac{\kappa^{*2}}{4} - \frac{\kappa^*\mu^*}{4} + \frac{R^*}{R}(\kappa^* - \mu^*) + \frac{R^{*2}}{R^2} + \frac{2R^{**}}{R}\right), \end{aligned} \tag{28}$$

$$\tilde{G}^1_0 = G^1_0 + e^{-\kappa}\left(\frac{\dot{\mu}'}{2} + \frac{\dot{\mu}\mu'}{4} - \frac{\omega'\dot{\mu}}{4} - \frac{\dot{\kappa}\mu'}{4}\right), \tag{29}$$

$$\begin{aligned} \tilde{G}^1_1 &= G^1_1 + e^{-\omega}\left(\frac{\ddot{\mu}}{2} + \frac{\dot{\mu}^2}{4} - \frac{\dot{\omega}\dot{\mu}}{4} + \frac{\dot{R}\dot{\mu}}{R}\right) - e^{-\kappa}\left(\frac{\mu'\omega'}{4} + \frac{\mu' R'}{R}\right) \\ &\quad + \epsilon e^{-\mu}\left(\frac{\omega^{**}}{2} + \frac{\omega^{*2}}{4} + \frac{R^{*2}}{R^2} + \frac{2R^{**}}{R} + \frac{R^*}{2R}(\omega^* - \mu^*) - \frac{\mu^*\omega^*}{4}\right), \end{aligned} \tag{30}$$

$$\begin{aligned} \tilde{G}^2_2 &= G^2_2 + e^{-\omega}\left(\frac{\dot{R}\dot{\mu}}{2R} - \frac{\dot{\omega}\dot{\mu}}{4} + \frac{\dot{\mu}\dot{\kappa}}{4} + \frac{\ddot{\mu}}{2} + \frac{\dot{\mu}^2}{4}\right) - e^{-\kappa}\left(\frac{R'\mu'}{2R} + \frac{\mu''}{2}\right. \\ &\quad \left. + \frac{\mu'^2}{4} + \frac{\omega'\mu'}{4} - \frac{\mu'\kappa'}{4}\right) + \epsilon e^{-\mu}\left(\frac{R^{**}}{R} + \frac{R^*\omega^*}{2R} + \frac{R^*\kappa^*}{2R} - \frac{R^*\mu^*}{2R}\right. \\ &\quad \left. + \frac{\omega^{**}}{2} + \frac{\omega^{*2}}{4} + \frac{\kappa^{**}}{2} + \frac{\kappa^{*2}}{4} + \frac{\kappa^*\omega^*}{4} - \frac{\kappa^*\mu^*}{4} - \frac{\mu^*\omega^*}{4}\right), \end{aligned} \tag{31}$$

$$\tilde{G}^3_3 = \tilde{G}^2_2 \,. \tag{32}$$

Now according to the field equations $\tilde{G}_{\mu\nu} = \tilde{T}_{\mu\nu}$. Here, for simplifying the problem we have taken $8\pi G = 1$. The stress energy tensor for fluid in equilibrium can be written as

$$\tilde{T}_{AB} = (\rho + p)\, \tilde{u}_A \tilde{u}_B - p\tilde{g}_{AB} \,. \tag{33}$$

As $\tilde{g}_{44} \sim O(\hbar^2)$ and $\tilde{u}_4\tilde{u}_4 \sim O(\hbar^2)$, we can consider that $\tilde{T}_{44} \sim 0$. Therefore the field equation will give us $\tilde{R}_{44} \sim 0$. This approximation will be used later for some of the calculations.

From the expressions of the Einstein's tensors it is clear that the expressions give us the stress energy tensor of a fluid in equilibrium which has $\tilde{u}_0 \neq 0$ and $\tilde{u}_1 \neq 0$ and $\tilde{u}_2 = \tilde{u}_3 = 0$. If $\tilde{u}_2$ or $\tilde{u}_3$ becomes nonzero then we should have got some expression for $\tilde{G}^0_2$ or $\tilde{G}^1_2$ etc. however that is not the case. Of course, there is no need to put any constrain on $\tilde{u}_4$ because that quantity is anyway very small and can be neglected in the classical limit.

Now according to the logic discussed in the previous section, the negative values of the expressions in the right hand side of Eq.(28) -Eq.(32) will have the terms having the properties of Dark matter and Dark energy. So let us consider these expressions as the stress energy tensor for the dark matter and the dark energy, and lets us represent them by $(T_g)^\mu_\nu$. Here g in the subscript is written just to say that the stress energy tensor is coming from the geometry and has nothing to do with the actual matter density. Now, if we define the density and the pressure from these geometric components by ρ_g and p_g, then these two components can be calculated from the stress energy tensor as

$$\rho_g = \tilde{T}^0_{g0} + \tilde{T}^1_{g1} - \tilde{T}^2_{g2} \,, \tag{34}$$

and

$$p_g = -\tilde{T}^2_{g2} \,. \tag{35}$$

The above expressions are coming from Eq.(33) and the fact that $\tilde{u}^1\tilde{u}_1 + \tilde{u}^2\tilde{u}_2 = 1$, as all the other components of velocity are zero.

Also it has been discussed before that $R_{44} \sim 0$ in the classical limit. Hence adding this term with any other expression will not change that expression. A little algebraic manipulation can show that if this expression is added to Eq.(35) then the expression for p_g will become highly simplified and more meaningful. Thus, expression for p_g is taken as

$$p_g = -\tilde{T}^2_{g2} + \tilde{R}^4_4 \,. \tag{36}$$

Now putting the expressions for $\tilde{T}^0_0$, $\tilde{T}^1_1$ and $\tilde{T}^2_2$ from Eq.(28), Eq.(30) and Eq.(31) into Eq.(34) and Eq.(36), the expressions for the density and pressure can be calculated as

$$\begin{aligned}\rho_g \;=\; & \frac{3}{2}\left(\frac{e^{-\kappa}\mu' R'}{R} - \frac{e^{-\omega}\dot{\mu}\dot{R}}{R}\right) + \frac{3}{2}\epsilon e^{-\mu}\left(\frac{R^*\mu^*}{R} - \frac{2R^{**}}{R}\right) - \epsilon e^{-\mu}\frac{R^{*2}}{R^2} + \epsilon e^{-\mu}\left(\frac{\omega^*\kappa^*}{4}\right) \\ & -\epsilon e^{-\mu}\frac{R^*}{2R}\left(\kappa^* + \omega^*\right) ,\end{aligned} \tag{37}$$

and

$$p_g \;=\; \frac{1}{2}\left(\frac{e^{-\kappa}\mu' R'}{R} - \frac{e^{-\omega}\dot{\mu}\dot{R}}{R}\right) + \frac{1}{2}\epsilon e^{-\mu}\left(\frac{R^*\mu^*}{R} - \frac{2R^{**}}{R}\right) + \epsilon e^{-\mu}\left(\frac{\omega^*\kappa^*}{4}\right) + \epsilon e^{-\mu}\frac{R^*}{2R}\left(\kappa^* + \omega^*\right) . \tag{38}$$

The above equations i.e. Eq.(37) and Eq.(38) clearly shows that there are four different types of components of the pressure and density in the above equations. These components can be separated as follows

$$\rho_{gr} = 3p_{gr} = \frac{3}{2}\left(\frac{e^{-\kappa}\mu' R'}{R} - \frac{e^{-\omega}\dot{\mu}\dot{R}}{R}\right) + \frac{3}{2}\epsilon e^{-\mu}\left(\frac{R^{*}\mu^{*}}{R} - \frac{2R^{**}}{R}\right), \tag{39}$$

$$\rho_{gd} = -\epsilon e^{-\mu}\frac{R^{*2}}{R^{2}}, \tag{40}$$

$$\rho_{gs} = p_{gs} = \epsilon e^{-\mu}\left(\frac{\omega^{*}\kappa^{*}}{4}\right), \tag{41}$$

$$\rho_{g\Lambda} = -p_{g\Lambda} = -\epsilon e^{-\mu}\frac{R^{*}}{2R}\left(\kappa^{*} + \omega^{*}\right). \tag{42}$$

The pressure and the density given by the Eq.(39) behave as that of the radiation. The pressure and density follows the relation $p = \frac{\rho}{3}$. Therefore, the quantity can be treated as some radiation like component of dark matter in the standard cosmology. The 2^{nd} component which is given by Eq.(40) behaves as the non-relativistic matter. For this component, the pressure is 0 and therefore this component fulfills all the properties of cold dark matter. The third component [Eq.(41)] is a more interesting component. For this component the pressure and the density are equal and hence it behaves as stiff matter. In the following section, it will be shown that this component will vanish in the FLRW line element. The final part which is given by Eq.(42) will satisfy the $\rho = -p$ relation. The component has a negative amount of pressure and hence this component behaves as dark energy of the standard cosmological model.

Therefore this section shows that if Machian gravity is correct and if cosmological model is derived using the General Theory of relativity, the hot dark matter, cold dark matter and dark energy will get inserted into the equations automatically from the geometry. Therefore, Machian gravity can provide a cosmological model exactly similar to that of the standard cosmological model without demanding any dark mater or dark energy.

In the standard cosmological model, it is also considered that the density of the dark matter and the dark energy are positive. If the density of some of the components in standard cosmological model is taken as negative then it will not give all the predictions properly. Therefore, we should demand that the sign of ρ_{gd} and $\rho_{g\Lambda}$ should be positive. This forces us to choose the sign of the background dimension to be negative. Therefore, the background dimension should be a space-like dimension.

IV. EFFECTS ON THE FLRW UNIVERSE

A five dimensional coordinate system is required to describe the Mach's principle, where the 5^{th} dimension is a quantifier of the background contribution. Also in the previous sections it has been discussed that when the five dimensional field equations are projected in a four dimensional space-time hyperspace, then the four dimensional field equations pick up some new extra terms having exactly the same properties as the dark matter and the dark energy. Now, in the standard cosmological model, the FLRW line element in the four dimensional space-time is used to describe the cosmology. Observations shows that the four dimensional FLRW cosmology with dark matter and dark energy can explain all the observational phenomenon to an extremely high accuracy. Therefore, it will be very convenient to demand that the four dimensional line element on the space-time hyperspace of the five dimensional universe is an FLRW line element.

Therefore, the line element in the five dimensional universe should be written as

$$ds^{2} = dt^{2} - a^{2}(t,\zeta)\left[dr^{2} + r^{2}\left(d\theta^{2} + \sin^{2}\theta d\phi^{2}\right)\right] - e^{\mu}d\zeta^{2}. \tag{43}$$

Here, the flat cosmological model is considered just for simplicity. If the open or closed model of the universe were considered then also the calculations would have been similar.

Now if the line element in Eq.(43) is compared to that of the line element of Eq.(11) then e^{ω} will be a constant, and hence ω^{*} will vanish. The Eq.(41) shows us that the energy density for the stiff matter will be zero which is indeed the case in standard cosmology. Eq(42) will then become

$$\rho_{g\Lambda} = -p_{g\Lambda} = e^{-\mu}\frac{R^{*}}{2R}\kappa^{*}. \tag{44}$$

Eq(43) shows us that e^{κ} is nothing but a^2 in this model and hence $\kappa^* = 2\frac{a^*}{a}$ and also R will be equivalent to $a(t,\zeta)r$ in this coordinate system.

Therefore, when the cosmological field equations from the Machian gravity model are projected in a four-dimensional space-time having a similar metric as that of FLRW, all the dark matter and the dark energy will arise from the geometry of the coordinate system. These dark matter and the dark energy has no relation with the actual matter and hence these are not directly observable and can only be detected by measuring the geodesic paths of different particles in freely falling frame.

V. CONCLUSION

The origin of the dark matter and the dark energy in the Machian gravity model is discussed. It is shown that the cosmology in this new model behaves exactly in the same way as that of the standard ΛCDM model, but in this case the dark matter and the dark energy emerges from the geometry and hence is not directly detectable. This solves all the problems that arise due to the non-detection of these components. The calculation in this paper also fixes the signature of the background dimension. The most important part is that calculations shown in this paper gives a sense of completeness to the standard ΛCDM model as all the components of the universe here arise from the theory.

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