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Lessons from classical gravity about the quantum structure of spacetime

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Abstract. I present the theoretical evidence which suggests that gravity is an emergent phenomenon like gas dynamics or elasticity with the gravitational field equations having the same status as, say, the equations of fluid dynamics/elasticity. This paradigm views a wide class of gravitational theories — including Einstein's theory — as describing the thermodynamic limit of the statistical mechanics of 'atoms of spacetime'. Strong internal evidence in favour of such a point of view is presented using the classical features of the gravitational theories with just one quantum mechanical input, viz. the existence of Davies-Unruh temperature of horizons. I discuss several conceptual ingredients of this approach.

1. How can *classical* gravity teach us anything about *quantum* spacetime?

In one sentence, the paradigm that we will explore [1] is the following: Gravity is an emergent phenomenon like gas dynamics or elasticity with the gravitational field equations having the same status as, say, the equations of fluid dynamics/elasticity. Historically, this paradigm originated with Sakharov [2] and was interpreted in different ways by Jacobson [3], Volovik [4], Bei-Lok Hu [5] and many others. (Analogue models [6] as well as the membrane paradigm [7] for black holes have some similarities with this approach. For a small sample of more recent work reassembling different aspects of these ideas, see [8]). I will now elaborate on this theme drawing mainly from the work I was involved in.¹

Part of this programme involves a “top-down” approach to quantum spacetime (in the sense of zooming in from the top to smaller and smaller spatial scales, like in a Google map) to learn key lessons (Sections 1–5), which are then used to provide a derivation of field equations from extremising spacetime entropy density (Section 6). Some people use the word “top-down” to mean exactly the opposite; I will use “top-down” the way I have defined it, viz. from classical to quantum domain.

One may find it surprising that such an attempt, to determine the features of the microscopic theory from knowing its properties at the macroscopic scales., is so successful. Of course, in the *strict* sense, classical theories cannot tell us anything about quantum dynamics; after all, classical physics, by definition, is independent of \hbar while quantum effects do depend on \hbar . But there is one effect, viz., the thermodynamics of spacetime horizons [9] which brings together the principles of quantum theory and gravity. This fact, along with a judicious choice for the questions to ask, allows one to make a fairly persuasive case for the structure of quantum

¹ I will use mostly positive signature with English alphabets covering $0, 1, \dots, D-1$ and Greek alphabets covering the spatial indices $1, 2, \dots, D-1$ of a D dimensional spacetime.

spacetime. To see such a “top-down” approach in context, let me describe at least three other — more conventional — examples in which the deeper, more exact, (‘bottom layer’) theory leaves a tell-tale signature on the ‘top layer’.

(i) *Electrons in a helium atom*: Suppose you manage to solve the Schrödinger equation for the two electrons in the helium atom and determine the energy eigenfunctions $\psi(\mathbf{x}_1, \mathbf{x}_2)$. Your experimental friend will tell you that only half of these wave functions [which are antisymmetric under the interchange $(\mathbf{x}_1 \Leftrightarrow \mathbf{x}_2)$] occur in the real world. Any amount of your staring at the Schrödinger equation for the helium atom will not tell you why nature requires this antisymmetry under pair exchange for electrons. The reason lies deep down in relativistic quantum theory but its residual effect remains as a tell-tale signature even in the $c = \infty$ limit of field theory, viz., the non-relativistic quantum mechanics.

(ii) *Boltzmann’s conjecture of atoms*: Classical thermodynamics of a gas/fluid uses variables like density, pressure etc. in the continuum description. But the fact that such a fluid can store and exchange heat energy *cannot* be understood within the continuum theory. Boltzmann had the insight to suggest that thermal phenomena *demand* the existence of microscopic degrees of freedom in matter. In fact, the law of equipartition, expressed as $E/[(1/2)k_B T] = N$ relates two thermodynamic variables E, T (which are well-defined for a continuum fluid) to N , the number density of microscopic degrees of freedom, which cannot be interpreted in the continuum limit at all. The Avogadro’s number, closely related to N , was determined even before we fully understood what exactly it counts and without any direct evidence for the molecular structure of matter. This is another example of our being able to say something about microscopic structure from the features of macroscopic theory.

(iii) *Equality of inertial and gravitational masses*: The most dramatic example is provided by Einstein’s use of principle of equivalence which, of course, was known for centuries. Einstein realized that $m_i = m_g$ is not a trivial algebraic accident which should be taken for granted, as others before him have done, but requires an explanation. This led him to the description of gravity in terms of the geometry of spacetime. The relation $m_i = m_g$ was a signature of the deeper theory discernible in the approximate (top-layer) description.

The lesson from these three examples is obvious. For a top down approach to be useful, you need to ask the right questions! One way is to pick up features of the theory that are usually taken for granted (‘algebraic accidents’) — or not even noticed — and demand deeper explanations for them. This is the procedure I will follow in this programme to probe the quantum structure of spacetime from known aspects of classical gravity.

2. The conceptual background

I will describe several peculiar features of classical gravitational theories from which we can obtain a broad picture regarding the quantum microstructure of the spacetime, in the form of a series of “lessons”. Most of these (starting from lesson 4!) will be specific mathematical features of the theory. But to provide the necessary backdrop, I will distill out of these mathematical features three conceptual points and describe them right at the outset.

Lesson 1: Providing a quantum description of spacetime structure is quite different from constructing a quantum theory of gravity.

In this approach, it is necessary to make a clear distinction between quantum description of spacetime structure and a theory of quantum gravity.

Classical field equations of gravity *also* happens to describe the classical dynamics of the spacetime because of the geometrical interpretation. In the emergent gravity paradigm, these field equations have a status similar to the equations of fluid mechanics or elasticity. So, if this

paradigm is correct, one should *not* expect quantizing a classical theory of gravity to lead us to the quantum structure of spacetime any more than quantizing the equations of elasticity or hydrodynamics will lead us to atomic structure of matter! Quantizing the elastic vibrations of a solid will lead only to phonon physics [5] just as quantizing a classical theory of gravity will lead to graviton physics. The latter could be quite different from a description of quantum structure of spacetime just as phonon physics is quite different from the physics of the atom.

Lesson 2: The guiding principle to use for understanding the quantum microstructure of the spacetime should be the thermodynamics of horizons.

Combining the principles of GR and quantum theory is not a technical problem that could be solved just by using sufficiently powerful mathematics. It is more of a conceptual issue and decades of failure of sophisticated mathematics in delivering quantum gravity indicates that we should try a different approach. This is very much in tune with item (iii) mentioned in Sec. 1. Einstein did not create a sophisticated mathematical model for m_i and m_g and try to interpret $m_i = m_g$. He used thought experiments to arrive at a conceptual basis in which $m_i = m_g$ can be embedded naturally so that $m_i = m_g$ will cease to be an algebraic accident. Once this is done, physics itself led him to the maths that was needed.

Of course, the key issue is what could play the role of a guiding principle similar to principle of equivalence in the present context. *For this, my bet will be on the thermodynamics of horizons.*[1, 10] A successful model will have the connection between horizon thermodynamics and gravitational dynamics *at its foundation* rather than this feature appearing as a result derived in the context of certain specific solutions to the field equations. We will see evidence for its importance throughout the discussion in what follows.

Lesson 3: Think beyond Einstein gravity, black hole thermodynamics and think off-shell.

There are four technical points closely related to the above conjecture (viz., thermodynamics of horizons should play a foundational role) which needs to be recognized if this approach has yield dividends:

- One must concentrate on the general context of observer dependent, local, thermodynamics associated with the local horizons, going beyond the *black hole* thermodynamics. Black hole horizons in the classical theory are far too special, on-shell, global constructs to provide a sufficiently general back-drop to understand the quantum structure of spacetime. The preoccupation with the black hole horizons loses sight of the conceptual fact that all horizons are endowed with temperature as perceived by the appropriate class of observers. Observer dependence [11] of thermal phenomena is a feature and not a bug!
- One should also think beyond Einstein's theory and use the structure of, say, Lanczos-Lovelock models of gravity [12] in exploring the microstructure of spacetime. Previous work (starting from ref. [13]) has shown that the interpretation of gravity as an emergent phenomenon transcends Einstein's theory and remains applicable to (at least) all Lanczos-Lovelock models. Exploiting this connection will allow us to discriminate between results of general validity from those which are special to Einstein's theory in $D = 4$. Irrespective of whether Lanczos-Lovelock models are relevant to real world, they provide a good test-bed to see which concepts and results are robust and general.
- A corollary is that one should *not* think of entropy of horizons as being proportional to their area. This result, which is true in Einstein's theory, fails for all higher order Lanczos-Lovelock models [14]. But all the general thermodynamic features still continue to remain valid. Because area brings in several other closely related geometrical notions,

restricting oneself to Einstein's theory leads to an incorrect view of what entropy and quantum microstructure of spacetime could be.

- The quantum features of a theory are off-shell features. But, fortunately, classical action principles provide a window to quantum theory because of the path integral formalism. Therefore any peculiar feature of classical action principle could give us insights into the underlying quantum theory much more than the structure of field equations. This suggests that we need to look at the off-shell structure of the theory using the form of action principles rather than tie ourselves down to field equations.

These ingredients, to a great extent, distinguish the approach I was developing from those of many others.

3. Lessons from the thermodynamics of horizons

Having outlined the broad conceptual features, I will now move on to specifics. There are four lessons one can learn from putting together well-known features of horizon thermodynamics in an appropriate manner.

Lesson 4: Temperature of horizons does not depend on the field equations of the theory and is just an indication that spacetimes, like matter, can be hot, but in a observer-dependent manner.

One can associate a temperature with any null surface that can act as horizon for a class of observers, in any spacetime (including flat spacetime). This temperature is determined by the behaviour of the metric close to the horizon and has *nothing to do with the field equations* (if any) which are obeyed by the metric.

The simplest situation is that of Rindler observers in flat spacetime with acceleration κ who will attribute a temperature $k_B T = (\hbar/c)(\kappa/2\pi)$ to the Rindler horizon — which is just a $X = T$ surface in the flat spacetime having no special significance to the inertial observers. While this result is usually proved for an eternally accelerating observer, they also hold in the (appropriately) approximate sense for an observer with variable acceleration [15]. In general, this result can be used to show that the vacuum state in a freely falling frame will appear to be a thermal state in the locally accelerated frame for high frequency modes if κ^{-1} is smaller than the local radius of (spacetime) curvature.

In the usual context of a bifurcation horizon that divides the spacetime into two causally disconnected regions R and L , the global vacuum state $|\text{vac}\rangle$ of a quantum field theory can be described by a vacuum functional $\langle \text{vac} | \phi_L, \phi_R \rangle$ in terms of the field configurations ϕ_L in L and ϕ_R in R . Using the Euclidean path integral representation for the ground state functional, one can express this functional in two different ways and obtain:

$$\langle \text{vac} | \phi_L, \phi_R \rangle \propto \int_{T_E=0; \phi=(\phi_L, \phi_R)}^{T_E=\infty; \phi=(0,0)} \mathcal{D}\phi e^{-A} \propto \int_{\kappa t_E=0; \phi=\phi_R}^{\kappa t_E=\pi; \phi=\phi_L} \mathcal{D}\phi e^{-A} \propto \langle \phi_L | e^{-(\pi/\kappa)H_R} | \phi_R \rangle \quad (1)$$

where H_R is the Hamiltonian describing the dynamics in one of the wedges [16] and κ is the acceleration. Both path integrals cover the upper-half of the Euclidean $X - T_E$ plane. The first path integral is in the global coordinate system (inertial, Kruskal) with time T_E running from $T_E = 0$ to $T_E = \infty$ with the boundary conditions at both limits as indicated. The second path integral is in the coordinate system adapted to region outside the horizon (Rindler, Schwarzschild) with the time coordinate behaving like a polar angle in the plane, going from $\kappa t_E = 0$ (the right wedge) to $\kappa t_E = \pi$ (the left wedge), with the fields taking appropriate boundary values in the two limits. Thus we get:

$$\langle \text{vac} | \phi_L, \phi_R \rangle \propto \langle \phi_L | e^{-(\pi/\kappa)H_R} | \phi_R \rangle \quad (2)$$

For describing the physics in the region outside the horizon, say in R , we will trace out the modes ϕ_L beyond the horizon. This gives a thermal density matrix for the observables in the right wedge:

$$\rho(\phi'_R, \phi_R) \propto \int \mathcal{D}\phi_L \langle \phi_L, \phi'_R | \text{vac} \rangle \langle \text{vac} | \phi_L, \phi_R \rangle \propto \langle \phi'_R | e^{-(2\pi/\kappa)H_R} | \phi_R \rangle \quad (3)$$

corresponding to the horizon temperature $T = \kappa/2\pi$. This result only depends on the near horizon geometry having the approximate form of a Rindler metric and is independent of the field equations of the theory.

Lesson 5: All thermodynamic variables are observer dependent.

An immediate consequence, not often emphasized, is that *all* thermodynamic variables must become observer dependent if vacuum acquires an observer dependent temperature. A “normal” gaseous system with “normal” thermodynamic variables (T, S, F etc.....) must be considered as a highly excited state of the inertial vacuum. It is obvious that a Rindler observer will attribute to this highly excited state different thermodynamic variables compared to what an inertial observer will attribute. Thus thermal effects in the accelerated frame brings in [11, 17] a new level of observer dependence even to *normal* thermodynamics. One need not panic if variables like entropy now acquire an observer dependence and lose their absolute nature.

Lesson 6: In sharp contrast to temperature, the entropy of horizons depends on the field equations of gravity and cannot be determined by using just QFT in a background metric.

One would have expected that if integrating out certain field modes leads to a thermal density matrix ρ , then the entropy of the system should be related to lack of information about the *same* field modes and should be given by $S = -\text{Tr } \rho \ln \rho$. This entropy, called entanglement entropy, (i) is proportional to area of the horizon and (ii) is divergent without a cut-off [18]. Such a divergence makes the result meaningless and thus we cannot attribute a unique entropy to horizon using just QFT in a background metric.² That is, while the temperature of the horizon can be obtained through the study of test-QFT in an external geometry, one cannot understand the entropy of the horizon by the same procedure.

This is because, unlike the temperature, the *entropy* associated with a horizon in the theory depends on the field equations of the theory, which we will briefly review. Given the principle of equivalence (interpreted as gravity being spacetime geometry) and principle of general covariance, one could still construct a wide class of theories of gravity. For example, if we take the action functional

$$A = \int d^D x \sqrt{-g} \left[L(R_{cd}^{ab}, g^{ab}) + L_{\text{matt}}(g^{ab}, \phi_A) \right] \quad (4)$$

where L_{matt} is the matter Lagrangian (for some matter variables denoted symbolically as ϕ_A) and vary the metric with appropriate boundary conditions, we will get the field equations (see e.g., chapter 15 of [20]):

$$\mathcal{G}_{ab} = P_a^{cde} R_{bcde} - 2\nabla^c \nabla^d P_{acdb} - \frac{1}{2} L g_{ab} \equiv \mathcal{R}_{ab} - \frac{1}{2} L g_{ab} = \frac{1}{2} T_{ab} \quad (5)$$

² In the literature, one often “regularizes” the expression for entanglement entropy by introducing a Planck scale cut-off by hand. This has no justification because of two reasons. First, a free quantum field theory in flat spacetime should not require any cut-off to give meaningful results. Second, in the conventional approach, there is no way a flat spacetime ($G = 0$) quantum field theory will know anything about Planck length. In fact, the divergence of entanglement entropy and the need for a Planck scale cut-off is an indication that there is no such thing as flat spacetime, just as there is no such thing as classical, continuum solid [19].

where $P^{abcd} \equiv (\partial L / \partial R_{abcd})$. A nice subclass of theories in which the field equations remain second order in the metric is obtained if we choose L such that $\nabla_a P^{abcd} = 0$. The most general scalar functionals $L(R_{cd}^{ab}, g^{ij})$ satisfying this condition are specific polynomials in curvature tensor which lead to the Lanczos-Lovelock models [12] with the field equations:

$$P_{ac}^{de} R_{de}^{bc} - \frac{1}{2} L \delta_a^b = \mathcal{R}_a^b - \frac{1}{2m} \mathcal{R} \delta_a^b = \frac{1}{2} T_a^b, \quad \mathcal{R}_a^b \equiv P_{ac}^{de} R_{de}^{bc}, \quad \mathcal{R} = \mathcal{R}_a^a \quad (6)$$

The second form of the equation is valid for the m -th order Lanczos-Lovelock model for which $\mathcal{R} = R^{abcd}(\partial L / \partial R^{abcd}) = mL$. In the simplest context of $m = 1$ we take $L \propto R = R/16\pi$ (with conventional normalization), leading to $P_{cd}^{ab} = (32\pi)^{-1}(\delta_c^a \delta_d^b - \delta_d^a \delta_c^b)$, we get $\mathcal{R}_b^a = R_b^a/16\pi$, $\mathcal{G}_b^a = G_b^a/16\pi$ and one recovers Einstein's equations. The structure of the theory is essentially determined by the tensor P_{cd}^{ab} which has the algebraic symmetries of curvature tensor and is divergence-free in all indices.

In any such generally covariant theory, the infinitesimal coordinate transformation $x^a \rightarrow x^a + q^a$ leads to the conservation of a Noether current J^a (which depends on q^a) given by:

$$J^a \equiv \left(2\mathcal{G}_b^a q^b + Lq^a + \delta_q v^a \right) = 2\mathcal{R}_b^a q^b + \delta_q v^a, \quad \nabla_a J^a = 0. \quad (7)$$

where $\delta_q v^a$ represents the boundary term in the action which arises for the variation of the metric in the form $\delta g^{ab} = (\nabla^a q^b + \nabla^b q^a)$. Given $\nabla_a J^a = 0$, we can introduce an anti-symmetric tensor J^{ab} by $J^a = \nabla_b J^{ab}$. For the Lanczos-Lovelock models, one can determine $\delta_q v^a$ and show that the J^{ab} and J^a can be expressed in the form

$$J^{ab} = 2P^{abcd} \nabla_c q_d; \quad J^a = 2P^{abcd} \nabla_b \nabla_c q_d \quad (8)$$

The field equations of Lanczos-Lovelock models, possess black hole solutions (with horizons) in asymptotically flat spacetime. Studying the physical processes occurring in such spacetimes, one can obtain an expression for the entropy of the horizon (called Wald entropy [14]) which is closely related to the Noether current J^a as follows:

$$S_{\text{Noether}} \equiv \beta \int d^{D-1} \Sigma_a J^a = \beta \int d^{D-2} \Sigma_{ab} J^{ab} = \frac{1}{4} \oint_{\mathcal{H}} (32\pi P_{cd}^{ab}) \epsilon_{ab} \epsilon^{dc} d\sigma \quad (9)$$

where $\beta^{-1} = \kappa/2\pi$ is the horizon temperature and J^a is the Noether current $q^a = \xi^a$ where ξ^a is the Killing vector corresponding to time translation symmetry of the asymptotically static black hole solution. In the final expression the integral is over any surface with $(D - 2)$ dimension which is a spacelike cross-section of the Killing horizon on which the norm of ξ^a vanishes, with ϵ_{ab} denoting the bivector normal to the bifurcation surface. Thus horizon entropy is given by an integral over the horizon surface of the P^{abcd} , which we may call the *entropy tensor* of the theory. Note that the Noether current J^a multiplied by $\beta_{loc} \equiv N\beta$, where N is the lapse function, can be thought of as the entropy current density.

In Einstein's theory, with $32\pi P_{cd}^{ab} = (\delta_c^a \delta_d^b - \delta_d^a \delta_c^b)$, the entropy will be one quarter of the area of the horizon. But in general, the entropy of the horizon is *not* proportional to the area and depends on the theory.³

This dichotomous situation as regards temperature versus entropy is the first indication that the thermodynamics of the horizon, probed by QFT in an external gravitational field, is just the tip of an iceberg. As we will see the emergent paradigm provides a better understanding of these features.

³ This feature again shows that the entanglement entropy cannot be identified with the entropy of the Lanczos-Lovelock models without modifying the regularization procedure. In the emergent paradigm one can argue that such a modification is indeed required. Then, using a generalisation of ideas described in ref.[21], one can possibly tackle this issue. I will not discuss this here; for more details, see ref. [19].

Lesson 7: The connection between horizon entropy and the conserved current arising from the diffeomorphism invariance demands deeper understanding.

Why should a current J^a , conserved due to diffeomorphism invariance of the theory have anything to do with a thermodynamical variable like entropy of horizons in the theory ?

In the conventional approach, which views $x^a \rightarrow x^a + q^a$ as a relabeling of coordinates, this question has no answer. In contrast, if we take the ‘active’ point of view, we notice that it shifts (virtually) the location of null surfaces and thus the information accessible to specific observers. The connection with entropy arises due to the cost of gravitational entropy involved in the virtual displacements of null horizons.

This idea can be made more precise in terms of entropy balance at local Rindler horizons [22]. Let us choose any event \mathcal{P} and introduce a local inertial frame (LIF) around it with Riemann normal coordinates $X^a = (T, \mathbf{X})$ such that \mathcal{P} has the coordinates $X^a = 0$ in the LIF. Let k^a be a future directed null vector at \mathcal{P} and we align the coordinates of LIF such that it lies in the $X - T$ plane at \mathcal{P} . We next transform from the LIF to a local Rindler frame LRF with acceleration κ along the X axis. Let ξ^a be the approximate Killing vector corresponding to translation in the Rindler time such that the vanishing of $\xi^a \xi_a \equiv -N^2$ characterizes the location of the local horizon \mathcal{H} in LRF. Usually, we shall do all the computation on a time-like surface infinitesimally away from \mathcal{H} with $N = \text{constant}$, called a “stretched horizon”. Let the time-like unit normal to the stretched horizon be r_a .

Consider an infinitesimal displacement of a local patch of the stretched horizon in the direction of r_a , by an infinitesimal proper distance ϵ , which will change the proper volume by $dV_{prop} = \epsilon \sqrt{\sigma} d^{D-2}x$ where σ_{ab} is the metric in the transverse space. The flux of energy through the surface will be $T_b^a \xi^b r_a$ and the corresponding entropy flux can be obtained by multiplying the energy flux by $\beta_{loc} = N\beta$. Hence the ‘loss’ of matter entropy to the outside observer because the virtual displacement of the horizon has engulfed some matter is $\delta S_m = \beta_{loc} \delta E = \beta_{loc} T^{aj} \xi_a r_j dV_{prop}$. Recalling from Eq. (9) that $\beta_{loc} J^a$ gives the gravitational entropy current, the change in the gravitational entropy is given by $\delta S_{grav} \equiv \beta_{loc} r_a J^a dV_{prop}$ where J^a is the Noether current corresponding to the local Killing vector ξ^a given by $J^a = 2\mathcal{G}_b^a \xi^b + L\xi^a$. As the stretched horizon approaches the true horizon, it can be shown that $Nr^a \rightarrow \xi^a$ and $\beta \xi^a \xi_a L \rightarrow 0$. Hence we get, in this limit: $\delta S_{grav} \equiv \beta \xi_a J^a dV_{prop} = 2\beta \mathcal{G}^{aj} \xi_a \xi_j dV_{prop}$. Comparing δS_{grav} and δS_m we see that the field equations $2\mathcal{G}_b^a = T_b^a$ can be interpreted as the entropy balance condition $\delta S_{grav} = \delta S_{matt}$ thereby providing direct thermodynamic interpretation of the field equations as local entropy balance in local Rindler frame.

In the emergent paradigm, the spacetime is analogous to a solid made of atoms and $x^a \rightarrow x^a + q^a(x)$ is analogous to the deformation of an elastic solid [23]. When such a deformation leads to changes in accessible information — like when one considers the virtual displacements of horizons — it costs some amount of gravitational entropy thereby providing a direct link between the transformation $x^a \rightarrow x^a + q^a(x)$ and spacetime entropy — a link that is lacking in the conventional approach. We will say more about this in Sec. 6.

4. Thermodynamic interpretation of field equations and action functionals

I stressed in Sec. 1 that for the top-down approach to be of use, we need to identify the ‘algebraic accidents’ in the top level description which are usually taken for granted without a demand for explanation. I will briefly summarize three such issues in classical gravity, which can give us clues about the microscopic theory.

Lesson 8: The gravitational field equations reduce to a thermodynamic identity on the horizon in a wide class of theories.

It can be shown that [24] the field equations in any Lanczos-Lovelock model, when evaluated on a static solution of the theory which has a horizon, can be expressed in the form of a thermodynamic identity $TdS = dE_g + PdV$. Here S is the correct Wald entropy of the horizon in the theory, E_g is a geometric expression involving an integral of the scalar curvature of the sub-manifold of the horizon and PdV represents the work function of the matter source. The differentials dS, dE_g etc. should be thought of as indicating the difference in S, E_g etc between two solutions in which the location of the horizon is infinitesimally displaced.

This equality between field equations on the horizon and the thermodynamic identity — originally obtained [25] for spherical horizons in Einstein's theory, has now been demonstrated for an impressively wide class of models [26] like stationary axisymmetric horizons and evolving spherically symmetric horizons in Einstein gravity, static spherically symmetric horizons and dynamical apparent horizons in Lanczos-Lovelock gravity, generic, static horizon in Lanczos-Lovelock gravity, three dimensional BTZ black hole horizons, FRW cosmological models in various gravity theories and even in the case Horava-Lifshitz gravity.

This result is non-trivial in the sense that the field equation on the horizon does not look very “thermodynamical” at first sight. For example, in the simplest context of spherically symmetric horizon in Einstein's theory [with $-g_{00} = g_{11}^{-1} = f(r)$ with $f(a) = 0$ determining the location of the horizon at $r = a$], the field equation on the horizon reduces to

$$\frac{c^4}{G} \left[\frac{\kappa a}{c^2} - \frac{1}{2} \right] = 4\pi P a^2 \quad (10)$$

where $\kappa = f'(a)/2$ is the surface gravity and P is the pressure of the source. As I said, this equation does not seem to have any thermodynamics in it. However, if we multiply it by da it can be re-written in the form:

$$\underbrace{\frac{\hbar}{c} \left(\frac{\kappa}{2\pi} \right)}_{k_B T} \underbrace{\frac{c^3}{G\hbar} d \left(\frac{1}{4} 4\pi a^2 \right)}_{k_B^{-1} dS} - \underbrace{\frac{1}{2} \frac{c^4 da}{G}}_{-dE_g} = \underbrace{Pd \left(\frac{4\pi}{3} a^3 \right)}_{P dV} \quad (11)$$

The only extra input we needed was the expression for the horizon temperature in terms of the surface gravity which needed introducing \hbar in the numerator and denominator. Similar miracle occurs in all the gravitational theories, much more general than Einstein's theory, in which entropy is no longer proportional to horizon area. As we discussed earlier, the temperature of the horizon knows *nothing* about the field equations of the theory but the entropy does. It is therefore remarkable that one obtains the correct combination TdS for a wide variety of theories showing that the information about the theory is encoded in the entropy functional, exactly as it would be for a macroscopic body.

There are significant differences between this identity $TdS = dE_g + PdV$ to which field equations reduce to and the so called Clausius relation $TdS = dE_m$ (used, for example, by Jacobson [3]) which need to be recognised:

- In addition to the obvious existence of the work term PdV , it should be stressed that E_m used in the Clausius relation $TdS = dE_m$ is related to *matter stress tensor* while E_g in the $TdS = dE_g + PdV$ is a purely geometrical construct built out of the metric. The origin of these differences can be traced to two different kinds of virtual displacements of the horizons considered in these two approaches to define the infinitesimal differences [27].

- More importantly, while $TdS = dE_g + PdV$ holds in widely different contexts, it has been found to be *impossible* to generalize $TdS = dE_m$ beyond Einstein's theory without introducing additional assumptions (like dissipation), the physical meaning of which remains unclear.

Incidentally, while Davies-Unruh temperature scales as \hbar the entropy scales as $1/\hbar$ (coming from inverse Planck area), thereby making TdS independent of \hbar ! This is reminiscent of the fact that in normal thermodynamics $T \propto 1/k_B, S \propto k_B$ making TdS independent of k_B . In both cases, the effects due to discrete microstructure (indicated by non-zero \hbar or k_B) disappear in the continuum limit thermodynamics. Thermal phenomena require microstructure but thermodynamical laws are independent of it! Similarly we expect the thermodynamic description of spacetime to be useful and independent of exact nature of the QG description. Any ('bottom-up') model for quantum gravity which leads to horizon thermodynamics and gives Davies-Unruh temperature for QFT in the semi-classical limit, must be consistent with the ('top-down') thermodynamic description merging together in the correct limit.

Lesson 9: Holographic structure of gravitational action functionals finds a natural explanation in the thermodynamic interpretation of the field equations.

If the gravitational dynamics and horizon thermodynamics are so closely related, with field equations becoming thermodynamic identities on the horizon, then the action functionals of the theory (from which we obtain the field equations) must contain information about this connection. This clue comes in the form of another unexplained algebraic accident related to the structure of the action functional and tells us something significant about the *off-shell structure* of the theory.

Gravity is the only theory known to us for which the natural action functional preserving symmetries of the theory contain second derivatives of the dynamical variables but still leads to second order differential equations. Usually, this is achieved by separating out the terms involving the second derivatives of the metric into a surface term which is either ignored or its variation is cancelled by a suitable counter-term. However, this leads to a serious conceptual mystery in the conventional approach when we recall the following two facts together: (a) The field equations can be obtained by varying the bulk term after ignoring (or by canceling with a counter-term) the surface term. (b) But if we evaluate the surface term on the horizon of any solution to the field equations of the theory, one obtains the entropy of the horizon! *How does the surface term, which was discarded before the field equations were obtained, know about the entropy associated with a solution to those field equations?!* In the conventional approach we need to accept it as another 'algebraic accident' without any explanation and, in fact, no explanation is possible within the standard framework.

The explanation lies in the fact that the surface and bulk term of the Lagrangian are related in a specific manner thereby duplicating the information about the horizon entropy [28]. One can show that there exists a relation of the form:

$$\sqrt{-g}L_{\text{sur}} = -\partial_a \left(g_{ij} \frac{\delta \sqrt{-g}L_{\text{bulk}}}{\delta (\partial_a g_{ij})} \right) \quad (12)$$

All Lanczos-Lovelock action functionals have this form [13]. In fact, this relation is crucial for an action with second derivatives of the dynamical variables to still lead to field equations which are only second order — a feature shared by all the Lanczos-Lovelock models. It can be shown that this result will be true for actions that can be separated into a surface term and a bulk term with the surface term being an integral over $\partial_a(q^A \pi_A^a)$ where q^A are the dynamical variables and π_A^a are the canonical momentum. This structure allows one to interpret all these action functionals,

including Einstein-Hilbert action, as providing the momentum space description (see p. 292 of [20]) of the theory.

This duplication of information also allows one relate the variation of the surface term to \mathcal{R}_b^a of the theory. From Eq. (7), it follows that:

$$\begin{aligned} \int_{\partial\mathcal{V}} d^{D-1}x \sqrt{h} n_a (\delta q^a) &= \int_{\mathcal{V}} d^D x \sqrt{-g} \nabla_a (\delta q^a) \\ &= \int_{\mathcal{V}} d^D x \sqrt{-g} \nabla_a (2\mathcal{R}_b^a q^b) = \int_{\partial\mathcal{V}} d^{D-1}x \sqrt{h} n_a (2\mathcal{R}_b^a q^b) \end{aligned} \quad (13)$$

Computing the corresponding variation of matter action under the change $\delta g^{ab} = \nabla^a q^b + \nabla^b q^a$, one can construct a variational principle to obtain the field equations, purely from the surface term [29]. More importantly, since the variation of the surface term gives the change in the gravitational entropy, we see that \mathcal{R}^{ab} essentially determines the gravitational entropy density of the spacetime. We will say more about this in sec. 6.

The duplication of information between surface and bulk term in Eq. (12) also allows one to obtain the full action [10] from the surface term alone using the entropic interpretation. In fact, in the the Riemann normal coordinates around any event \mathcal{P} the gravitational action reduces to a pure surface term, again showing that the dynamical content is actually stored on the boundary rather than in the bulk.

Lesson 10: Gravitational actions have a surface and bulk terms because they give the entropy and energy of static spacetimes with horizons, adding up to make the action the free energy of the spacetime.

This provides yet another, direct, physical interpretation for the structure of the gravitational action functionals analyzed above. The result is most easily seen for any Lanczos-Lovelock model by writing the time component of the Noether current in Eq. (7) for the Killing vector $q^a = \xi^a = (1, \mathbf{0})$ in the form:

$$L = \frac{1}{\sqrt{-g}} \partial_\alpha (\sqrt{-g} J^{0\alpha}) - 2G_0^0 \quad (14)$$

Only spatial derivatives contribute in the first term on the right hand side when the spacetime is static. Integrating over $L\sqrt{-g}$ to obtain the action it is is easy to see (using Eq. (9)) that the first term gives the entropy and the second term can be interpreted as energy [30].

Finally, I stress again that the real importance of these results arises from the fact that they hold for all Lanczos-Lovelock models in an identical manner.

5. The Avogadro number of the spacetime

The results described in the previous sections suggest that there is a deep connection between horizon thermodynamics and the gravitational dynamics. Because the spacetime can be heated up just like a body of gas, the Boltzmann paradigm (“If you can heat it, it has microstructure”) motivates the study of the microscopic degrees of freedom of the spacetime exactly the way people studied gas dynamics *before* they understood the atomic structure of matter. There exists, fortunately, an acid test of this paradigm which it passes with flying colours.

Lesson 11: Gravitational field equations imply the law of equipartition $\Delta E = (1/2)k_B T \Delta N$ in any static spacetime, allowing the determination of density of microscopic degrees of freedom. The result again displays holographic scaling.

Boltzmann’s conjecture led to the equipartition law $\Delta E = (1/2)k_B T \Delta N$ relating the number density ΔN of microscopic degrees of freedom required to store an energy ΔE at temperature

T and to the determination of Avogadro number of a gas. If our ideas are correct, we should be able to relate the E and T of a given spacetime to determine the number density of microscopic degrees of freedom of the spacetime. Remarkably enough, this can be done directly from the field equations [31]. In a hot spacetime, Einstein's equations *imply* the equipartition law

$$E = \frac{1}{2}k_B \int_{\partial V} \frac{\sqrt{\sigma} d^2x}{L_P^2} \left\{ \frac{Na^\mu n_\mu}{2\pi} \right\} \equiv \frac{1}{2}k_B \int_{\partial V} dn T_{\text{loc}} \quad (15)$$

(where $T_{\text{loc}} = (Na^\mu n_\mu/2\pi)$ is the local acceleration temperature and $\Delta n = \sqrt{\sigma} d^2x/L_P^2$) thereby allowing us to read off the number density of microscopic degrees of freedom. We again see that gravity is holographic in the sense that the number density Δn scales as the proper area $\sqrt{\sigma} d^2x$ of the boundary of the region rather than the volume. (In the case of a gas, we would have got an integral over the volume of the form $dV(dn/dV)$ rather than an area integral.) We also notice that, in Einstein's theory, the number density (dn/dA) is a constant with every Planck area contributing a single degree of freedom.

The true elegance of this result again rests on the fact that it holds true for all Lanczos-Lovelock models! For a Lanczos-Lovelock model with an entropy tensor P_{cd}^{ab} one gets the result

$$E = \frac{1}{2}k_B \int_{\partial V} dn T_{\text{loc}}; \quad \frac{dn}{dA} = \frac{dn}{\sqrt{\sigma} d^{D-2}x} = 32\pi P_{cd}^{ab} \epsilon_{ab} \epsilon^{cd} \quad (16)$$

where ϵ_{ab} is the binormal on the codimension-2 cross-section. All these gravitational theories are holographic and the density of microscopic degrees of freedom encodes information about the theory through the entropy tensor. *I consider these results as the most direct evidence for the emergent paradigm of gravity.*

Lesson 12: One can obtain the Wald entropy for a general theory directly from law of equipartition.

The density of microscopic degrees of freedom obtained in Eq. (16) suggests that the entropy associated with a *general surface* in Lanczos-Lovelock models (or the entropy associated with a horizon in a more general theory) will be proportional to an integral over $P_{cd}^{ab} \epsilon_{ab} \epsilon^{cd}$. That is,⁴

$$S \propto \int_{\partial V} dn \propto \int_{\partial V} 32\pi P_{cd}^{ab} \epsilon_{ab} \epsilon^{cd} \sqrt{\sigma} d^{D-2}x \quad (17)$$

This is precisely the expression for Wald entropy [14] *but we have obtained it using only the equipartition law and as a local statement!*

This comes about because the field equations have a specific relationship with Noether current. Further field equations imply equipartition law while Noether current is related to Wald entropy, thereby connecting all the three. Let me indicate how this comes about by a more direct analysis. In static spacetimes, we have a Killing vector ξ^a corresponding to time translation invariance. If we take $q^a = \xi^a$, the expression for the Noether current is quite simple and we get $J^a = 2\mathcal{R}_b^a \xi^b$. Using the relations $J^a \equiv \nabla_b J^{ab}$, $\xi^a = Nu^a$ and the antisymmetry of J^{ab} one can easily show that:

$$D_\alpha (J^{b\alpha} u_b) = 2N \mathcal{R}_{ab} u^a u^b \quad (18)$$

This is a generalization of the relation $D_\mu (Na^\mu) = 4\pi \rho_{\text{komar}}$ between the divergence of the acceleration and the Komar energy density in Einstein's theory, once again showing the role

⁴ Note that in Einstein's theory, we get $\Delta n = \Delta A/L_P^2$. One usually considers this as arising due to dividing the area ΔA into Δn patches of area L_P^2 . If we attribute f internal states to each patch, then the total number of microstates $\Delta\Omega$ will be $\Delta\Omega = f^{(\Delta n)}$ and $\Delta S = \ln \Delta\Omega \propto \Delta n$ which is how the extensivity $\Delta S \propto \Delta n$ arises. In a more general theory, we replace $\Delta n = \Delta A/L_P^2$ by the expression in Eq. (16).

of Noether potential J^{ab} in the dynamics. The integral version of this relation for a region \mathcal{V} bounded by $\partial\mathcal{V}$ is:

$$\int_{\partial\mathcal{V}} d^{D-2}x \sqrt{\sigma} (n_i u_b J^{bi}) = \int_{\partial\mathcal{V}} d^{D-2}x \sqrt{\sigma} (N n_\alpha J^{\alpha 0}) = \int_{\mathcal{V}} 2N \mathcal{R}_{ab} u^a u^b \sqrt{h} d^{D-1}x \quad (19)$$

where we have used $u_a = -N \delta_a^0$ and $J^{0\alpha} = -J^{\alpha 0}$. (The middle relation shows that the result is essentially an integral over $\partial\mathcal{V}$ of $J^{bi} d\sigma_{ib}$, where $d\sigma_{ib} = (1/2) n_{[i} u_{b]} \sqrt{\sigma} d^{D-2}x$.) Now consider a static spacetime with a bifurcation horizon \mathcal{H} given by the surface $N^2 \equiv -\xi^a \xi_a = 0$. The horizon temperature $T \equiv \beta^{-1} = \kappa/2\pi$ where κ is the surface gravity. Since the Wald entropy of the horizon is essentially the Noether charge (multiplied by β), we will interpret [22] the Noether charge *density* $\beta J_b u^b$ (multiplied by β) as the entropy density of the spacetime as perceived by the static observers with four velocity $u^a = \xi^a/N$, so that the total entropy is

$$S_{\text{grav}}[u^i] = \beta \int_{\mathcal{V}} J_b u^b \sqrt{h} d^{D-1}x \quad (20)$$

Using $J^a = 2\mathcal{R}_b^a \xi^b$ and Eq. (19) and integrating the expression over a region bounded by the $N = \text{constant}$ surface, it is easy to see that

$$S = \frac{1}{2} \beta E \quad (21)$$

which is a statement of equipartition, first obtained [32] in 2004 in the form of a relation $E = 2TS$ in Einstein's theory and is generalized to all Lanczos-Lovelock models in ref.[31]. Further, if we take $\partial\mathcal{V}$ to be the horizon \mathcal{H} and use $\beta T = 1$, we get the horizon entropy to be

$$S = \frac{1}{4} \int_{\mathcal{H}} dn = \frac{1}{4} \int_{\mathcal{H}} 32\pi P_{cd}^{ab} \epsilon_{ab} \epsilon^{cd} \sqrt{\sigma} d^{D-2}x \quad (22)$$

which is the standard expression for Wald entropy in a general theory thereby justifying the choice in Eq. (20). This ansatz in Eq. (20) also fixes the proportionality constant in Eq. (17) to be 1/4.

Our expressions for entropy and energy in differential form are given by $dE_{\text{komar}} = (1/2) T_{\text{loc}} (dn/dA) dA$, $dS = (1/4) (dn/dA) dA$. The resulting expression for TdS is essentially equivalent to what we found earlier in the case of first law, $TdS = dE_g + PdV$, applied to infinitesimal horizon displacements when the differentials appearing in the two expressions are properly related.

Lesson 13: Gravity is intrinsically quantum mechanical at all scales

The holographic nature of gravity which I have alluded to several times shows that area elements play a significant role in the microscopic description of the theory. This is directly related to the fact that the basic unit of the theory is the Planck *area* $\mathcal{A}_P \equiv (G\hbar/c^3)$. Only by taking a square root, rather artificially, one obtains the Planck *length*. Classical gravity, in fact, should be described using \mathcal{A}_P rather than using G with Newton's law of gravity written in the form $F = (\mathcal{A}_P c^3/\hbar)(m_1 m_2/r^2)$. This has the crucial consequence that one cannot really take $\hbar \rightarrow 0$ limit at fixed \mathcal{A}_P and call it classical gravity. Gravity is intrinsically quantum mechanical at all scales [33] because of the microstructure of spacetime.

As an aside, one may mention that, strictly speaking, normal matter is also intrinsically quantum mechanical at all scales due to the atomic structure. For example, one cannot study classical elasticity, say, by taking the strict, mathematical, limit $\hbar \rightarrow 0$ in a crystal lattice, because such a limit will also make all the electrons in the atom collapse! What we actually do is to keep \hbar nonzero at subatomic scales, ensuring the atomic stability and take the $\hbar \rightarrow 0$ limit for the lattice interactions in the continuum limit to obtain the laws of elasticity. We need to do something analogous to obtain classical spacetime from quantum spacetime.

6. Entropy density of spacetime and its extremisation

So far we have been faithfully following the ‘top-down’ philosophy of starting from known results in classical gravity and obtaining consequences which suggests an alternative paradigm. For example, the results in the last section were obtained by starting from the field equations of the theory, rewriting them in the form of law of equipartition and thus determining the density of microscopic degrees of freedom.

Ultimately, however, we have to start from a microscopic theory and obtain the classical results as a consequence. We know that the thermodynamical behaviour of a normal system can be described by an extremum principle for a suitable potential (entropy, free energy ...) treated as a functional of appropriate variables (volume, temperature, ...). If our ideas related to gravitational theories are correct, it must be possible to obtain the field equations by extremising a suitably defined thermodynamic potential. The fact that null surfaces block information suggests that this thermodynamic potential should be closely related to null surfaces in the spacetime. This expectation turns out to be correct [34].

Lesson 14: Gravitational field equations can be obtained from an alternative, thermodynamic, extremum principle.

Recall that ‘how gravity tells matter to move’ can be determined by demanding the validity of special relativistic laws for *all* locally inertial observers. Similarly, ‘how matter curves spacetime’ can be determined by demanding that the a suitable thermodynamic potential of the microscopic degrees of freedom of the spacetime should be an extremum *all* local Rindler observers. The physical content of this potential (free energy, entropy, enthalpy ...) will depend on the context but the argument works for any one of them. The mathematics involves associating with every null vector field $n^a(x)$ in the spacetime a thermodynamic potential $\mathfrak{S}(n^a)$ which is quadratic in n^a and given by:

$$\mathfrak{S}[n^a] = \mathfrak{S}_{grav}[n^a] + \mathfrak{S}_{matt}[n^a] \equiv - \left(4P_{ab}^{cd} \nabla_c n^a \nabla_d n^b - T_{ab} n^a n^b \right), \quad (23)$$

where P_{ab}^{cd} is a tensor having the symmetries of curvature tensor and is divergence-free in all its indices and T_{ab} is a divergence-free symmetric tensor. (Once we get the field equations we can read off T_{ab} as the matter energy-momentum tensor; the notation anticipates this result). We also know that the P^{abcd} with the assigned properties can be expressed as $P_{ab}^{cd} = \partial L / \partial R_{cd}^{ab}$ where L is the Lanczos-Lovelock Lagrangian and R_{abcd} is the curvature tensor [1]. This choice in Eq. (23) will also ensure that the equations resulting from the entropy extremisation do not contain any derivative of the metric which is of higher order than second. (More general possibilities exist which I will not discuss here.). We now demand that $\delta \mathfrak{S} / \delta n^a = 0$ for the variation of all null vectors n^a with the condition $n_a n^a = 0$ imposed by adding a Lagrange multiplier function $\lambda(x) g_{ab} n^a n^b$ to $\mathfrak{S}[n^a]$. Using

$$\frac{\partial \mathfrak{S}}{\partial (\nabla_c n^a)} = (-8P_{ab}^{cd} \nabla_d n^b); \quad \frac{\partial \mathfrak{S}}{\partial n^a} = 2[T_{ab} + \lambda(x) g_{ab}] n^b \quad (24)$$

the Euler-Lagrange equations reduce to:

$$\nabla_c \left[-8P_{ab}^{cd} \nabla_d n^b \right] = 2[T_{ab} + \lambda(x) g_{ab}] n^b \quad (25)$$

Because of the condition $\nabla_c P_{ab}^{cd} = 0$ and the antisymmetry $P_{ab}^{cd} = -P_{ab}^{dc}$ we find that all the derivatives disappear on the left hand side and an elementary calculation gives:

$$(2\mathcal{R}_b^a - T_b^a - \lambda \delta_b^a) n_a = 0, \quad (26)$$

where $\mathcal{R}_b^a \equiv F_{bi}^{jk} R_{jk}^{ai}$. We demand that Eq. (26) should hold for all null vector fields n^a . Using the generalized Bianchi identity and the condition $\nabla_a T_b^a = 0$ we obtain [1, 34] from Eq. (26) the equations

$$\mathcal{G}_b^a = \mathcal{R}_b^a - \frac{1}{2}\delta_b^a L = \frac{1}{2}T_b^a + \Lambda\delta_b^a \quad (27)$$

where Λ is a constant. These are precisely the field equations for gravity in a theory with Lanczos-Lovelock Lagrangian L (with an undetermined cosmological constant Λ which arises as an integration constant).

The thermodynamical potential can be obtained by integrating the density $\mathfrak{S}[n^a]$ over a region of space or a surface etc. depending on the context. The matter part of the \mathfrak{S} is proportional to $T_{ab}n^an^b$ which will pick out the contribution $(\rho + p)$ for an ideal fluid, which is the enthalpy density. If multiplied by $\beta = 1/T$, this reduces to the entropy density because of Gibbs-Duhem relation. When the multiplication by β can be reinterpreted in terms of integration over $(0, \beta)$ of the time coordinate, the corresponding potential can be interpreted as entropy and the integral over space coordinates can be interpreted as rate of generation of entropy. [This was the interpretation provided in the earlier works [1, 34] but the result is independent of this interpretation as long as suitable boundary conditions can be imposed]. One can also think of $\mathfrak{S}[n^a]$ as an effective Lagrangian for a set of collective variables n^a describing the deformations of null surfaces.

In addition to providing a purely thermodynamic extremum principle for the field equations of gravity, the above approach also has the following attractive features.

- The extremum value of the thermodynamic potential, when computed on-shell for a solution with static horizon, leads to the Wald entropy. This is a non-trivial consistency check on the approach because it was not designed to reproduce the Wald entropy. It also shows that when the field equations hold, the total entropy of a region \mathcal{V} resides on its boundary $\partial\mathcal{V}$ which is yet another illustration of the holographic nature of gravity.
- In the semi-classical limit, one can show [35] that the gravitational (Wald) entropy is quantized with $S_{\text{grav}}[\text{on-shell}] = 2\pi n$. In the lowest order Lanczos-Lovelock theory, the entropy is proportional to area and this result leads to area quantization. More generally, it is the gravitational entropy that is quantized. The law of equipartition for the surface degrees of freedom is closely related to this entropy quantization because both arise from the existence of discrete structures on the surfaces in question.
- The entropy functional in Eq. (23) is invariant under the shift $T_{ab} \rightarrow T_{ab} + \rho_0 g_{ab}$ which shifts the zero of the energy density. This symmetry allows any low energy cosmological constant, appearing as a parameter in the variational principle, to be gauged away thereby alleviating the cosmological constant problem to a great extent [36]. I will not discuss this issue here.

There is another way of interpreting Eq. (26) which is more in tune with the emergent perspective of gravity. Note that, while Eq. (26) holds for any vector field once the normalization condition is imposed through the Lagrange multiplier, the entropy was originally attributed to *null* vectors and hence it is natural to study Eq. (26) when $n^a = \ell^a$, the null normal of a null surface \mathcal{S} in the spacetime and project Eq. (26) onto the null surface. If ℓ is the normal to \mathcal{S} , then such a projection leads to the equations:

$$R_{mnl}\ell^m q_a^n = 8\pi T_{mn}\ell^m q_a^n; \quad R_{mnl}\ell^m \ell^n = 8\pi T_{mn}\ell^m \ell^n \quad (28)$$

where $q_{ab} = g_{ab} + \ell_a k_b + \ell_b k_a$ with k^a being another auxiliary null vector satisfying $\ell \cdot k = -1$. The metric q_{ab} with $q_{ab}\ell^b = 0 = q_{ab}k^b$ acts as a projector to \mathcal{S} (see ref. [37] for details). It is possible to rewrite the first equation in Eq. (28) in the form of a Navier-Stokes equation

thereby providing a hydrodynamic analogy for gravity. This equation, known in the literature as Damour-Navier-Stokes (DNS) equation [38], is usually derived by rewriting the field equations. Our analysis [37] provides an entropy extremisation principle for the DNS equation which makes the hydrodynamic analogy natural and direct.

It may also be noted that the gravitational entropy density — which is the integrand $\mathfrak{S}_{grav} \propto (-P_{ab}^{cd} \nabla_c \ell^a \nabla_d \ell^b)$ in Eq. (23) — obeys the relation:

$$\frac{\partial \mathfrak{S}_{grav}}{\partial (\nabla_c \ell^a)} \propto (-P_{ab}^{cd} \nabla_d \ell^b) \propto (\nabla_a \ell^c - \delta_a^c \nabla_i \ell^i) \quad (29)$$

where the second relation is for Einstein's theory. This term is analogous to the more familiar object $t_a^c = K_a^c - \delta_a^c K$ (where K_{ab} is the extrinsic curvature) that arises in the (1+3) separation of Einstein's equations. (More precisely, the projection to 3-space leads to t_a^c .) This combination can be interpreted as a surface energy momentum tensor in the context of membrane paradigm [39] because t_{ab} couples to δh^{ab} on the boundary surface when we vary the gravitational action (see, e.g., eq.(12.109) of [20]). Equation (29) shows that the entropy density of spacetime is directly related to t_a^c and its counterpart in the case of null surface. This term also has the interpretation as the canonical momentum conjugate to the spatial metric in (1+3) context and Eq. (29) shows that the entropy density leads to a similar structure. That is, the canonical momentum conjugate to metric in the conventional approach and the momentum conjugate to ℓ^a in S_{grav} are essentially the same.

Further, the *functional* derivative of the gravitational entropy in Eq. (23) has the form, in any Lanczos-Lovelock model:

$$\frac{\delta \mathfrak{S}_{grav}}{\delta \ell^a} \propto \mathcal{R}_{ab} \ell^b \propto J_a \quad (30)$$

Previous discussion has shown that the current $J_a = 2\mathcal{R}_{ab} \ell^b$ plays a crucial role in interpreting gravitational field equations as entropy balance equations. In the context of local Rindler frames, when ℓ^a arises as a limit of the time-like Killing vector in the local Rindler frame, J_a can be interpreted as the Noether (entropy) current associated with the null surface. In that case, the generalization of the two projected equations in Eq. (28) to Lanczos-Lovelock models will read as

$$J_a \ell^a = \frac{1}{2} T_{ab} \ell^a \ell^b; \quad J_a q_m^a = \frac{1}{2} T_{ab} \ell^a q_m^b \quad (31)$$

which relate the gravitational entropy density and flux to matter energy density and momentum flux. (The second equation in the above set becomes the DNS equation in the context of Einstein's theory.) All these results, including the DNS equation, will have direct generalization to Lanczos-Lovelock models which can be structured using the above concepts. We again see that all these ideas find a natural home in the emergent paradigm.

7. Concluding comments

As promised, I have presented the internal evidence hidden in the structure of classical gravitational theories which suggest that gravity is an emergent phenomenon. This evidence brings up the holographic nature of gravity in more than one way (surface density of microscopic degrees of freedom, structure of gravitational action functionals), provides a thermodynamic interpretation to field equations (field equations reducing to $TdS = dE_g + PdV$ on the horizons, entropy balance for virtual displacements of horizons, equipartition), allows one to explicitly determine the number density of microscopic degrees of freedom and — finally — derive the field equations from an entropy maximization procedure. The approach also clarifies several issues which have no explanation in conventional procedure and links several ideas together (like e.g. the relation between the diffeomorphism invariance and the entropy of null surfaces). All of these work in any Lanczos-Lovelock model seamlessly without we having to tinker anything.

It is worthwhile to list explicitly the questions which have natural answers in the emergent paradigm while have to be treated as algebraic accidents in the conventional approach:

- (i) While the temperature of the horizon can be obtained using QFT in curved spacetime, the *corresponding* entanglement entropy is divergent and meaningless. Why?
- (ii) The temperature of horizon is independent of the field equations of gravity but the entropy of the horizon depends explicitly on the field equations. What does this difference signify?
- (iii) The horizon entropy can be expressed in terms of the Noether current which is conserved due to diffeomorphism invariance. Why should an infinitesimal coordinate transformation $x^a \rightarrow x^a + q^a$ have anything to do with a thermodynamic variable like entropy?
- (iv) Why does gravitational field equations (which does not look very “thermodynamical”!) reduce to $TdS = dE_g + PdV$ on the horizon, picking up the correct expression for S for a wide class of theories?
- (v) How come all gravitational action principle have a surface and bulk term which are related in a specific manner (see Eq. (12))? Why do the surface and the bulk terms allow the interpretation as entropy and energy in static spacetimes?
- (vi) The field equations for gravity can be obtained from the bulk part of the action after discarding the surface term. But the surface term evaluated on the horizon of a solution gives the entropy of the horizon! How does the surface term — which was discarded before the field equations were obtained — know about the entropy of a solution?
- (vii) Why does the gravitational field equations reduce to the equipartition form, expressible as $\Delta E = (1/2)(k_B T)\Delta n$ allowing us to determine the analog of Avogadro’s number for the spacetime? And, why does the relevant microscopic degrees of freedom for a region reside on the boundary of the region?
- (viii) Finally, why is it possible to derive the field equations of any diffeomorphism invariant theory of gravity by extremizing an entropy functional associated with the null surfaces in the spacetime, without treating the metric as a dynamical variable?

Obviously, any alternative perspective, including the conventional approach, need to provide the answers for the above questions if they have to be considered a viable alternative to emergent paradigm. The explanations need to work for all Lanczos-Lovelock models and not for just Einstein’s theory. I think the emergent paradigm scores on all these counts and provides valuable insights into the deeper structure of the theory.

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