function coming from the Weyl tensor. In Kerr we find (using Boyer–Lindquist coordinates[11])

$$K_{pw} = (\alpha - i\beta)\,(r - ia\cos\theta) = K_2 - K_1$$

where $\alpha = (\kappa^0 f^1 - \kappa^1 f^0) + a\sin^2\theta(\kappa^1 f^3 - \kappa^3 f^1)$

and $\beta = (r^2 + a^2)\sin\theta(\kappa^3 f^2 - \kappa^2 f^3) - a\sin\theta(\kappa^0 f^2 - \kappa^2 f^0)$

Also we have **k.f** = 0, and **f** is only determined to within a multiple of **k** (so we can choose $f^0 = 0$ at any fixed point).

For a given disk model we can evaluate α and β at the point where a null ray leaves the disk, from a knowledge of the initial polarisation vector. We then have to solve three linear equations for the three space-like components of **f** at infinity. We find for orthonormal components

$$f^r(\infty) = 0, \quad f^\theta(\infty) = (SK_1 - TK_2)/(S^2 + T^2)$$

$$f^\Phi(\infty) = (-SK_2 - TK_1)/(S^2 + T^2)$$

where $S = (L_z/\sin\theta_0 - a\sin\theta_0)$

and $T = \mathrm{sgn}\,(k^\theta)_\infty (Q - L_z^2\cot^2\theta_0 + a^2\cos^2\theta_0)^{1/2}$

here Q and L_z are constants of motion along the ray. In this way we obtain, after integration over the disk, the results shown in Fig. 1.

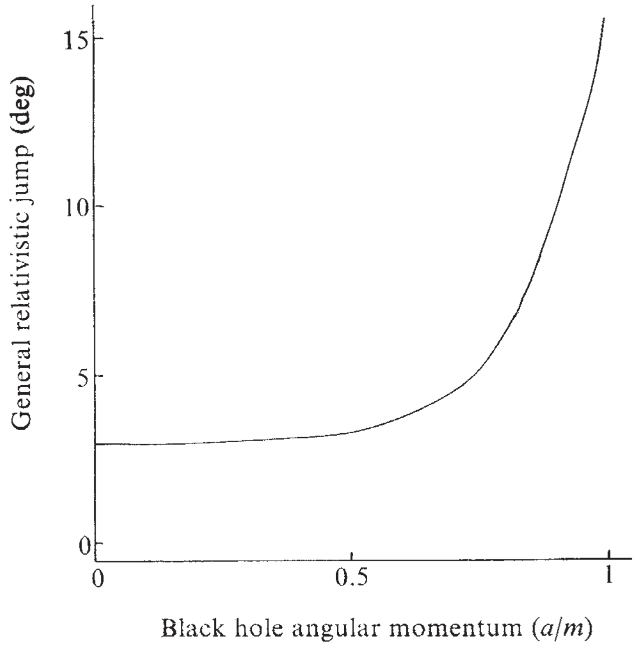

Fig. 3 General-relativistic jump for the two-temperature model, as a function of black hole angular momentum. Same parameters as Fig. 1.

Our previous numerical results for ϕ differ from the exact values by only about 1%.

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Quantum uncertainty in the final state of gravitational collapse

THE ratio of the action S to $\hbar$ (Planck's constant/2π) determines whether the physical system in question is to be treated classically or quantum mechanically. In the area of classical physics the ratio $S/\hbar$ is large compared with unity, and the governing equations are given by $\delta S = 0$. Quantum mechanics begins to be important when $S \lesssim \hbar$, and the definitive approach of classical physics is replaced by quantum uncertainty. We discuss here the behaviour of a physical system which is initially in the classical domain ($S \gg \hbar$) but whose later development may well take it into the region of quantum uncertainty. We consider a specific example of this—the gravitational collapse of a spherical dust ball. While classically such a dust ball ends up in a space–time singularity, the corresponding quantum mechanical result suggests a range of final states some of which are non-singular.

The explicit solution obtained here ignores pressures which have an important role in the gravitational equilibrium of stars. But classical relativity predicts that a space–time singularity develops in a body undergoing gravitational collapse in a fairly general and 'reasonable' set of physical conditions[1,2]. Thus pressures satisfying 'physically reasonable' equations of state are not able to prevent this fate for those stars which become black holes towards the end of their evolution. For such stars S eventually becomes small enough for the above quantum considerations to become important. In these cases the above solution gives a qualitative indication of the probable types of final states.

The classical relativistic equations are derived from the variation of an action given by

$$S = \frac{c^4}{16\pi G}\int_V R(-g)^{1/2}\,d^4x - \sum_a \int m_a\,da \qquad (1)$$

where c is the speed of light, G the gravitational constant, R the scalar curvature of space–time, g the determinant of the metric tensor g_{ik}. The second term of S is the action describing the behaviour of a system of particles labelled by a, with masses m_a. We may add other terms to S to include other possible physical interactions like electromagnetism. V is the 4-volume of space–time under consideration. In the classical approach, we may think of V as the region sandwiched between two chronologically ordered spacelike hypersurfaces Σ_1 and Σ_2. The specification on Σ_1 of a 3-geometry ${}^{(3)}G_1$, and ${}^{(3)}\dot{G}_1$, together with the field equations obtained from $\delta S = 0$ would lead to the determination of the 3-geometry ${}^{(3)}G_2$ on Σ_2 (see ref. 3).

The classical approach is a good one so long as $S \gg \hbar$, where $2\pi\hbar = h$ = Planck's constant. For most situations discussed in general relativity this condition is satisfied. But in the gravitational collapse of a compact object S may become small enough to be of the order $\hbar$ close to the singularity. It therefore becomes necessary to examine this problem from the quantum mechanical point of view. Here we adopt the path integral approach.

In this approach, non-classical geometries, that is, those not satisfying $\delta S = 0$ are permitted and the above sandwich problem is rephrased thus. Given two 3-geometries, ${}^{(3)}G_1$ on Σ_1 and ${}^{(3)}G_2$ on Σ_2 what is the probability amplitude for the system with the given action (1) to go from one to the other? The answer may be expressed as a propagator $K[{}^{(3)}G_2, \Sigma_2; {}^{(3)}G_1, \Sigma_1]$ which is obtained by the Feynman rule of sum over histories[4].

In practice the execution of this project is extremely difficult and has not been done in the general situation. It is, however, possible to obtain an exact solution under a very limited form. Since this

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form gives an insight into the gravitational collapse problem, it is described below in brief.

Following DeWitt[5] we take into consideration only the conformal degrees of freedom. Suppose the classical solution of $\delta S = 0$ is described by a metric $\bar{g}_{ik}$. Consider non-classical solutions which are conformal to the classical one

$$g_{ik} = \Omega^2 \bar{g}_{ik} \tag{2}$$

where Ω is the conformal function. From (2) we get

$$\int_V R(-g)^{1/2} d^4x = \int_V (\Omega^2 \bar{R} - 6\Omega_{;i}\Omega^{;i})(-\bar{g})^{1/2} d^4x + \text{surface term} \tag{3}$$

where on the right hand side the quantities refer to the classical metric. In the summing over histories we now have only the histories of Ω from Σ_1 to Σ_2.

The problem is further simplified for the case of a collapsing homogeneous ball of dust in an otherwise empty space–time[6]. In terms of the comoving coordinates $r\ (\leq r_b)$, θ, Φ, t, the classical solution is given by the line element

$$d\bar{s}^2 = c^2 dt^2 - Q^2(t)\left[\frac{dr^2}{1-\alpha r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{4}$$

in the interior of the ball. The scale factor $Q(t)$ is taken to be unity at the start of the collapse when the density was ρ_0. The Einstein field equations $\delta S = 0$ give

$$\dot{Q}^2 = c^2\alpha\left(\frac{1-Q}{Q}\right), \qquad \alpha = \frac{8\pi G\rho_0}{3c^2} \tag{5}$$

The singularity ($Q = 0$) develops in a t-interval $\pi/(2c\alpha^{1/2})$. We will denote this instant by $t = 0$. As we are interested in the final stages of collapse, we will approximate the solution of (5) by

$$Q = \left(\frac{3c}{2}\alpha^{1/2}\right)^{2/3} T^2, \qquad T = (-t)^{1/3} \tag{6}$$

Consider now the quantum fluctuations which preserve the symmetries of the problem. These are the conformal fluctuations for which Ω is a function of t only. We denote Σ_1 and Σ_2 by $t = t_1$, ($T = T_1$) and $t = t_2$, ($T = T_2$) respectively. Following the techniques of path integration[4] it is possible to compute K exactly. The answer is best expressed in terms of the departure from the classical solution ($\Omega = 1$)

$$\Phi = \Omega - 1 \tag{7}$$

Given $\Phi = \Phi_1$ on Σ_1 and $\Phi = \Phi_2$ on Σ_2 we have

$$K(\Phi_2, T_2; \Phi_1, T_1) = \left(\frac{3iV\rho_0 c^2}{4\pi\hbar}\frac{T_1^{\ 2}T_2^{\ 2}}{T_1 - T_2}\right)^{1/2} \times \exp\left\{\frac{3iV\rho_0 c^2}{4\hbar(T_1 - T_2)}[T_1^{\ 3}(T_1 - 2T_2)\Phi_1^{\ 2} + (T_2 - 2T_1)T_2^{\ 3}\Phi_2^{\ 2} + 2T_1^{\ 2}T_2^{\ 2}\Phi_1\Phi_2]\right\} \tag{8}$$

where V is the coordinate volume of the ball.

This kernel has the reproducing property and its significance can be seen in the following way. Suppose at Σ_1 the state of the dust ball is described by a wave packet of dispersion Δ_1 in Φ_1

$$\Psi(\Phi_1, \Delta_1) = \left(\frac{1}{2\pi\Delta_1^{\ 2}}\right)^{1/4} \exp\left(-\frac{\Phi_1^{\ 2}}{4\Delta_1^{\ 2}}\right) \tag{9}$$

where $\Delta_1 \ll 1$. Then at Σ_2, with $|t_2| \ll |t_1|$, the state is described by a wave packet with a mean $\langle\Phi_2\rangle = 0$ and a dispersion

$$\Delta_2 = \left(\frac{\hbar}{3V\rho_0 c^2 T_1 \Delta_1}\right)\left\{1 + \left(\frac{3V\rho_0\Delta_1^{\ 2}c^2}{\hbar}\right)^2 T_1^{\ 6}\right\}^{1/2} T_2^{\ -2} \tag{10}$$

Thus, however small Δ_1 may be, equation (10) shows that Δ_2 diverges as $T_2 \to 0$, ($t_2 \to 0$). In other words, although the wave function at t_2 continues to have the classical solution ($\Phi_2 = 0$) as the mean, the spread around this solution gets progressively larger as the so-called classical singular epoch is approached: (10) effectively tells us where the quantum uncertainty begins to dominate. For a solar mass dust ball with $\rho_0 = 1\ \text{g cm}^{-3}$, the linear shrinkage must factor 10^{-43} before the quantum uncertainty takes over.

Although the smallness of this ratio indicates the range over which the classical theory is valid, the ultimate dominance of quantum uncertainty as $t \to 0$ seems inescapable.

This solution can also be used to discuss the early stages of a Friedmann universe. A few years ago Hoyle and Narlikar[7] had suggested that the quantum fluctuations in a classical big bang model might produce models without particle horizons (which inhibit the transfer of information). It is interesting to note that the range of uncertainty indicated by Δ_2 permits such models.

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Origin of diffuse interstellar lines

SINCE their discovery over 40 years ago, the origin of the diffuse interstellar absorption lines has been a mystery although there have been many suggestions put forward. Developments in spectroscopy and astronomy have intensified rather than reduced the mystery. This letter takes into account some of these recent developments and suggests a rather definite (but unconfirmed) identification of the lines. Excellent surveys of the diffuse interstellar lines have been given by Herbig[1] and by Wu[2], so the many studies of these lines will not be mentioned again. For our purposes, it is sufficient to note that the strongest line which lies at 4,428 Å has a half width of 20 Å, and that 38 weaker lines of varying strengths and widths lie at longer wavelengths. The strength of the lines is strongly correlated with regions in space containing interstellar grains (particularly small grains) and there is strong evidence that all the lines arise from a single species or from closely related species. The only feature in the spectrum lying to the violet of 4,428 Å which seems to be related to the diffuse lines is a continuum in the 2,200 Å region but this must certainly involve much higher electronic states of the absorbing species.

The line widths of the diffuse lines are the unique characteristics which differentiate them from all identified interstellar lines. The line widths which vary between 1 and

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