Data analysis of continuous gravitational wave: Fourier transform-II

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Abstract

In this paper we obtain the Fourier Transform of a continuous gravitational wave. We have analysed the data set for (i) one year observation time and (ii) arbitrary observation time, for arbitrary location of detector and source taking into account the effects arising due to rotational as well as orbital motion of the earth. As an application of the transform we considered spin down and N-component signal analysis.

1 Introduction

Detection of gravitational waves (GW) mainly depends on the efficient Fourier analysis of the data of the output of the detector such as LIGO/VIRGO. In an earlier paper we presented Fourier analysis of one day observation data set of the response of a laser interferometer (Srivastava and Sahay, 2002 a). Hereafter the reference to this paper and its results are noted by the prefix I. We have seen that the amplitude and the frequency modulations result into a large number of side bands about the signal frequency f_o. Consequently, the maximum power lies in the frequency $f_o + 2f_{rot}$ with amplitude reduction by 74% to what one would have expected due to increased data interval. Here f_{rot} is the rotational frequency of the earth. Hence, for GW detection it is desirable to obtain FT for longer observation data. In the next section we investigate one year observation data. Incidently there exists correspondences and identifications of the present analysis to the analysis of paper-I. To facilitate analogous modifications we introduce corresponding quantities with

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tilde viz., $\tilde{\mathcal{C}}$ and $\tilde{\mathcal{D}}$ in place of $\mathcal{C}$ and $\mathcal{D}$. We obtain in section 3 generalisations of the results for arbitrary observation time. As an application of the results obtained we consider spin down and N-component signal analysis in sections 4 and 5. Last section contains the discussion and conclusions of the paper.

2 Fourier transform for one year integration

The phase of the GW signal of frequency f_o at time t is given via (I-25) i.e.

$$\Phi(t) \; = \; 2\pi f_o t + \mathcal{Z}\cos(w_{orb}t - \phi) + \mathcal{N}\cos(w_{rot}t - \delta) - \mathcal{R} - \mathcal{Q} \tag{1}$$

where

$$\left.\begin{aligned}
\mathcal{P} &= 2\pi f_o \tfrac{R_e}{c}\sin\alpha(\cos\beta_o(\sin\theta\cos\epsilon\sin\phi + \cos\theta\sin\epsilon) - \sin\beta_o\sin\theta\cos\phi)\,,\\
\mathcal{Q} &= 2\pi f_o \tfrac{R_e}{c}\sin\alpha(\sin\beta_o(\sin\theta\cos\epsilon\sin\phi + \cos\theta\sin\epsilon) + \cos\beta_o\sin\theta\cos\phi)\,,\\
\mathcal{N} &= \sqrt{\mathcal{P}^2 + \mathcal{Q}^2}\,,\\
\mathcal{Z} &= 2\pi f_o \tfrac{R_{se}}{c}\sin\theta\,,\\
\mathcal{R} &= \mathcal{Z}\cos\phi\,,
\end{aligned}\right\} \tag{2}$$

$$\left.\begin{aligned}
\delta &= \tan^{-1}\tfrac{\mathcal{P}}{\mathcal{Q}}\,,\\
\phi' &= w_{orb}t - \phi\,,\\
\xi_{orb} &= w_{orb}t \; = \; a\xi_{rot}; \quad a \; = \; w_{orb}/w_{rot} \; \approx \; 1/365.26\,,\\
\xi_{rot} &= w_{rot}t
\end{aligned}\right\} \tag{3}$$

where R_e, R_{se}, w_{rot} and w_{orb} represent respectively the earth's radius, average distance of earth's centre from the origin of Solar System Barycentre (SSB) frame, the rotational and the orbital angular velocity of the earth. θ and ϕ denote the celestial colatitude and celestial longitude of the source. ϵ and c represent the obliquity of the ecliptic and the velocity of light. α is the colatitude of the detector. Here t represents the time in seconds elapsed from the instant the sun is at the Vernal Equinox and β_o is the local sidereal time at that instant, expressed in radians.

The Fourier transform (FT) for one year observation time, T_{obs} is given as

$$\left[\tilde{h}(f)\right]_y = \int_0^{\bar{a}T} \cos[\Phi(t)]e^{-i2\pi ft}dt\;; \tag{4}$$

$$\bar{a} = a^{-1} = w_{rot}/w_{orb}\;; \quad T = \; one\ sidereal\ day\;; \tag{5}$$

$$T_{obs} = \bar{a}T \simeq 3.14 \times 10^7 \ sec. \tag{6}$$

This splits, similar to Eqs. (I-33-35), into two terms as

$$\left[\tilde{h}(f)\right]_y = I_{\bar{\nu}_-} + I_{\bar{\nu}_+} \ ; \tag{7}$$

$$\begin{aligned}
I_{\bar{\nu}_-} &= \frac{1}{2w_{orb}} \int_0^{2\pi} e^{i\left[\bar{\xi}\bar{\nu}_- + \mathcal{Z}\cos(\bar{\xi}-\phi) + \mathcal{N}\cos(\bar{a}\bar{\xi}-\delta) - \mathcal{R} - \mathcal{Q}\right]} d\bar{\xi}\,, && (8)\\
I_{\bar{\nu}_+} &= \frac{1}{2w_{orb}} \int_0^{2\pi} e^{-i\left[\bar{\xi}\bar{\nu}_+ + \mathcal{Z}\cos(\bar{\xi}-\phi) + \mathcal{N}\cos(\bar{a}\bar{\xi}-\delta) - \mathcal{R} - \mathcal{Q}\right]} d\bar{\xi}\,, && (9)\\
\bar{\nu}_{\mp} &= \frac{f_o \mp f}{f_{orb}}; \quad \bar{\xi} \ = \ \xi_{orb} \ = \ w_{orb} t && (10)
\end{aligned}$$

where f_{orb} is the orbital frequency of the earth. Hereafter, we neglect $I_{\bar{\nu}_+}$ as it oscillates rapidly and contributes very little to $\left[\tilde{h}(f)\right]_y$ and write $\bar{\nu}$ in place of $\bar{\nu}_-$. A careful comparison of Eq. (8) with (I-33) i.e.

$$I_{\nu_-} = \frac{1}{2w_{rot}} \int_0^{2\pi} e^{i[\xi\nu_- + \mathcal{Z}\cos(a\xi-\phi) + \mathcal{N}\cos(\xi-\delta) - \mathcal{R} - \mathcal{Q}]} d\xi \tag{11}$$

obtained for one day observation data reveals that the integrand of the equations are identical with following identifications and correspondences.

$$\left.\begin{array}{ccc}
\delta & \leftrightarrow & \phi\,, \\
\mathcal{Z} & \leftrightarrow & \mathcal{N}\,, \\
a & \leftrightarrow & \bar{a}.
\end{array}\right\} \tag{12}$$

We may, therefore, employ the results obtained there by introducing obvious corresponding quantities i.e. $\bar{\mathcal{B}}$, $\bar{\mathcal{C}}$, $\bar{\mathcal{D}}$ in place of $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ leaving $\mathcal{A}$ unchanged. Hence

$$\left[\tilde{h}(f)\right]_y \simeq \frac{\bar{\nu}}{2w_{orb}} \sum_{k=-\infty}^{k=\infty} \sum_{m=-\infty}^{m=\infty} e^{i\mathcal{A}} \bar{\mathcal{B}} [\bar{\mathcal{C}} - i\bar{\mathcal{D}}] \ ; \tag{13}$$

$$\begin{aligned}
\mathcal{A} &= \tfrac{(k+m)\pi}{2} - \mathcal{R} - \mathcal{Q}\,, \\
\bar{\mathcal{B}} &= \tfrac{J_k(\mathcal{N}) J_m(\mathcal{Z})}{\bar{\nu}^2 - (\bar{a}k+m)^2}\,, \\
\bar{\mathcal{C}} &= \sin 2\bar{\nu}\pi \cos(2\bar{a}k\pi - k\delta - m\phi) - \tfrac{\bar{a}k+m}{\bar{\nu}} \{\cos 2\bar{\nu}\pi \sin(2\bar{a}k\pi - k\delta - m\phi) + \sin(k\delta + m\phi)\}\,, \\
\bar{\mathcal{D}} &= \cos 2\bar{\nu}\pi \cos(2\bar{a}k\pi - k\delta - m\phi) + \tfrac{k\bar{a}+m}{\bar{\nu}} \sin 2\bar{\nu}\pi \sin(2\bar{a}k\pi - k\delta - m\phi) - \cos(k\delta + m\phi)
\end{aligned}$$

where J stands for the Bessel function of first kind. Now the FT of the two polarisation states can be written as

$$
\begin{aligned}
\left[\tilde{h}_+(f)\right]_y &= h_{o_+}\left[\tilde{h}(f)\right]_y \\
&\simeq \frac{\bar{\nu} h_{o_+}}{2w_{orb}} \sum_{k=-\infty}^{k=\infty} \sum_{m=-\infty}^{m=\infty} e^{i\mathcal{A}} \bar{\mathcal{B}}[\bar{\mathcal{C}} - i\bar{\mathcal{D}}]
\end{aligned} \tag{15}
$$

and

$$
\begin{aligned}
\left[\tilde{h}_\times(f)\right]_y &= -ih_{o_\times}\left[\tilde{h}(f)\right]_y \\
&\simeq \frac{\bar{\nu} h_{o_\times}}{2w_{orb}} \sum_{k=-\infty}^{k=\infty} \sum_{m=-\infty}^{m=\infty} e^{i\mathcal{A}} \bar{\mathcal{B}}[\bar{\mathcal{D}} - i\bar{\mathcal{C}}]
\end{aligned} \tag{16}
$$

The FT obtained contains double series of the Bessel functions of the orders k and m ranging from $-\infty$ to ∞. It is well known that the Bessel functions decrease rapidly as the order exceeds the argument. Hence possible range of k and m over which the summation of the series is to be considered depends on the arguments of Bessel functions i.e $\mathcal{Z}$ and $\mathcal{N}$. Referring to Eq. (2) it is found that

$$
\left.
\begin{aligned}
\mathcal{Z}_{max} &= 3133215\left(\tfrac{f}{1\,KHz}\right) \\
\mathcal{N}_{max} &= 134\left(\tfrac{f}{1\,KHz}\right)
\end{aligned}
\right\} \tag{17}
$$

The FT of a FM signal for

$$
\left.
\begin{array}{lll}
f_o = 50\ Hz\,, & h_{o_+} = h_{o_\times} = 1 & \\
\alpha = \pi/4\,, & \beta_o = 0\,, & \gamma = \pi\,, \\
\theta = \pi/18\,, & \phi = 0\,, & \psi = \pi/4.
\end{array}
\right\} \tag{18}
$$

is shown in Fig. (1). The spectrum resolution is kept equal to $1/T_o$ i.e. 3.17×10^{-8} Hz. We have convinced ourselves by plotting the FT at higher resolutions that the resolution of $1/T_o$ is sufficient to represent the relevant peaks. We notice that the drop in amplitude at the source frequency is about 98% which may be attributed to the presence of a large number of side bands.

The complete response $R(t)$ of the detector and its FT can be obtained similar to the one presented in section 4 of the paper-I. Taking the results straight away from (I-43, 44, 45) one may get

$$
\begin{aligned}
\left[\tilde{R}(f)\right]_y &= \left[\tilde{R}_+(f)\right]_y + \left[\tilde{R}_\times(f)\right]_y \\
&= e^{-i2\beta_o}\left[\tilde{h}(f + 2f_{rot})/2\right]_y \left[h_{o_+}(F_{1_+} + iF_{2_+}) + h_{o_\times}(F_{2_\times} - iF_{1_\times})\right] +
\end{aligned}
$$

$$
\begin{aligned}
&e^{i2\beta_o}\left[\tilde{h}(f-2f_{rot})/2\right]_y\left[h_{o_+}(F_{1_+}-iF_{2_+})-h_{o_\times}(F_{2_\times}+iF_{1_\times})\right]+\\
&e^{-i\beta_o}\left[\tilde{h}(f+f_{rot})/2\right]_y\left[h_{o_+}(F_{3_+}+iF_{4_+})+h_{o_\times}(F_{4_\times}-iF_{3_\times})\right]+\\
&e^{i\beta_o}\left[\tilde{h}(f-f_{rot})/2\right]_y\left[h_{o_+}(F_{3_+}-iF_{4_+})-h_{o_\times}(F_{4_\times}+iF_{3_\times})\right]+\\
&\left[\tilde{h}(f)\right]_y\left[h_{o_+}F_{5_+}-ih_{o_\times}F_{5_\times}\right] \qquad (19)
\end{aligned}
$$

Figure (2) shows the power spectrum of the complete response of the Doppler modulated signal. We have kept here all the parameters same as in FM. In this caee also we observe that maximum power is associated with f_o+2f_{rot} and the least lies in f_o-f_{rot}.

3 Fourier transform for arbitrary observation time

It is important to obtain the FT for arbitrary observation time. The results obtained will be employed to outline how spin down of a pulsar due to the gravitational radiation back reaction or due to some other mechanism can be taken into account.
The FT for a data of observation time T_o is given via

$$
\tilde{h}(f) \;=\; \int_0^{T_o}\cos[\Phi(t)]e^{-i2\pi ft}dt \qquad (20)
$$

which may be split

$$
\tilde{h}(f) \;= I_{\nu_-}+I_{\nu_+}\;; \qquad (21)
$$

$$
\begin{aligned}
I_{\nu_-} &= \frac{1}{2w_{rot}}\int_0^{\xi_o} e^{i[\xi\nu_-+\mathcal{Z}\cos(a\xi-\phi)+\mathcal{N}\cos(\xi-\delta)-\mathcal{R}-\mathcal{Q}]}d\xi\,, &(22)\\
I_{\nu_+} &= \frac{1}{2w_{rot}}\int_0^{\xi_o} e^{-i[\xi\nu_++\mathcal{Z}\cos(a\xi-\phi)+\mathcal{N}\cos(\xi-\delta)-\mathcal{R}-\mathcal{Q}]}d\xi\,, &(23)\\
\nu_\mp &= \frac{f_o\mp f}{f_{rot}};\quad \xi_o \;=\; w_{rot}T_o;\quad \xi \;=\; \xi_{rot} \;=\; w_{rot}t\,, &(24)
\end{aligned}
$$

[refer to Eqs. I (33 - 35)].
As I_{ν_+} contributes very little to $\tilde{h}(f)$ we drop I_{ν_+} and write ν in place of ν_-. Using the identity

$$
e^{\pm i\kappa\cos\vartheta} = J_o(\pm\kappa)+2\sum_{l=1}^{l=\infty} i^l J_l(\pm\kappa)\cos l\vartheta \qquad (25)
$$

we obtain

$$
\begin{aligned}
\tilde{h}(f) \;\simeq\; &\frac{1}{2w_{rot}}e^{i(-\mathcal{R}-\mathcal{Q})}\int_0^{\xi_o} e^{i\nu\xi}\left[J_o(\mathcal{Z})+2\sum_{k=1}^{k=\infty}J_k(\mathcal{Z})i^k\cos k(a\xi-\phi)\right]\\
&\times\left[J_o(\mathcal{N})+2\sum_{m=1}^{m=\infty}J_m(\mathcal{N})i^m\cos m(\xi-\delta)\right]d\xi \qquad (26)
\end{aligned}
$$

After performing the integration and proceeding in a straight-forward manner we have

$$\tilde{h}(f) \simeq \frac{\nu}{2w_{rot}} \sum_{k=-\infty}^{k=\infty} \sum_{m=-\infty}^{m=\infty} e^{i\mathcal{A}} \mathcal{B}[\tilde{\mathcal{C}} - i\tilde{\mathcal{D}}] \; ; \tag{27}$$

$$\left.\begin{array}{rcl} \mathcal{A} & = & \frac{(k+m)\pi}{2} - \mathcal{R} - \mathcal{Q} \\ \mathcal{B} & = & \frac{J_k(\mathcal{Z})J_m(\mathcal{N})}{\nu^2-(ak+m)^2} \\ \tilde{\mathcal{C}} & = & \sin\nu\xi_o\cos(ak\xi_o + m\xi_o - k\phi - m\delta) - \frac{ak+m}{\nu}\{\cos\nu\xi_o\sin(ak\xi_o + m\xi_o - k\phi - m\delta) \\ & & + \sin(k\phi + m\delta)\} \\ \tilde{\mathcal{D}} & = & \cos\nu\xi_o\cos(ak\xi_o + m\xi_o - k\phi - m\delta) + \frac{ka+m}{\nu}\sin\nu\xi_o\sin(ak\xi_o + m\xi_o - k\phi - m\delta) \\ & & - \cos(k\phi + m\delta) \end{array}\right\} \tag{28}$$

The FT of the two polarisation states of the wave can now be written as

$$\begin{array}{rcl} h_+(f) & = & h_{o_+}\tilde{h}(f) \\ & \simeq & \frac{\nu h_{o_+}}{2w_{rot}} \sum_{k=-\infty}^{k=\infty} \sum_{m=-\infty}^{m=\infty} e^{i\mathcal{A}} \mathcal{B}[\tilde{\mathcal{C}} - i\tilde{\mathcal{D}}] \; ; \end{array} \tag{29}$$

$$\begin{array}{rcl} \tilde{h}_\times(f) & = & -ih_{o_\times}\tilde{h}(f) \\ & \simeq & \frac{\nu h_{o_\times}}{2w_{rot}} \sum_{k=-\infty}^{k=\infty} \sum_{m=-\infty}^{m=\infty} e^{i\mathcal{A}} \mathcal{B}[\tilde{\mathcal{D}} - i\tilde{\mathcal{C}}] \end{array} \tag{30}$$

Now it is simple matter to obtain the FT of complete response. One gets

$$\begin{array}{rcl} \tilde{R}(f) & = & e^{-i2\beta_o}\tilde{h}(f+2f_{rot})/2\left[h_{o_+}(F_{1_+} + iF_{2_+}) + h_{o_\times}(F_{2_\times} - iF_{1_\times})\right] + \\ & & e^{i2\beta_o}\tilde{h}(f-2f_{rot})/2\left[h_{o_+}(F_{1_+} - iF_{2_+}) - h_{o_\times}(F_{2_\times} + iF_{1_\times})\right] + \\ & & e^{-i\beta_o}\tilde{h}(f+f_{rot})/2\left[h_{o_+}(F_{3_+} + iF_{4_+}) + h_{o_\times}(F_{4_\times} - iF_{3_\times})\right] + \\ & & e^{i\beta_o}\tilde{h}(f-f_{rot})/2\left[h_{o_+}(F_{3_+} - iF_{4_+}) - h_{o_\times}(F_{4_\times} + iF_{3_\times})\right] + \\ & & \tilde{h}(f)\left[h_{o_+}F_{5_+} - ih_{o_\times}F_{5_\times}\right] \end{array} \tag{31}$$

The FT of FM signal of a detector for a data of 120 days for

$$\left.\begin{array}{lll} f_o = 25\;Hz\,, & h_{o_+} = h_{o_\times} = 1 & \\ \alpha = \pi/6\,, & \beta_o = \pi/3\,, & \gamma = 2\pi/3\,, \\ \theta = \pi/9\,, & \phi = \pi/4\,, & \psi = \pi/4. \end{array}\right\} \tag{32}$$

is plotted in Fig. (3) with a resolution of $1/2T_o \approx 9.67 \times 10^{-8}$ Hz. The Power spectrum of the complete response is plotted in Fig. (4). In this case also the most of the power of the signal lies in the frequency $f_o + 2f_{rot}$ and the least with $f_o - f_{rot}$.

4 Spin down

Pulsars loose their rotational energy by processes like electro-magnetic breaking, emission of particles and emission of GW. Thus, the rotational frequency is not completely stable, but varies over a time scale which is of the order of the age of the pulsar. Typically, younger pulsars have the largest spin down rates. Current observations suggest that spin down is primarily due to electro-magnetic breaking (Manchester, 1992 and Kulkarni, 1992). Over the entire observing time, T_o the frequency drift would be small but it may be taken into account for better sensitivity. To account this aspect we consider the evaluation of FT in a sequence of time windows by splitting the interval $0 - T_o$ in M equal parts, each of interval $\triangle t$ ($T_o = M \triangle t$) such that the signal over a window may be treated as monochromatic. The strategy is to evaluate the FT over the window and finally to add the result. This process has been suggested by Brady and Creighton (2000) and Schutz (1998) in numerical computing and has been termed as *stacking* and *tracking*. For any such window the initial and final times may be taken respectively as $t_o + n \triangle t$ and $t_o + (n+1) \triangle t$ where t_o represents the starting of the data set and $0 \leq n \leq M-1$. The window under consideration is the n_{th} window. Let

$$
\begin{aligned}
I &= \int_{t_o+n\triangle t}^{t_o+(n+1)\triangle t} h(\bar{t}) e^{-i2\pi f \bar{t}} d\bar{t} \\
&= \int_0^{\triangle t} h(t + t_o \; + n \triangle t) e^{-i2\pi(t+t_o+n\triangle t)f} dt \; ; \qquad (33) \\
\bar{t} &= t + t_o + n \triangle t \qquad (34)
\end{aligned}
$$

Hence the FT for the data of the time-window under consideration is given via

$$
\left[\tilde{h}(f)\right]_s = \int_0^{\triangle t} \cos[\Phi(t + t_o + n \triangle t)] e^{-i2\pi(t+t_o+n\triangle t)f} dt \qquad (35)
$$

Taking the initial time of the data set

$$
t_o = 0 \qquad (36)
$$

and proceeding as in the previous section we obtain

$$
\begin{aligned}
\left[\tilde{h}(f)\right]_s \simeq\; & \frac{1}{2w_{rot}} e^{i[2\pi n(f_o-f)\triangle t - \mathcal{R} - \mathcal{Q}]} \int_0^{\triangle t} e^{i\nu\xi} \left[J_o(\mathcal{Z}) + 2\sum_{k=1}^{k=\infty} J_k(\mathcal{Z}) i^k \cos k(a\xi - \lambda) \right] \times \\
& \left[J_o(\mathcal{N}) + 2\sum_{m=1}^{m=\infty} J_m(\mathcal{N}) i^m \cos m(\xi - \zeta) \right] d\xi \qquad (37)
\end{aligned}
$$

After integration we get

$$
\left[\tilde{h}(f)\right]_s = \frac{\nu}{2w_{rot}} \sum_{k=-\infty}^{k=\infty} \sum_{m=-\infty}^{m=\infty} e^{i\mathcal{A}_s} \mathcal{B} \left[\mathcal{C}_s - i\mathcal{D}_s \right] \; ; \qquad (38)
$$

$$
\left.\begin{aligned}
\mathcal{A}_s &= \tfrac{(k+m)\pi}{2} + 2\pi n \triangle t(f_o - f) - \mathcal{R} - \mathcal{Q} \\
\mathcal{B} &= \tfrac{J_k(\mathcal{Z})J_m(\mathcal{N})}{\nu^2-(ak+m)^2}\,, \\
\mathcal{C}_s &= \sin(\nu\tau)\cos(ak\tau + m\tau - k\lambda - m\zeta) - \tfrac{ak+m}{\nu}\{\cos(\nu\tau)\sin(ak\tau + m\tau - k\lambda - m\zeta) \\
&\quad + \sin(k\lambda + m\zeta)\}\,, \\
\mathcal{D}_s &= \cos(\nu\tau)\cos(ak\tau + m\tau - k\lambda - m\zeta) + \tfrac{ak+m}{\nu}\sin(\nu\tau)\sin(ak\tau + m\tau - k\lambda - m\zeta) \\
&\quad - \cos(k\lambda + m\zeta)\,, \\
\lambda &= \phi - an\tau\,, \qquad \zeta \;=\; \delta - n\tau\,, \\
\tau &= w_{rot} \triangle t\,, \qquad n \;=\; 0, 1, 2, 3, ..., M - 1.
\end{aligned}\right\} \tag{39}
$$

The FT of the complete response would now be given via

$$
\begin{aligned}
\left[\tilde{R}(f)\right]_s \;=\;& e^{-i2\beta_o}\left[\tilde{h}(f + 2f_{rot})/2\right]_s \left[h_{o_+}(F_{1_+} + iF_{2_+}) + h_{o_\times}(F_{2_\times} - iF_{1_\times})\right] + \\
& e^{i2\beta_o}\left[\tilde{h}(f - 2f_{rot})/2\right]_s \left[h_{o_+}(F_{1_+} - iF_{2_+}) - h_{o_\times}(F_{2_\times} + iF_{1_\times})\right] + \\
& e^{-i\beta_o}\left[\tilde{h}(f + f_{rot})/2\right]_s \left[h_{o_+}(F_{3_+} + iF_{4_+}) + h_{o_\times}(F_{4_\times} - iF_{3_\times})\right] + \\
& e^{i\beta_o}\left[\tilde{h}(f - f_{rot})/2\right]_s \left[h_{o_+}(F_{3_+} - iF_{4_+}) - h_{o_\times}(F_{4_\times} + iF_{3_\times})\right] + \\
& \left[\tilde{h}(f)\right]_s \left[h_{o_+}F_{5_+} - ih_{o_\times}F_{5_\times}\right]
\end{aligned} \tag{40}
$$

5 N-component signal

The FT in Eqs. (27) and (31) are valid for a pulsar which emits GW signal at single frequency f_o. But there are known physical mechanisms which generate GW signals consisting of many components. An axially symmetric pulsar undergoing free precession, emits quadrupole GW at two frequencies, one equal to the sum of the spin frequency and the precession frequency, and the other twice of it (Zimmermann and Szedenits, 1979). The quadrupole GW from a triaxial ellipsoid rotating about one of its principal axes consists of one component only (Thorne, 1987). In this case the signal has frequency twice the spin frequency of the star. In general, if a star is non-axisymmetric and precesses, the GW signal consists of more than two components. Recently, new mechanisms e.g. r-mode instability of spinning neutron stars (Anderson, 1998; Lindblom et al., 1998; Owen, et al., 1998) and temperature asymmetry in the interior of the neutron star with miss-aligned spin axis (Bildsten, 1988) have been discussed in the literature.

In view of the above discussion CGW signal may consist of frequencies which are multiple of some basic frequencies. An analysis of the GW data of N-component of signal has been made recently by Jaranowski and Królak (2000). We do not continue this analysis as the requisite formalism is analogous to what we have presented in paper-I and the earlier sections of this paper

6 Discussion and conclusions

The analysis and results obtained in paper-I regarding FT of the response of a Laser Interferometer have been generalised in this paper. In this context following points must be noted.

1. For 120 days observation time data the resolution provided by FFT, which is equal to $1/T_o$, is sufficient to represent the structure of side bands.

2. In every case discussed, it turned out that the maximum power lies in the frequency $f_o + 2f_{rot}$. However, this is not established conclusively if this result is generic.

3. Throughout our analysis in paper-I and this paper we have employed following conditions.

   (i) The phase of the wave is zero at $t = 0$.

   (ii) The observation time of the data set is from $t = 0$ to $t = T_o$.

4. By trivial modifications we could get the general results with non-zero initial time and phase.

It is pointed out that in choosing the parameters for the source and the detectors we have been constrained by the computer memory and efficiency. Accordingly the choice of the parameters has been a matter of convenience. The application of the results obtained in the present and the earlier paper to the problem of all sky search for continuous gravitational wave sources will be made in our forthcoming article (Srivastava and Sahay, 2002 b).

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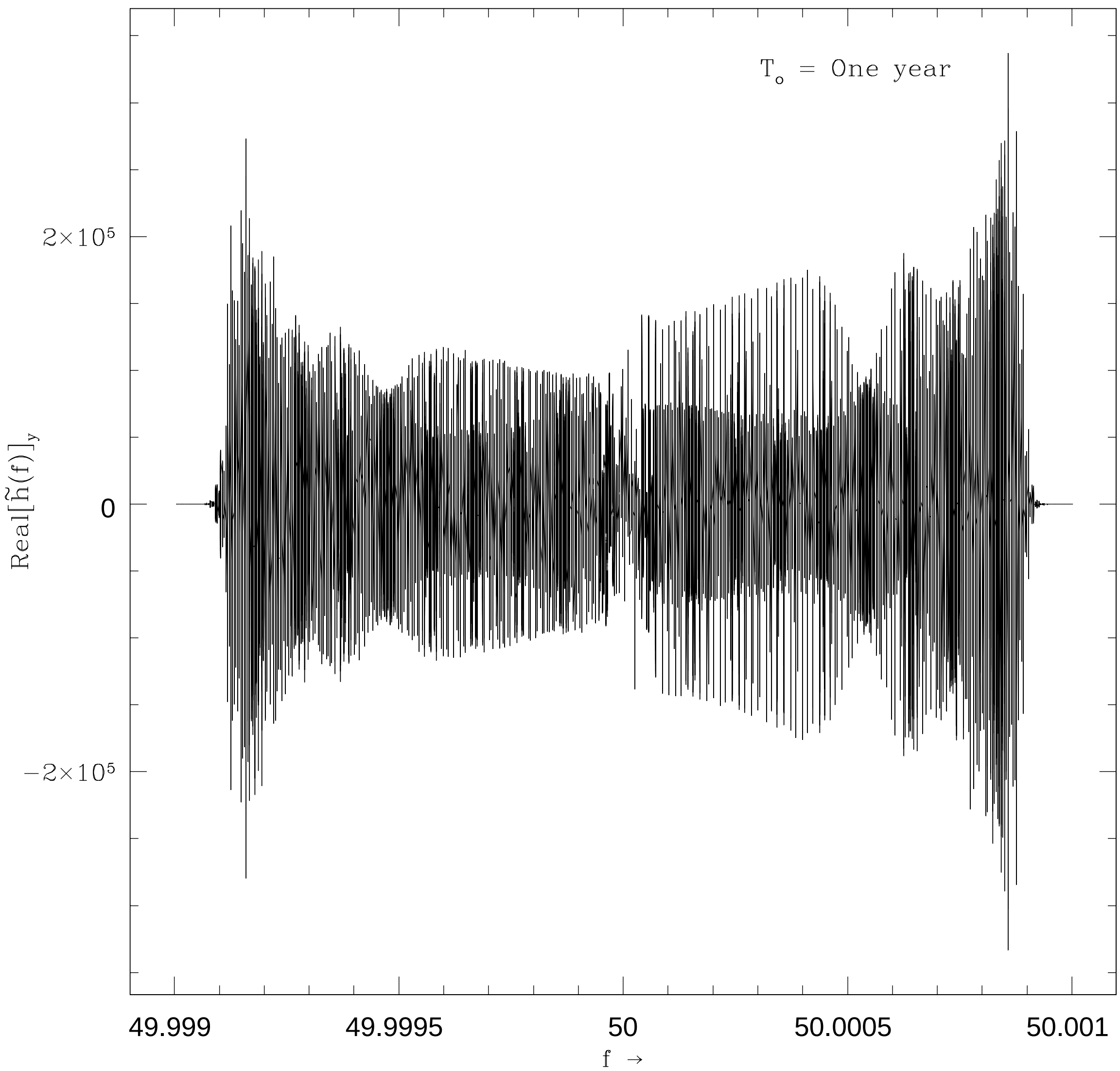

Figure 1: FT of a FM signal of frequency, $f_o = 50$ Hz from a source located at $(\pi/18, 0)$ with a resolution of 3.17×10^{-8}.

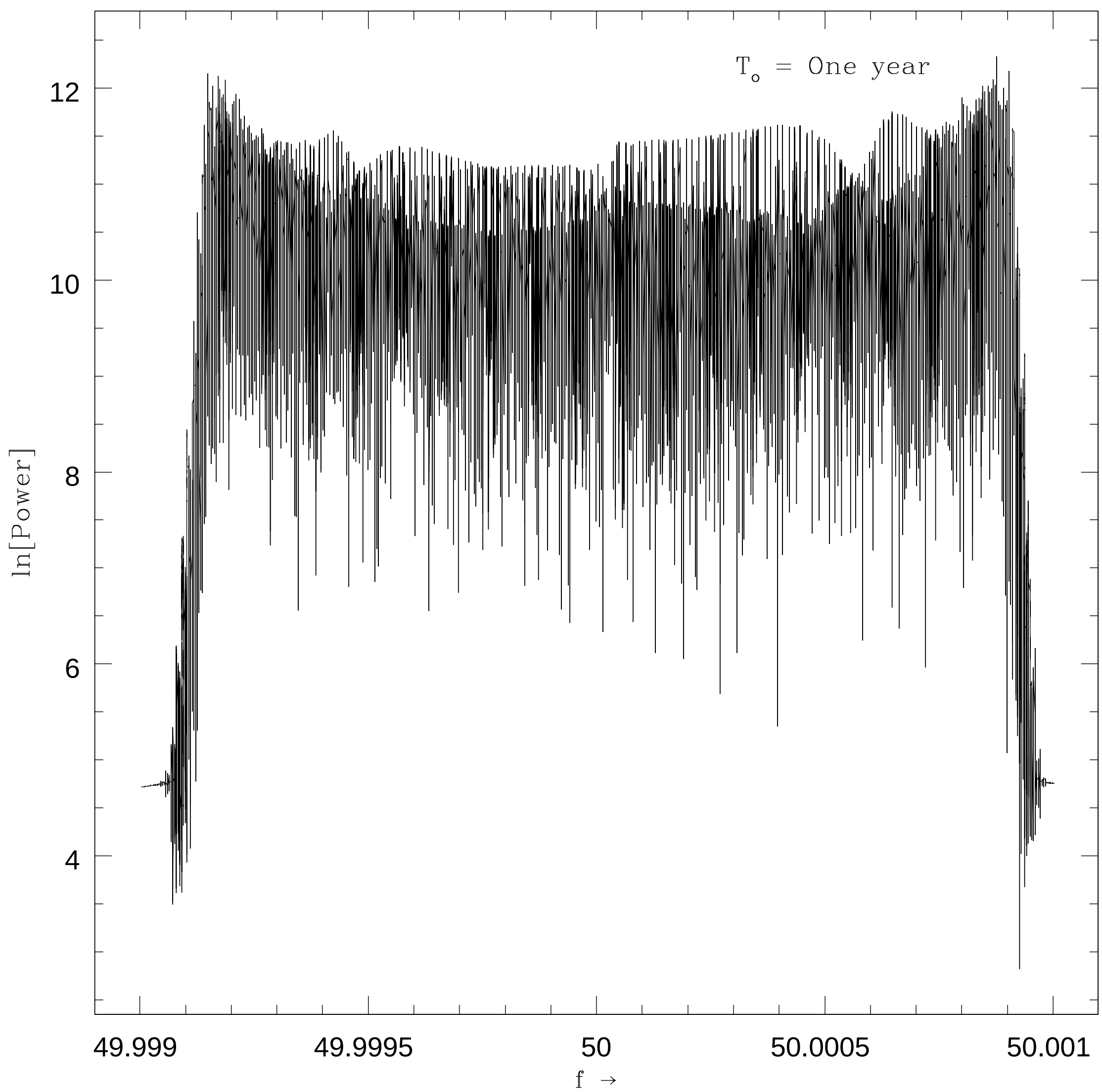

Figure 2: Power spectrum of the complete response of a modulated signal of frequency, $f_o = 50$ Hz from a source located at $(\pi/18, 0)$ with a resolution of 3.17×10^{-8}.

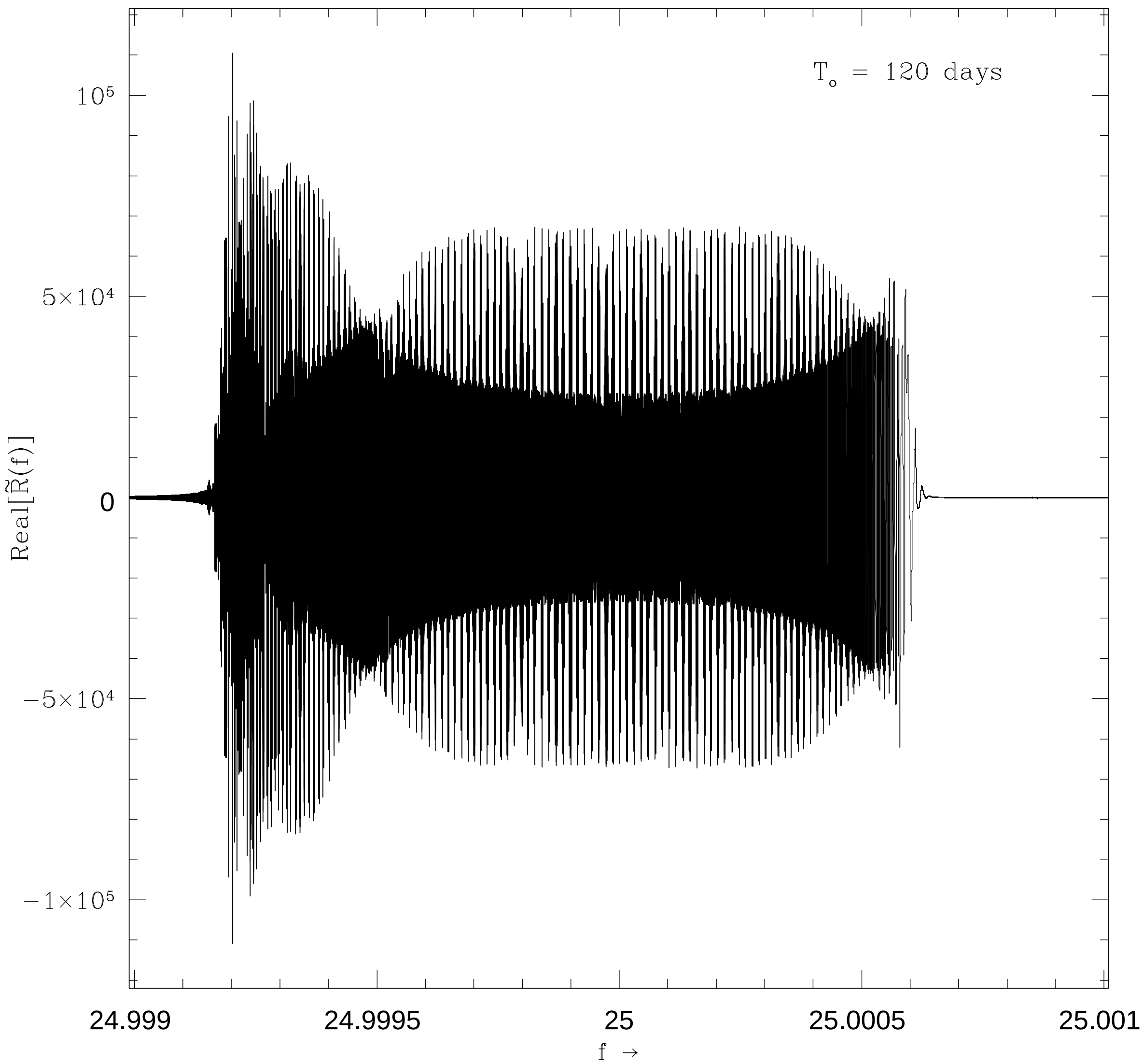

Figure 3: FT of a FM signal of frequency, $f_o = 25$ Hz from a source located at $(\pi/9, \pi/4)$ with a resolution of 9.67×10^{-8}.

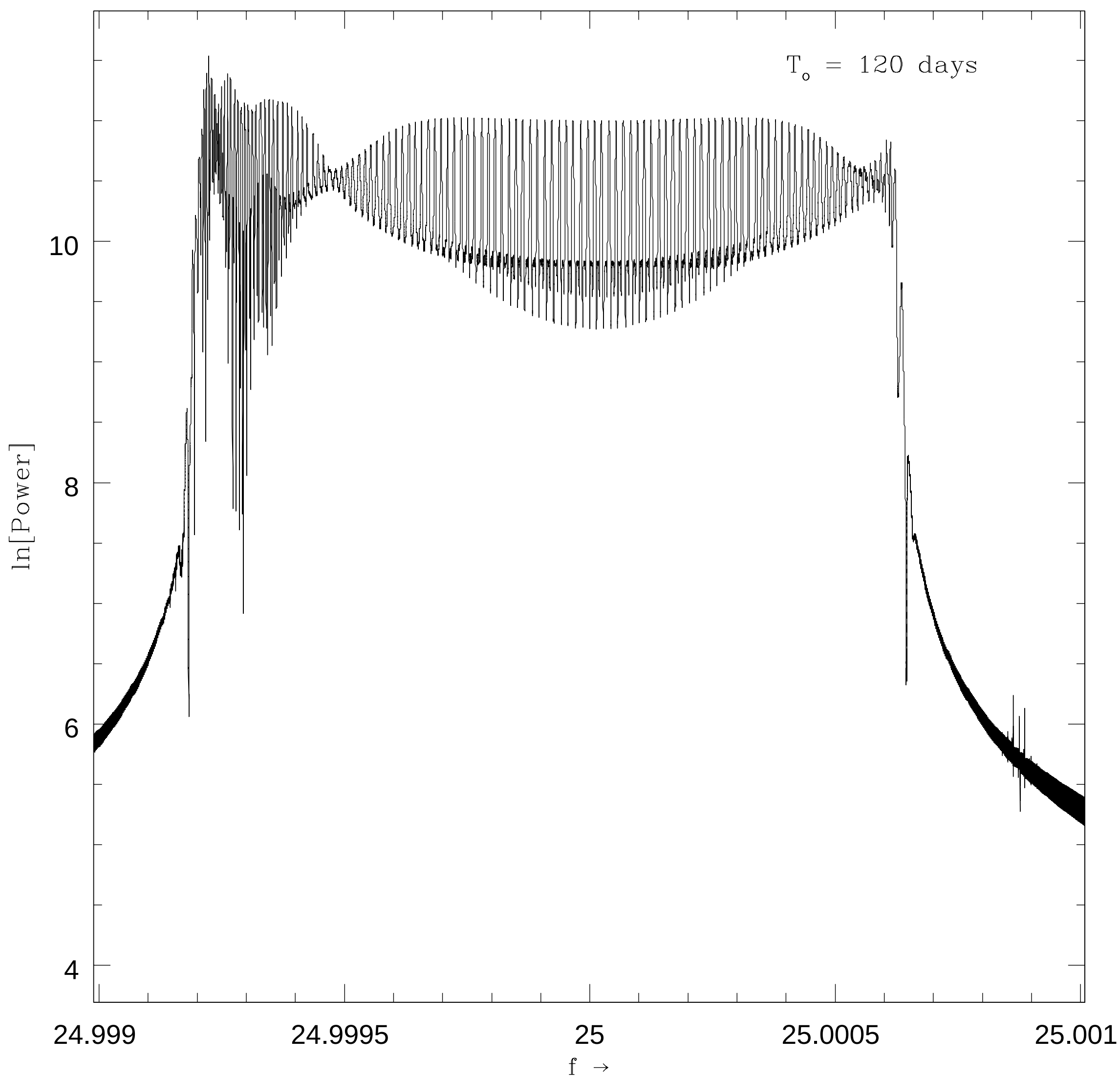

Figure 4: Power spectrum of the complete response of a Doppler modulated signal of frequency, $f_o = 25$ Hz from a source located at $(\pi/9, \pi/4)$ with a resolution of 9.67×10^{-8}.