

# Gravoelectric-dual of the Kerr solution

Naresh Dadhich\*

*Inter-University Center for Astronomy & Astrophysics,  
Post Bag 4, Ganeshkhind, Pune - 411 007, India.*

L.K. Patel

*Department of Mathematics,  
Gujarat University, Ahmedabad 380 009, Gujarat, India.*

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## Abstract

By decomposing the Riemann curvature into electric and magnetic parts, we define the gravoelectric duality transformation by interchange of active and passive electric parts which amounts to interchange of the Ricci and Einstein tensors. It turns out that the vacuum equation is duality-invariant. We obtain solutions dual to the Kerr solution by writing an effective vacuum equation in such a way that it still admits the Kerr solution but is not duality invariant. The dual equation is then solved to obtain the dual-Kerr solution which can be interpreted as the Kerr black hole sitting in a string dust universe.

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\*E-mail : [nkd@iucaa.ernet.in](mailto:nkd@iucaa.ernet.in)

## I. Introduction

In analogy with the Maxwell electromagnetic field, it is also possible to resolve gravitational field; i.e. the Riemann curvature into electric and magnetic parts relative to a timelike observer. In general relativity (GR), there are two kinds of gravitational charges; one non-gravitational energy distribution and the other gravitational field energy. Thus electric part would have further decomposition into active and passive parts corresponding to these two kinds [1-3]. Electromagnetic parts would be given by second rank tensors orthogonal to the resolving unit timelike vector. Electric parts are symmetric and account for 12 (6 each for active and passive) while magnetic part is trace free and accounts for the remaining 8 components of the Riemann curvature. The symmetric part of the magnetic part is equal to the Weyl magnetic part and antisymmetric part represents energy flux.

We consider a transformation which is interchange of active and passive electric parts [1] and we term it as electrogravity duality. It turns out that the vacuum equation is symmetric in active and passive parts and hence is duality invariant. The vacuum solutions would thus remain invariant modulo sign of constants of integration. It does in fact happen that the Riemann curvature for vacuum solutions changes sign under the duality transformation. That means  $GM \rightarrow -GM$  in the Schwarzschild solution. Thus vacuum solutions are self-dual modulo change of sign of  $G$ . This can however be understood as follows. The active part is anchored on the non-gravitational energy distribution while the passive part on the gravitational field energy [4]. For an attractive field the former is positive while the latter is negative and hence interchange of active and passive parts must naturally require  $G \rightarrow -G$ .

Now the question arises, can the symmetry of the equation be broken to get distinct dual solutions? It turns out that in obtaining the well-known and physically interesting black hole solutions, there always remains one equation unused which is implied by the others [4]. If we tamper this equation, vacuum solutions would remain undisturbed and the symmetry between active and passive parts would be broken. This is precisely what happens [1,3,4], and it is then possible to obtain distinct dual solutions. That is, it is possible to write an effective vacuum equation in such a way that it gives the same vacuum solutions but it is not duality invariant. Then the dual equation would yield distinct solutions. Following this method, solutions dual to Schwarzschild, Reissner-Nordström, and NUT solutions have been obtained

[1,3,5,6]. A dual solution is obviously non-vacuum and it is also asymptotically non-flat. It gives rise to energy momentum distribution which agrees with string dust distribution or that of a global monopole at large distance from the core. Thus duality transformation imbibes string dust or global monopole charge automatically, and dual spacetime describes the original source sitting in a string dust universe [7-8] or having a global monopole charge [9].

Note that the duality transformation interchanges active and passive electric parts which are respectively (double) projections of the Riemann and its double (left and right) dual onto a timelike observer. Since contraction of the Riemann is the Ricci while that of its double dual is the Einstein tensor, the duality transformation would hence imply interchange between the Ricci and Einstein tensors. The electrogravity duality implies the Ricci-Einstein duality. Thus the dual equation would result when the Ricci components are replaced by the Einstein components in the effective vacuum equation.

In this paper by application of the duality transformation we wish to obtain spacetime dual to the Kerr rotating black hole. As expected it would be quite involved and different from the other cases. Firstly, manipulation of equations is very complicated and secondly the dual equation, unlike the other cases, admits more than one solutions. That is dual solution is not unique. Dual spacetimes could represent the Kerr black hole in a string dust universe, as the stresses generated by the duality conform with that of string dust. They admit a horizon but it would not be a  $r = \text{const.}$  but instead be  $r = f(\theta)$  surface. When string dust density is switched off they reduce to the Kerr black hole.

In Sec.II we decompose the Riemann curvature in electromagnetic parts and then write the vacuum equation and consider the duality transformation. This is followed in Sec.III by solving the dual equation for the axially symmetric metric to obtain dual solutions. In Sec.IV we discuss the string dust interpretation and conclude in Sec.V with discussion on the general aspects of the duality transformation.

## II. Electromagnetic decomposition and duality

We first resolve the Riemann curvature relative to a timelike unit vector into electromagnetic parts as follows [1],

$$E_{ac} = R_{abcd}u^b u^d, \tilde{E}_{ac} = *R *_{abcd} u^b u^d \quad (1)$$

$$H_{ac} = *R_{abcd}u^b u^d = H_{(ac)} - H_{[ac]} \quad (2)$$

where

$$H_{(ac)} = *C_{abcd}u^b u^d, \quad H_{[ac]} = \frac{1}{2}\eta_{abce}R_d^e u^b u^d. \quad (3)$$

Here  $C_{abcd}$  is the Weyl conformal curvature,  $\eta_{abcd}$  is the 4-dimensional volume element. Note that  $E_{ab}$  and  $\tilde{E}_{ab}$  are symmetric,  $H_{ab}$  is trace-free and they are all orthogonal to the resolving unit timelike vector  $u^a$ . In terms of them, the Ricci curvature reads as

$$R_{ab} = E_{ab} + \tilde{E}_{ab} + (E + \tilde{E})u_a u_b - \tilde{E}g_{ab} + H^{mn}(\eta_{acmn}u_b + \eta_{bcmn}u_a)u^c \quad (4)$$

Then the vacuum equation for any timelike unit resolving vector would imply

$$E \text{ or } \tilde{E} = 0, H_{[ab]} = 0, E_{ab} + \tilde{E}_{ab} = 0 \quad (5)$$

which is symmetric in  $E_{ab}$  and  $\tilde{E}_{ab}$ .

We define the electrogravity duality transformation by

$$E_{ab} \leftrightarrow \tilde{E}_{ab}, H_{[ab]} = H_{[ab]}. \quad (6)$$

Obviously the vacuum equation is symmetric in  $E_{ab}$  and  $\tilde{E}_{ab}$  and hence would remain invariant under the duality transformation. It would give rise to the same vacuum solution. It turns out that the Weyl curvature changes sign under the duality [2], which means the constants of integration representing physical parameters like mass, angular momentum and NUT charge in the vacuum solutions must change sign. It is equivalent to  $G \rightarrow -G$ . It can in fact be verified for the Kerr solution by looking at the Riemann components as given in [10]. The vacuum solutions would modulo sign of  $G$  be self dual. It turns out that while obtaining the Schwarzschild solution, there remains one equation unused, which is implied by the others. In particular,  $H_{[ab]} = 0$ ,  $\tilde{E} = 0$  and  $E_{22} + \tilde{E}_{22} = 0$  determine the solution completely leaving  $E_{11} + \tilde{E}_{11} = 0$  free. In terms of the Ricci components, this is equivalent to  $R_{01} = 0 = R_{22}, R_0^0 = R_1^1$  with  $R_{00} = 0$  being free, which is implied

by the others. That is the set  $R_{01} = 0 = R_{22}, R_0^0 = R_1^1$  suffices to give the unique Schwarzschild solution. We take this as the effective vacuum equation which also yields the Schwarzschild solution uniquely. The set dual to the effective vacuum equation would be obtained by changing Ricci to Einstein, viz  $G_{01} = 0 = G_{22}, G_0^0 = G_1^1$ . This dual set then admits the unique solution which can be interpreted as the Schwarzschild black hole with a global monopole charge [9] or sitting in a string dust universe [7-8]. By this method of the duality transformation, the solutions dual to the Reissner-Nordström and the NUT solutions have also been obtained [1,3,5,6]. In the next section we wish to apply this method to axially symmetric space-time to obtain solution dual to the Kerr solution. It turns out that in this case the solution is not unique and we find two distinct solutions..

### III. Dual solutions

We consider an axially symmetric line element in the form [10],

$$ds^2 = 2(du + g \sin \alpha d\beta)dt - M^2(d\alpha^2 + \sin^2 \alpha d\beta^2) - 2L(du + g \sin \alpha d\beta)^2. \quad (7)$$

Here  $M$  and  $L$  are functions of  $u$ ,  $\alpha$  and  $t$  and  $g$  is a function of  $\alpha$  only. We use  $u$ ,  $\alpha$ ,  $t$  and  $\beta$  as coordinates. Introducing the tetrads

$$\begin{aligned} \theta^1 &= du + g \sin \alpha d\beta, & \theta^2 &= M d\alpha, \\ \theta^3 &= M \sin \alpha d\beta, & \theta^4 &= dx - L\theta^1 \end{aligned}$$

we can express the metric (7) in the form

$$ds^2 = 2\theta^1\theta^4 - (\theta^2)^2 - (\theta^3)^2 = g_{(ab)}\theta^a\theta^b. \quad (8)$$

The components  $R_{ab}$  of the Ricci tensor for the metric (7) were obtained by Vaidya *et al* [11]. We reproduce them below for ready reference, and they are given in the basis frame as follows:

$$\begin{aligned} R_{23} &= 0 \\ R_{44} &= \frac{2}{M} \left[ M_{xx} - \frac{f^2}{M^3} \right] \end{aligned}$$

$$\begin{aligned}
R_{24} &= \frac{g}{M} \left[ \left( \frac{M_x}{M} \right)_y - \left( \frac{f}{M^2} \right)_u \right] \\
R_{34} &= -\frac{g}{M} \left[ \left( \frac{M_x}{M} \right)_u - \left( \frac{f}{M^2} \right)_y \right] \\
R_{14} &= \frac{2}{M} \left[ M_{xu} + (LM_x)_x + \left( \frac{Lf^2}{M^3} \right) \right] + L_{xx} \\
R_{12} &= LR_{24} + \frac{g}{M} \left[ \left( L_x + \frac{M_u}{M} \right)_y + \left( \frac{2fL}{M^2} \right)_u \right] \\
R_{13} &= LR_{34} + \frac{g}{M} \left[ - \left( L_x + \frac{M_u}{M} \right)_u + \left( \frac{2fL}{M^2} \right)_y \right] \\
R_{22} &= R_{33} = \frac{1}{M^2} \left[ g^2 \left( \frac{M_u}{M} \right)_u + g^2 \left( \frac{M_y}{M} \right)_y - 1 + 2f \left( \frac{M_y}{M} \right) \right. \\
&\quad \left. + 4 \left( \frac{f^2 L}{M^2} \right) - (M^2)_{ux} - (L(M^2))_{xx} \right] \\
R_{11} &= L^2 R_{44} + \frac{1}{M^2} \left[ g^2 (L_{uu} + L_{yy}) + 2fL_y + 2L_u M M_x + 4L M M_{xu} \right. \\
&\quad \left. - 2L_x M M_u + 2M M_{uu} \right].
\end{aligned} \tag{9}$$

In the above equations, the variable  $y$  replaces  $\alpha$ , the defining relation being

$$gd\alpha = dy. \tag{10}$$

Here and in what follows a suffix denotes partial differentiation, eg.,  $g_\alpha = \partial g / \partial \alpha$ ,  $L_y = \partial L / \partial y$ ,  $M_{xu} = \partial^2 M / \partial x \partial u$  etc and  $x = t$ . The symbol  $2f$  stands for the expression  $g_\alpha + g \cot \alpha$ .

In the case of spherical symmetry it was  $R_{00} = 0$  was free while in this case for the metric (7) it is  $R_{14} = 0$  is free. That is it is implied by the others and is not used in obtaining the vacuum solution. We shall thus consider the effective vacuum equation as

$$R_{ab} = 0, \text{ except } R_{14}. \tag{11}$$

It can be verified from (9) that solution of the rest of the equation automatically implies  $R_{14} = 0$  giving the vacuum Kerr solution.

The dual equation would be obtained by letting  $R_{ab} \rightarrow G_{ab}$  and hence it would be

$$G_{ab} = 0, \text{ except } G_{14}, \quad (12)$$

which would imply

$$R_{ab} = 0, \text{ except } R_{22} = R_{33}. \quad (13)$$

To find the dual solution we have to solve the set (13) which would read as

$$R_{44} = 0, \quad R_{24} = 0, \quad R_{34} = 0 \quad (14)$$

$$R_{14} = 0 \quad (15)$$

$$R_{12} = 0, \quad R_{13} = 0 \quad (16)$$

$$R_{22} = -\rho \quad (17)$$

$$R_{11} = 0. \quad (18)$$

We proceed as follows.

Eqns. (14-18) yield the general solutions

$$M^2 = \frac{f}{Y}(X^2 + Y^2) \quad (19)$$

and

$$2L = -\frac{Y_u}{Y}X + 2G + \frac{2AX + 2BY}{X^2 + Y^2}, \quad (20)$$

where  $X$  is a function of  $t$ ,  $u$  and  $y$  and  $Y$ ,  $A$ ,  $B$  and  $G$  are functions of  $u$  and  $y$  satisfying the relations

$$X_t = -1, \quad X_y = Y_u, \quad X_u = -Y_y \quad (21)$$

$$B = -2YG - YY_y \quad (22)$$

and

$$B_u = A_y, \quad B_y = -A_u. \quad (23)$$

Eqn. (19) determines

$$\rho = -\frac{1}{X^2 + Y^2} \left[ 2G + \frac{Y}{f} \left( \frac{1}{2} g^2 \nabla^2 \log \left( \frac{Y}{f} \right) - f_y + 1 + 3f \frac{Y_y}{Y} \right) \right], \quad (24)$$

where  $\nabla^2 \equiv \partial^2/\partial u^2 + \partial^2/\partial y^2$ .

Now we assume that  $Y$  is a function of  $y$  only. This leads from eqn.(21) to

$$X = au - t, \quad Y = -ay + b, \quad (25)$$

where  $a$  and  $b$  are constants of integration, no additional constant is added in  $X$  because such a constant can always be incorporated in the  $t$ -coordinate.

Eqns. (22), (23) and (25) will then lead to

$$Y\nabla^2 G - 2aG_y = 0 \quad (26)$$

of which we take the particular solution

$$2G = \text{constant} = c. \quad (27)$$

Eqns. (22) and (25) then lead to

$$B = (a - c)Y, \quad A = a(a - c)u + m, \quad (28)$$

where  $m$  is again a constant of integration.

We now introduce the variable  $\theta$  and a function  $h(\theta)$  as follows:

$$\left(\frac{f}{Y}\right)^{1/2} d\alpha = d\theta, \quad \left(\frac{f}{Y}\right)^{1/2} \sin \alpha = h(\theta). \quad (29)$$

Using (10), (25) and (29) we can show that

$$\frac{Y}{f} \left\{ g^2 \nabla^2 \log \left( \frac{Y}{f} \right) - f_y + 1 + 3f \frac{Y_y}{Y} \right\} = -2a - \frac{h_{\theta\theta}}{h}. \quad (30)$$

The Kerr solution would follow if we take  $h_{\theta\theta}/h = -1$ , which would imply  $f = Y$  and give

$$2L = c + \frac{2X[a(a - c)u + m] + 2(a - c)Y^2}{X^2 + Y^2}. \quad (31)$$

So far we have not used eqn.(18), which would now imply  $a = c$ , and finally we shall have

$$\rho = \frac{1 - a}{X^2 + Y^2}, \quad (32)$$

where  $X$  and  $Y$  are given by (25).

Using eqns. (10), (25) and (29) we can write

$$2hY = (g \sin \alpha)_\theta, \quad hY_\theta = -ag \sin \alpha, \quad (33)$$

where  $h = \sin \theta$ . These relations together then give the following differential equation for  $Y$ :

$$Y_{\theta\theta} + \frac{h_\theta}{h} Y_\theta + 2aY = 0. \quad (34)$$

This integrates to give  $Y = k \cos \theta$  for  $a = 1$ , when  $\rho$  would vanish giving the Kerr solution. This is the Legendre equation which could give non-vacuum dual solution only when the integer  $a \neq 1$  but the solution would not include the Kerr solution as a particular case. This would however be a dual solution for different integer values of  $a$ .

If we wish to have a dual solution that includes the Kerr solution as a particular case, we will have to give up the relation  $f = Y$ . In that case we can have the following two different solutions:

$$Y = k \cos \theta, \quad h = \sin^{2a-1} \theta, \quad g \sin \alpha = \frac{k}{a} \sin^{2a} \theta, \\ \rho = \frac{a-1}{X^2 + Y^2} [1 - 2(2a-1) \cot^2 \theta]. \quad (35)$$

and

$$Y = k \cos^a \theta, \quad h = \sin \theta \cos^{a-1} \theta, \quad g \sin \alpha = k \sin^2 \theta, \\ \rho = \frac{a-1}{X^2 + Y^2} [2 - (a-2) \tan^2 \theta]. \quad (36)$$

These are two distinct solutions. Thus dual solution to the Kerr solution is not unique. Of course each has stresses corresponding to a string dust distribution of density  $\rho$ , which will diverge at  $\theta = 0$  for the former and at  $\theta = \pi/2$  for the latter. Both however reduce to the Kerr solution when  $a = 1$ . The metric would read as

$$\begin{aligned}
ds^2 = & 2(du + g \sin \alpha d\beta)dt - (R^2 + Y^2)(d\alpha^2 + H^2(\theta)d\beta^2) \\
& - \left[ a + \frac{2mR}{R^2 + Y^2} \right] (du + g \sin \alpha d\beta)^2,
\end{aligned} \tag{37}$$

where we have defined  $X = R = (a-1)t - ar$  as a new radial coordinate. For the two dual solutions  $Y, h, g \sin \alpha$  as given above in (35-36). The solutions go over to the Kerr solution when  $a = 1$  with  $m$  and  $k$  as mass and specific angular momentum.

The dual solutions do however admit horizon defined by the equation,

$$ah^2(R^2 + Y^2) - 2mRh^2 + (ag \sin \alpha)^2 = 0 \tag{38}$$

which would for the solution (35) give the horizon as

$$R_+ = \frac{m}{a} \left( 1 + \sqrt{1 - (k^2/m^2)(a^2 \cos^2 \theta + \sin^2 \theta)} \right). \tag{39}$$

Thus horizon has unlike the Kerr black hole  $\theta$  dependence. This is so for the other solution as well. In other dual solutions [1,3,5], the basic character of the field remained unaltered while here it is not so as indicated by the  $\theta$  dependence of the horizon. In the next section we shall show that these solutions could be interpreted as rotating black hole sitting in a string dust universe with the string dust density  $\rho$  given by (35-36). It is true that the solutions obtained are rather complicated and unfortunately cannot be transformed to the Boyer-Lindquist form to gain more physical insight. Since dual solution is not unique, it may be possible to find a simpler and physically more transparent solution. The search is on.

#### IV. String dust interpretation

In terms of the electromagnetic parts, the effective vacuum equation (11) will take the form

$$\tilde{E} = 0, H_{[ab]} = 0, E_{ab} + \tilde{E}_{ab} = -(E + \tilde{E})w_a w_b \tag{40}$$

which is no longer symmetric in active and passive parts. Here  $w_a$  is a unit spacelike vector orthogonal to  $u_a$  and in the direction of acceleration vector. It would admit the same vacuum solution but it is now not invariant under the duality transformation (6).

The dual equation (12) would read as

$$E = 0, H_{[ab]} = 0, E_{ab} + \tilde{E}_{ab} = -(E + \tilde{E})w_a w_b \quad (41)$$

which is equivalent to the set (14-18). This is what we have solved for dual solution and it gives rise to the only surviving stress component

$$T_{14} = \rho \quad (42)$$

where  $\rho$  is given by (35) or (36).

On the other hand a string dust distribution is characterized by [7,8],

$$T_{ab} = \rho(u_a u_b - w_a w_b). \quad (43)$$

For the metric (7) we choose the timelike and spacelike vectors as follows:

$$u_{(a)} = (1, 0, 0, \frac{1}{2}) \quad w_{(a)} = (1, 0, 0, -\frac{1}{2}). \quad (44)$$

Then eqn. (43) would imply the distribution (42). Thus our dual solutions could represent a Kerr black hole sitting in a string dust universe, and the string density which is equal to radial tension is given by (35) and (36) for the two solutions. When string density is switched off, the Kerr solution follows.

## V. Discussion

The very first time one encounters the duality transformation is in electrodynamics where it prescribes a relation between electric and magnetic fields under which the vacuum Maxwell equation remains invariant. A similar kind of relationship between active and passive electric parts and magnetic part of the gravitational field in fact leads to the Einstein vacuum equation [2]. This is because electromagnetic parts for gravity stem from the Riemann curvature and hence contain second derivative of the metric and consequently dynamics of the field. In the Maxwell theory, they contain first derivative of the gauge potential, and to get to the equation of motion they need to be differentiated once. There is thus a basic difference between electromagnetic parts of the gravitational field and that of the Maxwell field. This basic difference should always be kept in mind.

A dual is only defined for an antisymmetric tensor by the Hodge dual. The duality transformation represents in general a rotation. However we are

here considering a relation between active and passive electric parts which are symmetric second rank 3-tensors. Thus our electrogravity duality transformation does not represent a duality rotation [12] and is therefore different in character from the duality transformations considered in other context in GR [13] as well as in other theories including the string theory [14]. The duality is at the center stage of the current field theory research and has played very important role in connecting different theories and situations.

At any rate, our duality though different from the usual duality, it is however a relation between active and passive electric parts and marks a symmetry of the vacuum equation. It is thus a valid statement. The remarkable feature of this is in finding new dual solutions which imbibe global monopole or string dust automatically. Its connection with production of topological defects is rather intriguing and interesting, and this feature permeates in lower and higher dimensions as well as in scalar tensor theories [15-17]. In the case of the Schwarzschild field, the duality simply restores the gauge freedom in choosing zero of the potential one had in the Newtonian theory. The vacuum equation does not permit this freedom because the spacetime is asymptotically flat and the potential could only vanish at infinity. This means that topological defects (dual solutions) thus do not disturb the basic character of the field at the Newtonian level. However we do not yet fully understand the physical meaning and import of the electrogravity duality.

Physical features of the black hole with global monopole charge [9] have been considered by several authors [18-21]. It has been argued that since the global monopole solution (and so are the dual solutions in general), is not asymptotically flat, hence its asymptotic regions would be curved. Note that positivity of ADM mass is proved only for isolated system that generates asymptotic Minkowski geometry. Objects with negative mass may therefore generate non-flat asymptotic regions [20]. This suggests an association of duality with negative mass. Recall that we have discussed in the Introduction that duality would imply gravitational constant turning negative which means gravitational mass turning negative. This is because interchange of active and passive parts would imply interchange of positive non-gravitational matter energy and negative gravitational field energy. This seems consistent with the strange and unusual thermodynamical behaviour of black holes with gauge cosmic strings and global monopoles [20-21]. There appears to be a deep connection between the electrogravity duality and the topological

defects. It calls for a comprehensive study to probe it further and deeper.

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