

International Journal of Modern Physics D
© World Scientific Publishing Company

**Physical properties of Tolman-Bayin solutions:
some cases of static charged fluid spheres in general relativity**

Saibal Ray*

*Department of Physics, Barasat Government College,
North 24 Parganas, Kolkata 700 124, India
saibal@iucaa.ernet.in*

Basanti Das

*Belda Prabhati Balika Vidyapith, Belda,
West Midnapur 721 424, W.B., India*

Farook Rahaman

*Department of Mathematics, Jadavpur University,
Kolkata 700 032, W.B., India*

Subharthi Ray

*Inter-University Centre for Astronomy and Astrophysics,
Post Bag 4, Ganeshkhind, Pune 411 007, India
sray@iucaa.ernet.in*

In this article, Einstein-Maxwell space-time has been considered in connection to some of the astrophysical solutions as previously obtained by Tolman (1939) and Bayin (1978). The effect of inclusion of charge into these solutions has been investigated thoroughly and also the nature of fluid pressure and mass density throughout the sphere have been discussed. Mass-radius and mass-charge relations have been derived for various cases of the charged matter distribution. Two cases are obtained where perfect fluid with positive pressures give rise to *electromagnetic mass* models such that gravitational mass is of purely electromagnetic origin.

Keywords: general relativity; charged dusts; charged stars.

1. Introduction

The so-called Schwarzschild interior solutions obtained by using Einstein's field equations corresponding to various spherically symmetric static perfect fluid distributions usually suffer from the well-known problem of singularity. An uncharged incompressible fluid sphere of mass m cannot be held in equilibrium below certain radius $a = 9m/4$ and even demands a larger value for a related to physically reasonable equation of state ¹. One way to overcome this singularity due to gravitational

*Associate, IUCAA, Pune 411 007, India

collapsing of a spherically symmetric material distribution is to include charge to the neutral case. It is observed that gravitational collapse can be averted in the presence of charge where gravitational attraction is counter balanced by the electrical repulsion in addition to the pressure gradient^{2,3,4}

But even then many questions came up regarding the stability of the charged sphere and also about the amount of charge. Bonnor⁵ worked on this type of charge inclusion model and showed that a dust cloud of arbitrarily large mass and small radius can remain in equilibrium if it has an electric charge density related to the mass density by $\sigma = \pm\rho$. According to Stettner⁶, a fluid sphere of uniform density with a net surface charge is more stable than without charge. However, Glazer⁷ by considering radial pulsations showed that Bonnor⁵ model is electrically unstable. He also explicitly established the effects of electric charge upon dynamical stability⁸. The work of Whitman and Burch⁹, for arbitrary charge and mass distribution showed how charged analogue cases give more stability. They also applied the pulsation equations to the charged solution of Pant and Sah¹⁰, which represents the charged analogue of Tolman¹¹ solution of type VI, and got unsatisfactory results in connection to the boundary condition which is incompatible with densities and pressures. de Felice et al.² proposed a model for charged perfect fluid and concluded that inclusion of charge inhibits the growth of space-time curvature which has a great role to avoid singularities. However, very recently Ray et al.¹² and Ghezzi¹³ have studied the effect of electric charge on compact stars assuming the charge distribution is proportional to the mass density. Both the group have argued that with the huge amount of charge and strong electric field these type of stars would be unstable and eventually would form charged black holes.

Therefore, starting from the very well-known Weyl-Majumdar-Papapetrou^{14,15,16} type general relativistic charged dust solutions, via Bonnor⁵ and several others (some of which have been mentioned in the above introductory discussions), one can arrive at the realm of charged compact stellar objects. But a genuine question can be raised in this context that: what about the normal stars, do they carry electric charge and if at all, what will be the stability of their configurations? As such regarding the effect of charge and possibility of holding charge by the stars are not unavailable in the past literature^{17,18,19,20}. As a continuation of such study Shvartsman²¹ argued that while astrophysical systems are usually thought to be electrically neutral, this may not be always true in the real situation. His analysis is based on the exchange processes between stars and the surrounding medium. Very recently, Neslušan²² has reminded about the existence of global electrostatic field of the sun and other normal stars. He has given a general charge-mass relation $q_r = [2\pi\epsilon_0 G(m_p - m_e)m_r]/q$ where ϵ_0 is the primitivity of vacuum, G is the gravitational constant, m_r is the stellar mass in the sphere of radius r , m_p and m_e are respectively the mass of proton and electron having charge q , while q_r is the global electrostatic charge inside the star. He got it to be $q_r = 77.043m_r$ when q_r in Coulomb and m_r in solar masses corresponding to an ideally quiet, perfectly spherical, non-rotating star.

So, motivated by all the above facts regarding the charged sphere in connection to normal stars, we have considered here charge analogue of Tolman VI¹¹ and Bayin²³ type solutions which represent very important class of astrophysical solutions. However, in a series of work Ray and Das^{24,25,26} have already been studied some specific aspects of these type of solutions in different context. In the present work we analyze different physical properties of the static charged stellar model. We show that some cases provide *electromagnetic mass model* (EMMM) where all the physical parameters, including the gravitational mass, vanish due to vanishing charge density^{27,28}. It has also shown that EMMM do exist even with positive pressure which clearly contradicts the observation done by Ivanov⁴ that EMMM are always associated with repulsive pressure. The effect of charge inclusion in these EMMM, along with other solutions, has been investigated thoroughly in connection to mass-radius and mass-charge relations. The nature of fluid pressure and mass density throughout the fluid sphere have also been discussed.

2. The Einstein-Maxwell field equations

Let us consider a static spherically symmetric matter distribution corresponding to the line element

$$ds^2 = A^2 dt^2 - B^2 dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

where A and B are function of the radial coordinate r only.

Then the set of Einstein-Maxwell field equations, in the co-moving coordinates, for the above line element may be explicitly written²⁴ as

$$\frac{1}{B^2} \left(\frac{2B'}{Br} - \frac{1}{r^2} \right) + \frac{1}{r^2} = 8\pi\rho + \frac{q^2}{r^4}, \quad (2)$$

$$\frac{1}{B^2} \left(\frac{2A'}{Ar} + \frac{1}{r^2} \right) - \frac{1}{r^2} = 8\pi p - \frac{q^2}{r^4}, \quad (3)$$

$$\frac{1}{B^2} \left[\frac{A''}{A} - \frac{A'B'}{AB} + \frac{1}{r} \left(\frac{A'}{A} - \frac{B'}{B} \right) \right] = 8\pi p + \frac{q^2}{r^4}, \quad (4)$$

where the total charge within a sphere of radius r , in terms of the 4-current J^i , can be given by

$$q = 4\pi \int_0^r J^0 r^2 AB dr. \quad (5)$$

3. The solutions to the field equations

3.1. Bayin's class of solution

Now, assuming $A'/Ar = C(r)$ and then equating (3) and (4), we get Bernoulli equation for $B(r)$ and $C(r)$ as follows

$$\frac{dB}{dr} = \left[\frac{1 - \frac{2q^2}{r^2}}{(C + \frac{1}{r^2}) r^3} \right] B^3 + \left[\frac{C^2 r - \frac{1}{r^3} + \frac{dC}{dr}}{C + \frac{1}{r^2}} \right] B. \quad (6)$$

4 *Ray, Das, Rahaman & Ray*

Therefore, choosing $q(r) = Kr^n$ we get the solutions for $p(r)$ and $\rho(r)$ corresponding to different values of the parameter n (for detail results vide ref. ²⁴).

Case I: For $n = 1$

$$8\pi p = \frac{K^2}{r^2} - \frac{1}{r^2} + \left\{ \frac{a_0 + 3a_1r}{(a_0 + a_1r)r^2} \right\} \times \left\{ 1 + B_0r^2 - \frac{2r}{C_0} + \frac{4r^2}{C_0^2} \ln \frac{D}{r} - K^2 - 4K^2r^2 \left(\frac{2}{C_0^2} \ln \frac{D}{r} - \frac{1}{C_0r} + G_1 \right) \right\}, \quad (7)$$

$$8\pi\rho = \frac{K^2}{r^2} + \frac{4}{C_0} \left(\frac{1}{r} + \frac{1}{D} \right) - 3B_0 + \frac{4K^2}{rD} - \frac{12}{C_0^2} \ln \frac{D}{r} - 8K^2r \left(\frac{2}{C_0^2} \ln \frac{D}{r} - \frac{1}{C_0r} + G_1 \right). \quad (8)$$

where C_0 and G_1 are the constants of integration. Here and in what follows D has been substituted for $(C_0 + 2r)$.

Case I: For $n = 3$

$$8\pi p = K^2r^2 - \frac{1}{r^2} + \frac{a_0 + 3a_1r}{(a_0 + a_1r)r} \left[1 + B_0r^2 - \frac{2r}{C_0} + \frac{4r^2}{C_0^2} \ln \frac{D}{r} - 2Kr^4 - \frac{K^2r^2}{2} \left(\frac{D^2}{2} - 2C_0D + C_0^2 \ln D + G_2 \right) \right], \quad (9)$$

$$8\pi\rho = -11K^2r^2 + \frac{4}{C_0} \left(\frac{1}{r} + \frac{1}{D} \right) - 3B_0 + \frac{4K^2r^3}{D} - \frac{12}{C_0^2} \ln \frac{D}{r} - K^2 \left[\frac{D^2}{2} + 2C_0(C_0 + r) + C_0^2 \ln D + 8G_2 \right]. \quad (10)$$

where G_2 is the constant of integration.

Case II: For $n = 1$

$$8\pi p = \frac{1}{W_0^2} \left[2(C_1r^2 + W_0^2 - r^4 - 2W_0^2K^2)^{1/2} - r^2 + C_1 \right] - \frac{K^2}{r^2}, \quad (11)$$

$$8\pi\rho = \frac{1}{W_0^2} (5r^2 - 3C_1) + \frac{K^2}{r^2}. \quad (12)$$

where C_1 is the constant of integration.

Case II: For $n = 3$

$$8\pi p = \frac{1}{W_0^2} \left[2 \{ C_1 r^2 + W_0^2 - (1 - 2W_0^2 K^2) r^4 \}^{1/2} - r^2 + C_1 \right] + 3K^2 r^2, \quad (13)$$

$$8\pi\rho = \frac{1}{W_0^2} (5r^2 - 3C_1) - 11K^2 r^2. \quad (14)$$

Case III: For $n = 1$

$$8\pi p = \frac{1}{r^2} [K^2 - 1 + (2K^2 - 1)(C_3 r - 2)C_3 r], \quad (15)$$

$$8\pi\rho = \frac{1}{r^2} \left[\frac{1 - K^2}{r^2} - \frac{2C_3 r + (2K^2 - 1)(C_3 r - 2)^3}{(C_3 r - 2)r^2} \right]. \quad (16)$$

where C_3 is the constant of integration.

3.2. Tolman VI type class of solution

In their work Ray & Das²⁴ considered the solution of Pant & Sah¹⁰ to discuss about the electromagnetic mass model (EMMM) while in another paper²⁵ they have discussed the role for equation of state regarding the EMMM. However, in the present work we would like to find out some other features of the solutions, specially the condition to have positive pressure. The solutions given by Pant & Sah¹⁰ corresponding to the metric (Eq. 1), where $e^{\nu/2} = A$, $e^{\lambda/2} = B$ and $K = E^2/8\pi$ in the present mathematical nomenclature, become

$$\rho = \frac{1}{16\pi r^2} [1 - c(n - 1)^2], \quad (17)$$

$$p = \frac{1}{16\pi r^2} [c(n + 1)^2 - 1] \quad (18)$$

where

$$c = \left[1 - \frac{2m}{a} + \frac{q^2}{a^2} \right] = \left[1 - \frac{2q^2}{a^2} \right] (1 + 2n - n^2)^{-1}. \quad (19)$$

The above set of solutions, in view of c , with $\Lambda = 0$ and $B = 0$ represents the charged analogue of Tolman's¹¹ solution VI and thus in the absence of the total charge q reduces to the neutral one (the sub-case C of uncharged fluid sphere in Pant & Sah¹⁰). The corresponding total gravitational mass $m(r = a)$, can be obtained as

$$m = \frac{na^2(2 - n) + 2q^2}{2(1 + 2n - n^2)a}. \quad (20)$$

4. Detailed study of some of the solutions

4.1. Bayin's solutions

Case I: For $n = 1$

From the expressions for pressure $p(r)$ and density $\rho(r)$, as given in Eqs. (7) and (8), for the central values, viz., $r = 0$, we have $p_0/\rho_0 = -1$ which can be written as $p_0 + \rho_0 = 0$. This equation of state gives the vacuum fluid case. Also this solution shows a pressure distribution which goes to negative infinity as r approaches zero and density approaches to positive infinity at the centre. According to Bayin²³, this solution can be used to represent portions of stars, but can not provide a physically acceptable model for the entire star.

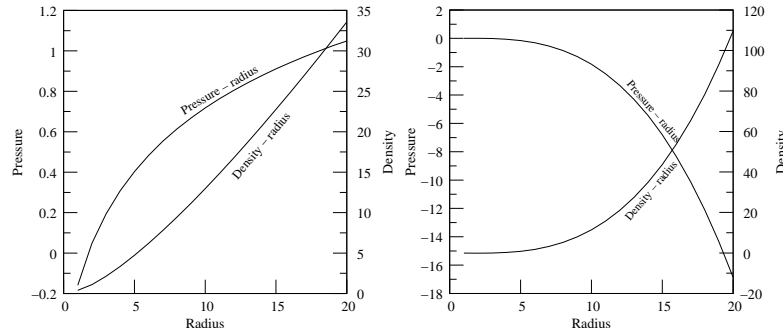


Fig. 1. Bayin's solutions showing pressure - density - radius plot for $n=1$ (left) and $n=3$ (right) (Case I).

Case I: For $n = 3$

At the origin, $r = 0$, we get $p_c/\rho_c = -1/3$ which turns into $3p_c + \rho_c = 0$. This equation of state gives the radiation case. Also the solution for $p(r)$ shows a pressure distribution which gives negative infinity pressure at the centre. So it can be used also to represent a portion of a star. Here mass density is positive infinity at the origin.

Case II: For $n = 1$

At the origin it gives $p_c/\rho_c = -1$ and we get $p_c + \rho_c = 0$. This equation of state represents vacuum fluid case. From the expression of $p(r)$ we see pressure goes to negative infinity as r approaches zero. Likewise the above case it can be used to represent a portion of the star where mass density is positive infinity at the origin.

Case II: For $n = 3$

At the origin, we get the pressure and density as follows,

$$8\pi p_c = \frac{2W_0 + C_1}{W_0^2}, \quad (21)$$

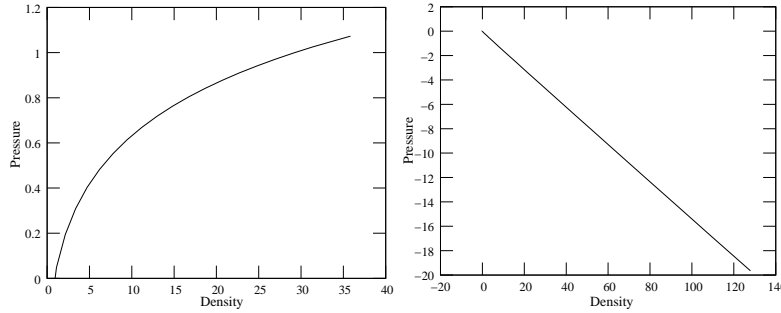


Fig. 2. The pressure vs density (equation of state) plot for $n=1$ (left) and $n=3$ (right) for Bayin's solutions (Case I).

$$8\pi\rho_c = -\frac{3C_1}{W_0^2}. \quad (22)$$

Here we see that, at the centre, pressure and density have finite values. In order to have positive pressure and density at the origin we must have the following conditions: $C_1 < 0$ and $W_0 > |C_1|/2$ where C_1 and W_0 are constants.

Now, from the Eqs (21) and (22) we get

$$\frac{p_c}{\rho_c} = -\frac{2W_0}{3C_1} - \frac{1}{3}. \quad (23)$$

This is exactly the same result as Bayin²³ got in his non-charge case. Therefore, it is very interesting to note that for both the cases, corresponding to static neutral-fluid sphere and static charged-fluid sphere, we get the same result for central pressure and density.

In Bayin's²³ paper we see $\rho(r)$ increases as r goes from centre to surface which, indeed, is not to be a physically reasonable case. But in our expression we have an extra part $11K^2r^2/8\pi$ which is arising due to inclusion of charge where K is a constant. So we can choose suitable values for K such that $\rho(r)$ can be a decreasing function of radius from centre to surface and hence the solution can provide a physically valid case. Therefore, for $\rho(r)$ to be a decreasing function of r from centre to surface we must have, from the Eqs. (14) and (22), the condition

$$\left(\frac{5r^2}{W_0^2} - \frac{3C_1}{W_0^2} - 11K^2r^2\right) < \left(-\frac{3C_1}{W_0^2}\right), \quad (24)$$

which gives $W_0^2K^2 > 5/11$. Thus, altogether we get three conditions as follows: $C_1 < 0$, $W_0 > |C_1|/2$ and $W_0^2K^2 > 5/11$.

Again, for $p(r)$ to be a decreasing function of radius from centre to surface of

8 *Ray, Das, Rahaman & Ray*

the star, we get the condition, from the Eqs. (13) and (21), as

$$\begin{aligned} & \left[2 \{ C_1 r^2 + W_0^2 - (1 - 2W_0^2 K^2) r^4 \}^{1/2} - r^2 + C_1 \right] \\ & < [2W_0 + C_1 - 3K^2 r^2 W_0^2]. \end{aligned} \quad (25)$$

This gives

$$r^2 < \left[\frac{4(C_1 - W_0 + 3K^2 W_0^2)}{(5 - 9W_0^2 K^2)(1 - W_0^2 K^2)} \right]. \quad (26)$$

The above result is obtained on the condition $5/9 < W_0^2 K^2 < 1$. If we apply this condition for radius of the star, for $r = a$, then we can conclude that through-out the star the pressure and density have finite values and they are decreasing function of radius from centre to surface.

Now, we can get the expression for radius of the star considering $p(r = a) = 0$ at the boundary as

$$\begin{aligned} a^2 = & -\frac{3C_1}{9W_0^2 K^2 - 5} \pm \frac{1}{2} \left[\left(\frac{6C_1}{9W_0^2 K^2 - 5} \right)^2 \right. \\ & \left. - \frac{4(C_1^2 - 4W_0^2)}{(9W_0^2 K^2 - 5)(W_0^2 K^2 - 1)} \right]^{1/2}. \end{aligned} \quad (27)$$

By matching interior and exterior solutions on the boundary, $r = a$, we can evaluate the integration constants C_1 and C_2 in terms of mass m and radius a of the star which can be expressed as follows:

$$C_1 = \left(1 - 2W_0^2 K^2 + \frac{K^2}{W_0^2} \right) a^2 - \frac{2m}{a^3} W_0^2, \quad (28)$$

$$\begin{aligned} C_2 = & \frac{1}{2} \ln \left(1 - \frac{2m}{a} + K^2 a^4 \right) - \frac{1}{(1 - 2W_0^2 K^2)^{1/2}} \times \\ & \sin^{-1} \left[\frac{C_1 - 2a^2(1 - 2W_0^2 K^2)}{C_1^2 + 4W_0^2(1 - 2W_0^2 K^2)^{1/2}} \right]. \end{aligned} \quad (29)$$

Therefore, after substituting the above value of C_1 we can get a relation between m and a as

$$\begin{aligned} m^2 + \frac{a^5}{W_0^2} \left(2 - W_0^2 K^2 - \frac{K^2}{W_0^2} \right) m \\ - \left(4 - 8W_0^2 K^2 + \frac{8K^2}{W_0^2} - 10K^4 \right. \\ \left. + 7W_0^4 K^4 + \frac{K^4}{W_0^4} \right) \frac{a^{10}}{4W_0^4} + \frac{a^6}{W_0^2} = 0. \end{aligned} \quad (30)$$

The equation of state for this case which is valid throughout the sphere is given by

$$\begin{aligned} \rho + p = & \frac{1}{4\pi W_0^2} \left[\{ C_1 r^2 + W_0^2 - (1 - W_0^2 K^2) r^4 \}^{1/2} \right. \\ & \left. + 2r^2 - C_1 \right] - \frac{K^2 r^2}{\pi}. \end{aligned} \quad (31)$$

Case III : For $n = 1$

When $K^2 < 5/9$ pressure is negative infinity but density is positive infinity at the centre of the sphere. Again, if we take the condition $K^2 > 1$ then the result will be reverse one. We get, from the boundary condition, the value of the constant $C_3 = (1 + K^2)/m$. Equating pressure to zero at the boundary, $r = a$, we get the relation between m and a of the star as follows

$$a = \alpha m \quad (32)$$

where

$$\alpha = \frac{1}{1 + K^2} \left[1 + \left(1 + \frac{1 - K^2}{2K^2 - 1} \right)^{1/2} \right]. \quad (33)$$

This represents the charge analogue to Bayin's²³ case III where the relation between the radius and mass was $a = m$ for the value $K = 0$. Again, for our present charged case even putting $K = 1$ we can recover the same result $a = m$. In this connection it would be very interesting to note that, from the relation $q(r) = Kr^n$, for $K = 1$ and $n = 1$ we get the total charge of the sphere as $q = a$. As a result, $q = m$ which provides the stability condition regarding a charged fluid sphere. On the choice of suitable value for K we can get different relation between q and m . A detail study of these type of relation $a = \alpha m$ have been performed by several worker in different context^{29,2}.

In Figs. 1 & 2, we have made a graphical study of Bayin's solutions for Case I, setting all the constants to unity, with the only variable being radius, and for the different n values. This shows a general trend of the nature of the curve, but fails to match with the realistic stars (compact and charged). In a future work, we will match conditions to realistic stars, estimating the exact values of the parameters, where we can hopefully give a better insight of the solutions. It is also noteworthy to mention that not all of the Bayin's solutions are physically feasible to match realistic astrophysical objects, nevertheless they are of academic importance.

4.2. Tolman's solutions**Case I: For $n = 0$**

If we now make the specific choice $n = 0$ for the parameter n appearing in the above solution set then one get the following expressions.

$$\rho = \frac{1}{8\pi r^2} \left[\frac{q^2}{a^2} \right], \quad (34)$$

$$p = -\frac{1}{8\pi r^2} \left[\frac{q^2}{a^2} \right], \quad (35)$$

This gives the equation of state

$$\rho + p = 0 \quad (36)$$

10 *Ray, Das, Rahaman & Ray*

and the mass-radius relation

$$m = \frac{q^2}{a}. \quad (37)$$

We see that it gives EMMM for imperfect fluid case. Now if we look at the pressure profile, we see it is negative infinity at the centre and increases from centre to surface. The pressure has a finite value at the boundary.

Case II: For $n = 0.5$

We are in favor of this case because of some historical reasons. This choice was originally done by Tolman¹¹ himself in his uncharged version. In this case the gravitational mass becomes

$$m = \frac{3a^2 + 8q^2}{14a}. \quad (38)$$

The equation of state, by virtue of Eqs. (17) and (18), can be written as

$$\rho + p = \frac{1}{4\pi r^2} \left[\frac{n(a^2 - 2q^2)}{(1 + 2n - n^2)a^2} \right], \quad (39)$$

which for the present case reduces to

$$\rho + p = \frac{1}{14\pi r^2} \left[1 - \frac{2q^2}{a^2} \right]. \quad (40)$$

$$p = \frac{1}{56\pi r^2} \left[1 - \frac{9q^2}{a^2} \right] \quad (41)$$

In this case also pressure is negative infinity at the centre. But if we put the condition ($q/a < \pm 1/3$) we see pressure becomes positive throughout the sphere. So, finally we get this case as a perfect fluid case having positive pressure but not EMMM. At the boundary, equating pressure to zero, we get $a^2 = 9q^2$. So, from Eq. (38) one can obtain $m = (5/6)q$ which gives $q/m > 1$.

Case III: For $n = 1$

For this choice of n , the gravitational mass becomes

$$m = \frac{a^2 + 2q^2}{4a}, \quad (42)$$

whereas equation of state is

$$\rho + p = \frac{1}{8\pi r^2} \left[1 - \frac{2q^2}{a^2} \right]. \quad (43)$$

$$p = \frac{1}{16\pi r^2} \left[1 - \frac{4q^2}{a^2} \right] \quad (44)$$

In this case to have positive pressure everywhere in the sphere we must have the condition ($q/a < \pm 1/2$). It is also a perfect fluid case having positive pressure but not EMMM. At the boundary, equating pressure to zero, we get $a^2 = 4q^2$. So, from

Eq. (42) we have $m = (3/4)q$ which gives $q/m > 1$.

Case IV: For $n = 1.5$

In this case the gravitational mass is

$$m = \frac{3a^2 + 8q^2}{14a}, \quad (45)$$

and the equation of state becomes

$$\rho + p = \frac{3}{14\pi r^2} \left[1 - \frac{2q^2}{a^2} \right]. \quad (46)$$

The gravitational mass for this case is the same as that of $n = 0.5$ case.

$$p = \frac{1}{56\pi r^2} \left[9 - \frac{25q^2}{a^2} \right] \quad (47)$$

To have positive pressure throughout the sphere we get the condition $(q/a < \pm 3/5)$. At the boundary, equating pressure to zero, one can get $a^2 = (25/9)q^2$. So, from Eq. (45) we get $m = (7/10)q$ which gives $q/m > 1$.

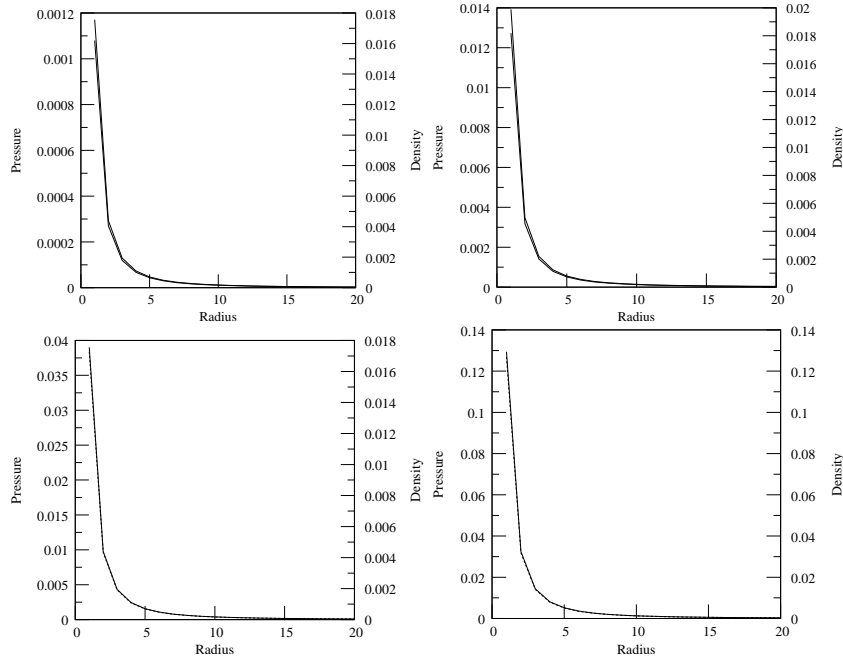


Fig. 3. Tolman's solutions, showing pressure - density - radius plot for $n=0.5$ (top left), $n=1.0$ (top right), $n=1.5$ (bottom left) & $n=2.0$ (bottom right) for cases II - V respectively.

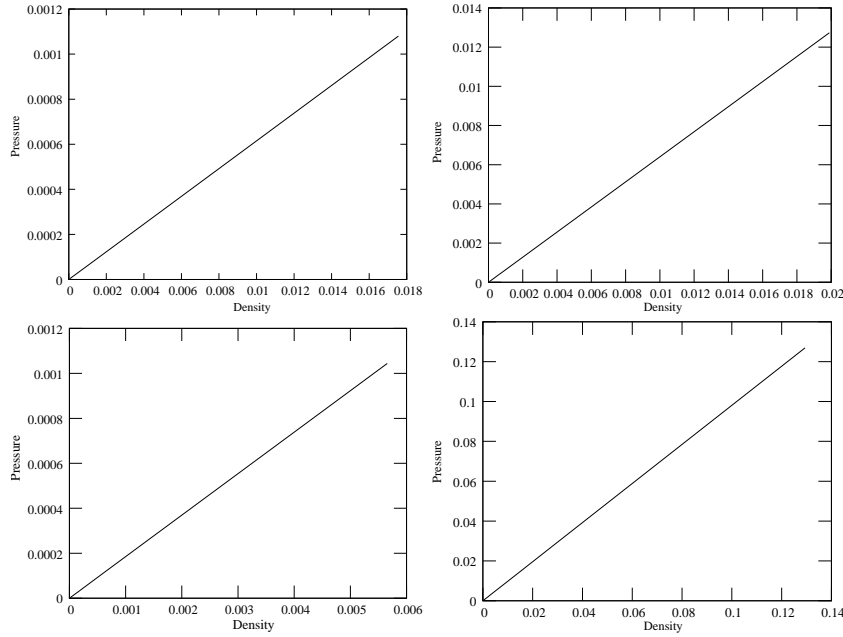


Fig. 4. Equation of states for $n=0.5$ (top left), $n=1.0$ (top right), $n=1.5$ (bottom left) & $n=2.0$ (bottom right) for Tolman's solutions for cases II - V respectively.

Case V: For $n = 2$

The gravitational mass in this case becomes

$$m = \frac{q^2}{a}. \quad (48)$$

This mass expression is exactly the same as that of the $n = 0$ case of our previous work (Ray & Das, 2004) which vanishes for the vanishing electric charge and thus provides 'electromagnetic mass' model (Lorentz, 1904; Feynman et al., 1964). However, the equation of state for the present situation differs from that of the $n = 0$ case and is given by

$$\rho + p = \frac{1}{2\pi r^2} \left[1 - \frac{2q^2}{a^2} \right]. \quad (49)$$

$$p = \frac{1}{8\pi r^2} \left[4 - \frac{9q^2}{a^2} \right] \quad (50)$$

To have positive pressure throughout the sphere the condition is $q/a < \pm 2/3$. It is also an interesting case that this condition gives us it to be a perfect fluid case having positive pressure and also it gives EMMM. Otherwise we can generalize all above conditions to one condition which is $q/a < \pm 1/3$ and by this we get positive

pressure for all cases and they are perfect fluid in nature. At the boundary, equating pressure to zero, we get $q/a = 2/3$. So, from Eq. (48) we get $m = (2/3)q$ which gives $q/m > 1$. In Figs. 3 & 4, we have plotted the equation of state of the cases described in Tolman's solutions for the 4 cases ($n=0.5, 1.0, 1.5$ & 2) where we have set the uniform condition $q/a < \pm 1/3$ for all the four cases. For all the above cases mass density is positive infinity at the centre and decreases from centre to surface. It is again to be stressed that the plots are shown here to give an implication of the nature or behavior of the solutions, and matching the variables to real units needs some normalization factors, which is not the primary purpose of this paper and will be shown elsewhere.

5. Conclusions

From discussion in section IV (A), we do not have any physically viable results for Bayin's solutions for case I, $n = 1, 3$ and case II, $n = 1$. All three cases give negative infinity pressure at the centre. In this context we would like to mention that idea of negative pressure is not new in astrophysics, especially in the realm of cosmology. The equation of state, viz., $\rho + p = 0$ for positive mass density provides a repulsive type negative pressure^{30,31,32,33}. In that sense, negative pressure in our solution is not unrealistic. However, for case II, $n = 3$ we get physically interesting results and hence have carried out detailed study. At origin, the equation of state is the same as that Bayin's²³ with neutral case. Throughout the sphere the equation of state is very complicated. Moreover, for Bayin's case pressure was a increasing function of radius from centre to surface and hence according to him it was not a physically reasonable case. But in our case due to inclusion of charge we get a condition by which pressure becomes a decreasing function of radius from centre to surface and thus becomes a physically acceptable case. Also, this perfect fluid case provides EMMM with positive pressure. Therefore, it gives an example which goes in contradiction to Ivanov's⁴ conclusion that "Electromagnetic mass models are subcases, often spoiled by negative pressure". For this case, we have also found out a relation between total mass and radius. For case III, $n = 1$, we get two conditions: one for negative infinity pressure with positive infinity density and the other one with reverse features, both at the origin of the sphere. For this case, we get also a mass-radius relation which gives again the same result in the absence of charge related to Bayin²³ case. It is interesting to note that for a particular value of $K = 1$, we get total mass of the spherical system equals the total charge. Unfortunately, here the fluid pressure becomes negative infinity in the centre. Otherwise, for other values of K we get finite values for the ratio of total charge to total mass.

In the Pant & Sah¹⁰ cases, which are charge analogue to Tolman VI type, we have a series of conditions to get positive as well as finite pressure and density throughout the sphere. However, from this series of conditions one can easily arrive at a general condition which is $q/a < \pm 1/3$. One particular case, for $n = 2$ gives EMMM with positive pressure under the perfect fluid condition. Again it is an

example which goes in contradiction to Ivanov's conclusion. We also notice that for $n = 0.5, 1.0, 1.5, 2.0$, total charge to total mass ratio becomes constant in each case and it is greater than unity. It reminds the general theorem given by de Felice et al. ³⁴ that *if the total electric charge of a perfect fluid ball is smaller than its total mass, then there is no regular static configuration having a radius arbitrarily close to the size of the external horizon*. Therefore, the above four cases again give support in favor of the stability of the charged spherical models in connection to normal stars.

Acknowledgements

One of the authors (SR1) is thankful to the authority of Inter-University Centre for Astronomy and Astrophysics, Pune, India, for providing him Associateship programme under which a part of this work was carried out. Support under UGC grant (No. F-PSN-002/04-05/ERO) is also gratefully acknowledged.

References

1. H. A. Buchdhal, Phys. Rev. **116**, 1027 (1959).
2. F. de Felice, Y. Yu and J. Fang, Mon. Not. R. Astron. Soc. **277**, L17 (1995).
3. R. Sharma, S. Mukherjee and S. D. Maharaj, Gen. Rel. Grav. **33**, 999 (2001).
4. B. V. Ivanov, Phys. Rev. D **65**, 104001 (2002).
5. W. B. Bonnor, Mon. Not. R. Astron. Soc. **129**, 443 (1965).
6. R. Stettner, Ann. Phys. (N.Y.) **80**, 212 (1973).
7. I. Glazer, Ann. Phys. **101**, 594 (1976).
8. I. Glazer, Astrophys. J. **230**, 899 (1979).
9. P. G. Whitman and R. C. Burch, Phys. Rev. D **24**, 2049 (1981).
10. D. N. Pant and A. Sah, J. Math. Phys. **20**, 2537 (1979).
11. R. C. Tolman, Phys. Rev. **55**, 364 (1939).
12. S. Ray, A. L. Espindola, M. Malheiro, J. P. S. Lemos & V. T. Zanchin, Phys. Rev. D **68**, 084004 (2003); astro-ph/0307262.
13. C. R. Ghezzi, Phys. Rev. D, **72**, 104017 (2005); gr-qc/0510106.
14. H. Weyl, Ann. Physik **54**, 117 (1917).
15. S. D. Majumdar, Phys. Rev. D **72**, 390 (1947).
16. A. Papapetrou, Proc. R. Irish Acad. **81**, 191 (1947).
17. A. Pannekoek, Bull. Astron. Insts. Netherlands, **1**, 107 (1922).
18. S. Rosseland, Mon. Not. R. Astron. Soc. **84**, 720 (1924).
19. A. S. Eddington, The Internal Constitution of the Stars (Cambridge Univ. Press, N. Y., 1926).
20. T. G. Cowling, Mon. Not. R. Astron. Soc. **90**, 140 (1929).
21. V. F. Shvartsman, Soviet Physics - JETP **33**, 475 (1971).
22. L. Neslusan, Astron. Astrophys. **372**, 913 (2001).
23. S. S. Bayin, Phys. Rev. **D18**, 2745 (1978).
24. S. Ray and B. Das, Astrophys. Space Sci. **282**, 635 (2002).
25. S. Ray and B. Das, Mon. Not. R. Astron. Soc. **349**, 1331 (2004).
26. S. Ray, astro-ph/0409527 (2004).
27. H. A. Lorentz, Proc. Acad. Sci. Amsterdam **6** (1904) (Reprinted in Einstein et al., The Principle of Relativity, Dover, INC, p. 24, 1952).

28. R. P. Feynman, R. B. Leighton and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Palo Alto, Vol. II, Chap. 28, 1964).
29. F.I. Cooperstock and V. De La Cruz, *Gen. Rel. Grav.* **9**, 835 (1978).
30. C. W. Davies, *Phys. Rev.* **D30**, 737 (1984).
31. J. J. Blome and W. Priester, *Naturwissenschaften* **71**, 528 (1984).
32. C. Hogan, *Nature* **310**, 365 (1984).
33. N. Kaiser and A. Stebbins, *Nature* **310**, 391 (1984).
34. F. de Felice, S. Liu and Y. Yu, *Class. Quan. Grav.* **16**, 2669 (1999).