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SQUEEZING AND DUAL RECYCLING IN LASER INTERFEROMETRIC GRAVITATIONAL WAVE DETECTORS

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ABSTRACT

We calculate the response of an ideal Michelson interferometer incorporating both dual recycling and squeezed light to gravitational waves. The photon counting noise has contributions from the light which is sent in through the input ports as well as the vacuum modes at sideband frequencies generated by the gravitational waves. The minimum detectable gravity wave amplitude depends on the frequency of the wave as well as the squeezing and recycling parameters. Both squeezing and the broadband operation of dual recycling reduce the photon counting noise and hence the two techniques can be used together to make more accurate phase measurements. The variance of photon number is found to be time-dependent. oscillating at the gravity wave frequency but of much lower order than the constant part.

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Laser-interferometric gravitational wave detectors [1] operate by sensing the difference in phase shifts imposed on the laser light in the two orthogonal arms of a Michelson type of interferometer by a gravitational wave. This phase shift manifests itself in the observed intensity change of the interference pattern. The sensitivity of the detector is determined by two fundamental sources of quantum mechanical noise : the photon counting error and the error originating due to fluctuations in radiation pressure on the mirrors. At the present level of the laser power available, the smallest detectable signal is limited by the photon counting statistics and various efforts have been made to increase the sensitivity level of these interferometers.

Caves[2] first realized that the photon number fluctuations at the output could be understood due to the interference of the vacuum fluctuations of light which enters through the unused input port of the beam splitter with the ingoing laser light. He suggested that instead a squeezed photon state could be injected through this port to reduce the photon counting noise. For a Michelson interferometer operating on a dark fringe, most of the light escapes towards the laser source. Therefore, it had been suggested [3] that this light can be recycled by putting a mirror in front of the source to enhance the sensitivity of the interferometer. This technique is known as Power recycling. Brillet et al[4] argued that squeezing and power recycling are compatible with each other and that both can be used together to improve the signal-to-noise ratio.

Gravitational waves modulate the phase of the laser light, thus generating sidebands which, travel towards the photodetector in an interferometer operating at the dark fringe [5]. These sidebands comprise the signal which can also be recycled by another mirror placed in front of the photodetector. The above method used in conjunction with power recycling is known as dual recycling[6].

It is, therefore, important to attempt an analysis of the quantum mechanical noise present in a dual recycling interferometer that also uses squeezed light and to investigate how well these two techniques work together. In this letter we report our results obtained for interferometers operating in the broadband mode. We arrive at a complete expression for the variance of the photon number fluctuations which is found to have a time-dependent component. The presence of sidebands significantly alters the noise. We calculate the minimum detectable gravitational wave amplitude as a function of its frequency as well as squeezing and recycling parameters and conclude that that the broad-band operation of dual recycling is compatible with the squeezed light technique and can therefore be used to enhance the sensitivity.

We first evaluate the minimum detectable phase difference with both squeezing and dual recycling without considering gravitational waves. Referring to Fig.1, monochromatic light beams (of angular frequency ω_0) in coherent and squeezed vacuum states enter ports 1 and 2 respectively. The annihilation operators a and b represent light in the coherent and squeezed modes respectively. One may now write down equations for the intra-cavity electric field operators, $E_{a'}$ and $E_{b'}$. We assume that the distance between the recycling mirrors and the 50:50 beam-splitter has been adjusted such that $E_{a'}$ and $E_{b'}$ add in phase with the ingoing modes, E_a and E_b respectively. We also assume that the beam-splitter introduces no phase shift upon reflection for a wave incident on the side of port 2 and a phase shift π for a wave reflected on the side of port 1. The quantities t_1 (t_2) and r_1 (r_2) represent the transmission and reflection coefficients of the power (signal) rec

One can obtain expressions for the annihilation operators a' and b' in terms of the input modes a and b. Then the annihilation operator describing the mode at the output of port 2 can be given as

$$\text{Out2} := t_2 b' - r_2 b = \frac{1}{M}[iat_1t_2\sin\theta + b\{t_2^2(\cos\theta - r_1) - r_2 M\}], \tag{1}$$

where

$$M = 1 + r_1r_2 - (r_1 + r_2) = (1 - r_1)(1 - r_2) \tag{2}$$

and θ is the phase difference of light between the two arms of the interferometer (at dark fringe, $\theta = 0$). Then the mean and the rms value of the photon number at the output port

2 are found to be

$$N = \frac{t_1^2 t_2^2 \sin^2\theta}{M^2}\bar{n} \tag{3}$$

and

$$\Delta N = \sqrt{\bar{n}}\frac{t_1 t_2 \sin\theta}{M^2}\left[t_1^2 t_2^2 \sin^2\theta + (t_2^2\cos\theta - r_1 t_2^2 - r_2 M)^2 e^{-2r}\right]^{1/2} \tag{4}$$

respectively, where $\bar{n}$ is the mean number of photons in the coherent beam and r is the squeeze factor. In these expressions, we have neglected terms with coefficients $\sinh^2 r$ since $\bar{n} \gg \sinh^2 r$. At the dark fringe most of the laser light escapes towards port 1. However, for a very small phase shift $\delta\theta$, we obtain a very small change in the mean number of photons, $\delta N(\theta)$ at the output port 2. Equating the change to the rms value, we. therefore obtain the minimum detectable phase $\delta\theta$ at a dark fringe to be

$$\delta\theta = \frac{e^{-r}}{\sqrt{\bar{n}}}\left[\frac{t_2^2(1-r_1) - r_2 M}{2t_1 t_2}\right]. \tag{5}$$

As can be easily seen, for values of r_1 and r_2 close to unity and a large squeeze factor, $\delta\theta$ is considerably reduced. This shows that squeezing and recycling are compatible with each other and can be used together to increase the sensitivity of the interferometer.

We now examine the case when a gravity wave of dimensionless amplitude. $h(t) = h_0 \sin\omega_g t$, propagating along the z-axis impinges on an interferometer whose arms are oriented along the x and y axes. If the gravity wave interacts with the laser beam of frequency ω_0, propagating along the y-axis for a time τ then the phase picked up by light as a function of time is

$$\delta\phi(t) = \frac{\omega_0 h_0}{2\omega_g}\int_{t-\tau}^{t}\sin(\omega_g t)\,dt = \epsilon_g \sin\omega_g(t - \frac{\tau}{2}), \tag{6}$$

where ω_0 is the laser light frequency and

$$\epsilon_g = \frac{\omega_0 h_0}{2\omega_g}\sin\frac{\omega_g \tau}{2}. \tag{7}$$

Due to the quadrupolar nature of the gravity wave the phase acquired by the laser beam travelling along the x-axis is $(-\delta\phi)$. The gravity wave thus modulates the phase of light in the two arms which gives rise to a time-dependent intensity [5].

The positive frequency part of the electric field operator propagating along the y-axis can be written as

$$E_+(t) = \int_0^\infty \frac{d\omega}{2\pi}\left(\frac{\hbar\omega}{2\epsilon_0 A_0}\right)^{1/2} a(\omega)e^{-i\omega t}e^{i\epsilon_g \sin\omega_g t}, \tag{8}$$

where ϵ_0 is the permitivity constant and A_0 is the cross-sectional area of the quantization volume. One can now show[7] that after modulation, the positive frequency part of the electric field operator can be written as :

$$E_+(t) = \int_0^\infty \frac{d\omega}{2\pi}\left(\frac{\hbar\omega}{2\epsilon_0 A_0}\right)^{1/2} e^{-i\omega t}\sum_{n=-\infty}^{+\infty}(1+\frac{n\omega_g}{\omega_0})^{1/2}J_n(\epsilon_g)a(\omega+n\omega_g), \tag{9}$$

where $a(\omega+n\omega_g)$ are the annihilation operators at newly-generated frequencies $\omega \pm \omega_n$ and $J_n(\epsilon_g)$ are the ordinary Bessel functions.

So, for any wave originally present with frequency ω, after modulation, one gets sidebands at $\omega \pm n\omega_g$. Since $J_n(\epsilon_g) \sim (\epsilon_g)^n$ for small ϵ_g and we are interested in terms of order $\mathcal{O}(\epsilon_g)$, we consider in our calculation only the first sideband on both sides. The sideband modes $a(\omega \pm \omega_g)$ and $b(\omega \pm \omega_g)$ which enter the interferometer through the ports 1 and 2 respectively are all in their vacuum states.

So, the intracavity electric field at the port 1 can be written as

$$E_{a'}(t') = \frac{e^{i\psi_1}}{2}\Big[(t_1E_a + r_1E_{a'} + r_2E_{b'} + t_2E_b)e^{-i\delta\phi(t)}e^{-i\theta}$$
$$+ (t_1E_a + r_1E_{a'} - r_2E_{b'} - t_2E_b)e^{i\delta\phi(t)}e^{i\theta}\Big], \tag{10}$$

where θ is the constant phase offset between the two arms of the interferometer; $t' = t+L/c$ and L is twice the arm length. The phase ψ_1 is that acquired by light in traversing twice the path between the beam-splitter and the power recycling mirror.

We can now write the Fourier transform of each electric field and then pull in the time dependent phase factor inside the integral to obtain

$$\int \tilde{E}_{a'}(\Omega)e^{-i\Omega t'}\,d\Omega = \frac{e^{i\psi_1}}{2}\Bigg[\Bigg(t_1\int \tilde{E}_a(\Omega)e^{-i\Omega t}e^{-i\delta\phi(t)}\,d\Omega + r_1\int \tilde{E}_{a'}(\Omega)e^{-i\Omega t}e^{-i\delta\phi(t)}\,d\Omega$$
$$+ r_2\int \tilde{E}_{b'}(\Omega)e^{-i\Omega t}e^{-i\delta\phi(t)}\,d\Omega + t_2\int \tilde{E}_b(\Omega)e^{-i\Omega t}e^{-i\delta\phi(t)}\,d\Omega\Bigg)e^{-i\theta}$$
$$+\Bigg(t_1\int \tilde{E}_a(\Omega)e^{-i\Omega t}e^{i\delta\phi(t)}\,d\Omega + r_1\int \tilde{E}_{a'}(\Omega)e^{-i\Omega t}e^{i\delta\phi(t)}\,d\Omega$$
$$- r_2\int \tilde{E}_{b'}(\Omega)e^{-i\Omega t}e^{i\delta\phi(t)}\,d\Omega - t_2\int \tilde{E}_b(\Omega)e^{-i\Omega t}e^{i\delta\phi(t)}\,d\Omega\Bigg)e^{i\theta}\Bigg]. \quad (11)$$

Now, we can easily see that the presence of $e^{-i\delta\phi(t)}$ factor inside each integral sign leads to phase modulation and subsequently to the generation of sidebands. We confine our attention only to the first sideband on both sides. The constant phase factor $\theta = \Omega\Delta L$ is different for different frequencies, but, for simplicity, we assume it to be the same for all frequencies since the difference is very small ($\omega_g \ll \omega_0$). We consider the coefficients r_1, r_2, t_1, t_2 to be independent of frequency. Since $\omega_g \ll \omega_0$, we set the factor $(1+\omega_g/\omega) \to 1$ in Eq.(9).

Now, one can write equations for the outgoing (primed) annihilation operators for $\Omega = \omega,\ \omega \pm \omega_g$ after dividing throughout by the same normalization constant.

$$\begin{aligned} a'(\Omega)\exp[-i(\Omega L/c + \psi_1(\Omega))] &= Ae^{-i\theta} + Be^{+i\theta},\\ b'(\Omega)\exp[-i(\Omega L/c + \psi_2(\Omega))] &= Ae^{-i\theta} - Be^{+i\theta}, \end{aligned} \quad (12)$$

where

$$\begin{aligned} A =& t_1\sum_n a(\Omega+n\omega_g)J_n(-h) + r_1\sum_n a'(\Omega+n\omega_g)J_n(-h)\\ &+ r_2\sum_n b'(\Omega+n\omega_g)J_n(-h) + t_2\sum_n b(\Omega+n\omega_g)J_n(-h),\\ B =& t_1\sum_n a(\Omega+n\omega_g)J_n(+h) + r_1\sum_n a'(\Omega+n\omega_g)J_n(+h)\\ &- r_2\sum_n b'(\Omega+n\omega_g)J_n(+h) - t_2\sum_n b(\Omega+n\omega_g)J_n(+h), \end{aligned} \quad (13)$$

where the index n can take values 0 and ±1. The important quantity in the above equation is the phase factor $(\Omega L/c + \psi_i(\Omega))$ that appears on the left hand side. The phase ψ_i will be different for different frequencies Ω. However, the point to be noted here is that we adjust the distance between the recycling mirrors and beamsplitter in such a way that these phase factors become unity – a condition called 'on resonance'. This esssentially means that the laser light as well as the sidebands are resonant with the cavities formed by the recycling mirrors. This is termed as the broad-band operation of dual recycling.

We now have six coupled equations for a' and b' at three different frequencies (i.e. ω and $\omega \pm \omega_g$). One can arrange these equations in the following matrix form

$$P_{ki} A'_i = Q_{kj} A_j, \tag{14}$$

where P_{ki} and Q_{kj} are two 6×6 matrices and

$$\begin{aligned} A'_i \equiv & \text{Transpose}(a'_0,\, a'_-,\, a'_+,\, b'_0,\, b'_-,\, b'_+), \\ A_j \equiv & \text{Transpose}(a_0,\, a_-,\, a_+, b_0,\, b_-,\, b_+\,), \end{aligned} \tag{15}$$

where (and from now onwards) indices 0, $(-)$ and $(+)$ correspond to $n = 0$, -1 and $+1$ respectively. So, for example, $a_0 \equiv a(\omega_0)$, $a_- \equiv a(\omega_0 - \omega_g)$, $b_+ \equiv b(\omega_0 + \omega_g)$ etc.

So, all the six equations can be solved and the six primed annihilation operators can be written in terms of the six input (unprimed) annihilation operators through a 6×6 matrix, $P^{-1}_{ki} Q_{kj}$. If we define a_μ and b_μ as two $3\tilde{1}$ column vectors, i.e., a_μ = Transpose(a_0, a_-, a_+) and a_μ = Transpose(a_0, a_-, a_+), one can write the output fields at port 2 in a simple form

$$\begin{aligned} c_\alpha =& t_2 b'_\alpha - r_2 b_\alpha \\ =& X_{\alpha\mu} a_\mu + Y_{\alpha\nu} b_\nu, \end{aligned} \tag{16}$$

where $X_{\alpha\mu}$ and $Y_{\alpha\nu}$ are two 3×3 matrices and (from now onwards) the greek indices take values 0, $(-)$ and $(+)$. The values of different components of the 3×3 matrices $X_{\alpha\mu}$ and $Y_{\alpha\nu}$ are given below

$$X_{00} = X_{--} = X_{++} = -i \frac{t_1 t_2 \sin\theta}{(1 - r_1)(1 - r_2)},$$

$$Y_{00} = Y_{--} = Y_{++} = 1, \tag{17}$$

$$X_{+-} = X_{-+} = Y_{+-} = Y_{-+} = 0.$$

The components of order $\mathcal{O}(\epsilon_g)$ are

$$\begin{aligned} X_{-0} = -X_{0+} = X_{0-} = -X_{-0} &= \epsilon_g \frac{t_1 t_2}{(1-r_1)(1-r_2)}, \\ Y_{0+} = -Y_{+0} = Y_{-0} = -Y_{0-} &= i\epsilon_g \frac{t_2^2(1+r_1)\sin\theta}{(1-r_1)(1-r_2)} \end{aligned} \qquad (18)$$

We essentially follow references [7,8] for the expression for the time-dependent photocurrent, $\hat{N}$.

$$\begin{aligned} \hat{N} &= \sum_{\mu,\nu} c_\mu^\dagger c_\nu \\ &= X_{\mu\alpha}^* X_{\nu\beta} a_\alpha^\dagger a_\beta + Y_{\mu\alpha}^* Y_{\nu\beta} b_\alpha^\dagger b_\beta + X_{\mu\alpha}^* Y_{\nu\beta} a_\alpha^\dagger b_\beta + Y_{\mu\alpha}^* X_{\nu\beta} b_\alpha^\dagger a_\beta. \end{aligned} \qquad (19)$$

The mean number of photons, $\bar{N}$ is made up of a constant part, $\bar{N}_0 = \sum_\alpha c_\alpha^\dagger c_\alpha$ and the time-dependent part $\delta\bar{N}(t) = \sum_{\alpha\neq\beta} c_\alpha^\dagger c_\beta$. The latter is essentially due to the beating of modes of two different frequencies which gives rise to the time-dependent part at ω_g. There is also a time-dependent part at $2\omega_g$ but of order $\mathcal{O}(\epsilon_g^2)$ and so we neglect it. Hence we get

$$<\hat{N}> = \bar{n}_0 + \delta I(t) = \bar{N}\frac{t_1^2 t_2^2 \sin\theta}{(1-r_1)^2(1-r_2)^2}[\sin\theta + 4\epsilon_g \sin\omega_g t]. \qquad (20)$$

The variance in photon number is given by

$$\begin{aligned} (\Delta\bar{N})^2 &= <\hat{N}^2> - <\hat{N}>^2 \\ &= \bar{n}\left[\frac{3t_1^4 t_2^4 \sin^4\theta}{(1-r_1)^4(1-r_2)^4} + \frac{2t_1^2 t_2^2 \sin^2\theta}{(1-r_1)^2(1-r_2)^2} + \frac{t_1^2 t_2^2 \sin^2\theta}{(1-r_1)^2(1-r_2)^2}\epsilon^{-2r}\right] \\ &+ \bar{n}\epsilon_g \sin\omega_g t\left[\frac{20 t_1^4 t_2^4 \sin^3\theta}{(1-r_1)^4(1-r_2)^4} + \frac{8t_1^2 t_2^2 \sin\theta}{(1-r_1)^2(1-r_2)^2} - \frac{4t_1^2 t_2^4 (1+r_1)\sin^3\theta}{(1-r_1)^3(1-r_2)^3}\right. \\ &\left. + 4\epsilon^{-2r}\left(\frac{t_1^2 t_2^2 \sin\theta}{(1-r_1)^2(1-r_2)^2} - \frac{t_1^2 t_2^2(1+r_1)\sin^3\theta}{(1-r_1)^3(1-r_2)^3}\right)\right]. \end{aligned} \qquad (21)$$

The terms appearing within the brackets of the constant part of Eq.(21) can be explained as follows: the first term is due to the coherent excitations (a_0) superposed with coherent fluctuations as well as with the vacuum fluctuations of a_- and a_+. The second term is coherent excitations superposed with vacuum fluctuations from b_- and b_+, whereas the third term is due to the interference between squeezed and coherent light. The time-dependence of $\Delta\bar{N}$ arises due to the beating of the time-dependent part of $\bar{N}$ with its constant part. Everywhere, we have neglected terms representing squeezed fluctuations being superposed on all the vacuum fluctuations since $\bar{n} \gg \sinh^2 r$. The variance being time-dependent would mean that the frequencies separated by ω_g are correlated although the spectrum is white. This has been referred to in the literature [9] as modulated shot noise.

The minimum detectable gravity wave amplitude h_0 is now obtained by setting equal the maximum value of the 'signal', $\delta\bar{N}(t)$ to the maximum value of $\Delta\bar{N}$. This gives us a quadratic equation in h_0

$$h_0^2 = \frac{4\omega_g^2}{\bar{n}\omega_0^2 \sin^2(\omega_g\tau/2)} \left[\frac{3}{16}\sin^2\theta + \frac{(1-r_1)^2(1-r_2)^2}{8t_1^2t_2^2} + \frac{(1-r_1)^2(1-r_2)^2}{t_1^2t_2^2}\frac{e^{-2r}}{16}\right]$$
$$+ \frac{h_o}{\bar{n}}\frac{2\omega_g}{\omega_0\sin(\omega_g\tau/2)}\left[\frac{5}{4}\sin\theta + \frac{1}{2}\frac{(1-r_1)^2(1-r_2)^2}{t_1^2t_2^2\sin\theta} - \frac{(1-r_2)\sin\theta}{4}\right.$$
$$\left. + e^{-2r}\left(\frac{(1-r_1)^2(1-r_2)^2}{4t_1^2t_2^2} - \frac{(1-r_1)\sin\theta}{4t_2^2}\right)\right]. \tag{22}$$

Since h_0 is already very small, we neglect the term $h_0/\bar{n}$. We finally arrive at the expression for h_0

$$h_0 = \frac{2\omega_g}{\omega_0\sqrt{\bar{n}}\sin(\omega_g\tau/2)}\sqrt{\frac{3}{16}\sin^2\theta + \frac{(1-r_1)^2(1-r_2)^2}{8t_1^2t_2^2} + \frac{(1-r_1)^2(1-r_2)^2}{t_1^2t_2^2}\frac{e^{-2r}}{16}}. \tag{23}$$

At the dark fringe ($\theta = 0$) the first term is negligibly small. For a large squeeze factor r and values of reflection coefficients r_1 and r_2 close to unity, h_0 can be considerably reduced.

If losses are introduced in the recycling mirrors, it would be possible to optimize h_0 in terms of the parameters r, r_1, r_2. Experimentalists usually implement internal [10], external [11] phase modulation (at MHz frequency) so that measurements of intensity can be made at sufficiently high frequency where the noise is really shot noise limited. Using the equations described above, it should be possible to include both internal (or external) and gravity wave modulation (including losses) and calculate the minimum detectable gravity wave amplitude. Work in this direction is currently being pursued and will be communicated in future[12].

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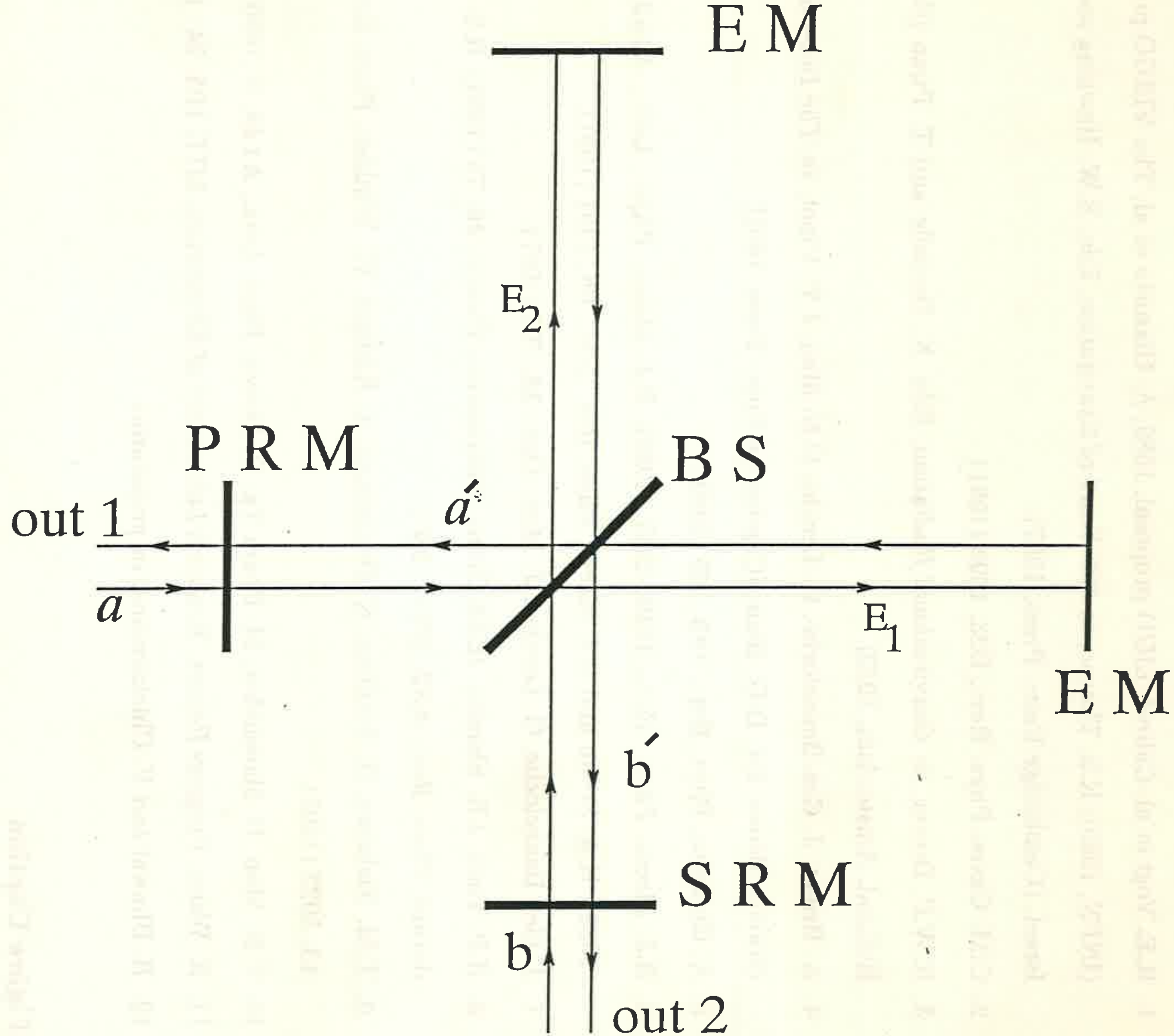
Figure Caption

Fig.1: A schematic diagram for dual recycling. BS- Beam-Splitter, EM- End Mirror, PRM- Power Recycling Mirror, SRM- Signal Recycling Mirror.


E M
E M
P R M
B S
S R M
out 1
a
a´
E2
E1
b
b´
out 2