MOTION OF TEST PARTICLES AROUND MONOPOLES

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Abstract :

We present a detail analysis of the motion of test particles around gauge and global monopoles using the Hamilton-Jacobi (H-J) formalism. We find that particles cannot be trapped by gauge monopoles while there may exist bound orbits for global monopoles under certain conditions.

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1. Introduction

Monopoles are point-like topological objects that may arise during phase transitions in the early universe [1,2]. In particular ,if $\pi_2(\mu) \neq I$ (μ is the vacuum manifold) i.e. if μ contains surfaces which can not be continuously shrunk to a point [1] then monopoles are formed. When monopoles are formed due to a gauge symmetry breaking , then they are termed as gauge monopoles. They are similar to elementary particles with finite energy and its mass is concentrated in a very tiny core.Usually,grand unified theories predict such monopoles [3] . Global monopoles on the other hand ,result from a global symmetry breaking . They have a linearly divergent mass, due to the long - range Nambu-Goldstone field[4,5].

In this paper,we study the motion of test particles in the gravitational field of gauge and global monopoles using H-J formalism and examine whether bound orbits are possible or not.

2. The gauge monopoles

The metric ansatz for gauge monopole is [4]

$$ds^2 = -dt^2 + dr^2 + (1 - 8G\pi\eta^2)r^2(d\theta^2 + sin^2\theta d\phi^2). \quad (1)$$

It describes a spherically symmetric space-time with a deficit solid angle.

We consider a relativistic particle with mass m moving in the field of the gauge monopole (1). The H-J equation is

$$-\left(\frac{\partial S}{\partial t}\right)^2 + \left(\frac{\partial S}{\partial r}\right)^2 + \frac{(1-8\pi G\eta^2)^{-1}}{r^2}\left[\left(\frac{\partial S}{\partial \theta}\right)^2 + Cosec^2\theta\left(\frac{\partial S}{\partial \theta}\right)^2\right] + m^2 = 0 \quad (2)$$

As there are no explicit dependence of t and ϕ, so a natural form for $S(r,\theta,\phi,t)$ will be

$$S(r,\theta,\phi,t) = -E.t + S_1(r) + S_2(\theta) + J.\phi, \tag{3}$$

Where the constants E and J are identified as the energy and angular momentum of the particle. If we substitute the ansatz (3) for S in the $H-J$ equation(2) then we get the following expressions (in integral form) for the unknown functions S_1 and S_2 :

$$S_1(r) = \epsilon \int \left[E^2 - m^2 - p^2/(1-8\pi G\eta^2)r^2\right]^{1/2} dr \tag{4}$$

and

$$S_2(\theta) = \epsilon \int \left[p^2 - J^2.Cosec^2\theta\right]^{1/2} d\theta. \tag{5}$$

Here $\epsilon = \pm 1$, stands for the sign changing whenever r (or θ) passes through a zero of the integrand [6] in (4) (or 5) and p is a constant .

For determination of the trajectory of the particle following $H-J$ method, we consider [7]

$$\frac{\partial S}{\partial E} = constant,$$

$$\frac{\partial S}{\partial J} = constant,$$

and

$$\frac{\partial S}{\partial p} = constant.$$

(Without loss of generality one can consider the constants to be zero.)

Hence we get

$$t = \epsilon \int \frac{E}{\sqrt{E^2 - m^2 - \frac{\alpha^2}{r^2}}} dr, \alpha^2 = p^2/(1 - 8\pi G\eta^2), \tag{6}$$

$$\phi = \epsilon \int \frac{J Cosec^2\theta}{\sqrt{p^2 - J^2 Cosec^2\theta}} d\theta, \tag{7}$$

and

$$Cos^{-1}(\frac{Cos\theta}{\gamma}) = \int \frac{\alpha^2}{pr^2(E^2 - m^2 - \alpha^2/r^2)^{1/2}} dr, \gamma^2 = 1 - \frac{J^2}{p^2}. \tag{8}$$

From (6) the radial velocity of the particle is

$$\frac{dr}{dt} = \frac{1}{E}(E^2 - m^2 - \alpha^2/r^2)^{1/2}. \tag{9}$$

The turning points of the trajectory are given by $dr/dt = 0$ and as a consequence the potential curves are [7]

$$\frac{E}{m} = (1 + \alpha^2/r^2)^{1/2}. \tag{10}$$

As the radicand in the above expression is a monotonically decreasing function, so it does not have extremals. So particles can not be trapped by gauge monopoles. This is expected from the result of Vilenkin [4] that monopole described by metric (1) exerts no gravitational force on the matter around it.

3. The global monopoles

The line element describing global monopole can be taken to be

$$ds^2 = -F(r)dt^2 + H(r)dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2, \qquad (11)$$

where

$$F(r) = \{1 - 8(1+\beta)\pi G\eta^2 - Gm/r\}$$

and

$$H(r) = (1 + 8\pi G\eta^2 + Gm/r).$$

(notations are same as in [8] .)

We now examine whether the previous results persist for the global monopole.In this case ,the H-J equation is

$$F(r)^{-1}\left(\frac{\partial S}{\partial t}\right)^2 + H(r)^{-1}\left(\frac{\partial S}{\partial r}\right)^2 + r^{-2}\left(\frac{\partial S}{\partial \theta}\right)^2 + r^{-2}Cosec^2\theta\left(\frac{\partial S}{\partial \phi}\right)^2 + m_0^2 = 0, \qquad (12)$$

with m_0 ,the mass of the relativistic particle. As before , we use the method of separation of variables to solve the above partial differential equation ($H-J$ equation) and we write

$$S(r,\theta,\phi,t) = -Et + S_1(r) + S_2(\theta) + J\phi, \qquad (13)$$

with

$$S_1(r) = \epsilon \int \left(\frac{H}{F}\right)^{1/2} \left[E^2 - F(r)(m_0^2 + p^2/r^2)\right]^{1/2} dr, \qquad (14)$$

and

$$S_2(\theta) = \epsilon \int \sqrt{p^2 - J^2 cosec^2\theta} d\theta. \tag{15}$$

Following H-J method ,the equation of the trajectories can be written as

$$t = \epsilon \int \left(\frac{H}{F}\right)^{1/2} \frac{E}{[E^2 - F(r)(m_0^2 + p^2/r^2))]^{1/2}} dr, \tag{16}$$

$$\phi = \epsilon \int J cosec \frac{d\theta}{(p^2 sin^2\theta - J^2)^{1/2}}, \tag{17}$$

and

$$Cos^{-1}(\frac{Cos\theta}{\gamma}) = \int \{F(r)H(r)\}^{1/2} \frac{p}{r^2} \frac{dr}{(E^2 - F(r)m_0^2 + p^2/r^2))^{1/2}}. \tag{18}$$

From (16) ,the radial velocity has the expression

$$E\frac{dr}{dt} = \left(\frac{F}{H}\right)^{1/2} \{E^2 - F(r)(m_0^2 + p^2/r^2)\}^{1/2}, \tag{19}$$

So at the turning points $(dr/dt = 0)$

$$\frac{E}{m_0} = [F(r)(1 + \frac{p^2}{m_0^2 r^2})]^{1/2}, \tag{20}$$

and determine the corresponding potential curves.

We note that if

$$\frac{p}{m_0} > 3GM\{1 + 8(1+\beta)\pi G\eta^2\},$$

then the above radicand has no real extremals. Hence there is no bound state of the trajectory and the particle can not be trapped by the monopole. But if

$$\frac{p}{m_0} < 3GM\{1 + 8(1+\beta)\pi G\eta^2\},$$

then there are two real extremals at

$$r = \frac{p^2(1 - S(1+\beta)\mu)}{Gmm_0^2}\left[1 \pm \sqrt{1 - \frac{3m_0^2G^2m^2}{p^2(1-16(1+\beta)\pi G\eta^2)}}\right], \tag{21}$$

and the trajectory of the test particle is bounded i.e. particle can be trapped by global monopole.

Finally, we note that $r = GM/(1 - 8(1+\beta)\pi G\eta^2) = r_0$ corresponds to an event horizon where spatial and temporal co-ordinates interchange their roles and the regions $r < r_0$ and $r > r_0$ can not influence each other.

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