Stabilization of branes in a cosmological setting

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We study some cosmological consequences of the five dimensional, two brane Randall-Sundrum scenario. We integrate over the extra dimensions and in four dimensions the action reduces to that of scalar tensor gravity. The radius of the compact extra dimension is taken to be time dependent. It is shown that the radius of the extra dimension rapidly approaches a constant nonzero separation of branes. A radion dominated universe cannot undergo accelerated expansion in the absence of a potential. It is shown that a simple quadratic potential with minimum at zero leads to constant nonzero separation of branes in a similar level but now accelerated expansion is possible. After stabilization the quadratic potential contributes an effective cosmological constant term. We show that with a suitable tuning of parameters the requirements for solving the hierarchy problem and getting an effective dark energy can be satisfied simultaneously.

I. INTRODUCTION

In this paper, we study some cosmological implications of five-dimensional warped geometry proposed by Randall and Sundrum [1]. The Randall-Sundrum model consists of two four dimensional branes which are defects in a five dimensional anti-deSitter background. One of the branes is a positive tension Planck brane and the other is the brane on which standard model particles are confined; this has a negative tension and is called the TeV brane. The hierarchy between the four dimensional Planck scale and the fundamental scale of the theory is resolved because of the presence of the exponential warp factor.

The five-dimensional spacetime is a slice of anti-deSitter geometry, where we have a negative cosmological constant. Two 3-branes are located at fixed points of orbifold S^1/Z_2. In other words, the extra fifth dimension is a circle with opposite points identified. We take the two orbifold points to be situated at $y = 0$ and $y = 1/2$ [2]; the positive tension Planck brane is located at $y = 0$ and the negative tension TeV brane is situated at $y = 1/2$.

The action for the five dimensional anti-deSitter spacetime is given by

$$\begin{aligned} S &= 2\int d^4x \int_0^{1/2} dy\sqrt{-G}\left(M^3R - l\right) \\ &+ \int d^4x\sqrt{-g^{(+)}}\left(L^+ - V^+\right) \\ &+ \int d^4x\sqrt{-g^{(-)}}\left(L^- - V^-\right) \end{aligned} \tag{1}$$

where

$$V^+ = -V^- = 12m_0M^3, \quad l = -12m_0^2M^3, \tag{2}$$

the five dimensional Ricci scalar is denoted by R, the bulk cosmological constant is given by l and M is the five dimensional Planck mass. The $(+)$ sign denotes the Planck brane and $(-)$ sign represents the negative tension TeV brane. The matter fields on the positive and negative tension branes are L^+ and L^- respectively, while $V^\pm$ represent the brane tensions on positive and negative tension branes respectively.

The metrics on the two four-dimensional branes are therefore given by

$$\begin{aligned} g_{\mu\nu}^{(+)} &= G_{\mu\nu}(x^\mu, y = 0) \ \text{ and} \\ g_{\mu\nu}^{(-)} &= G_{\mu\nu}(x^\mu, y = 1/2) \end{aligned} \tag{3}$$

The five-dimensional Einstein equations are solved by the metric

$$ds^2 = e^{-2m_0r_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2dy^2 \tag{4}$$

where $\eta_{\mu\nu}$ represents the flat four dimensional 3-brane while r_c is the radius of the extra dimension. This model solves the mass hierarchy problem in particle physics and has therefore been a subject of extensive study.

To study cosmology of the brane worlds, the radius of the extra dimension is taken to be time dependent. It was shown that in order that the brane world models be consistent with observations, the separation between the branes, the radion, should be a constant [3]. In general, the presence of bulk scalar fields achieves this constant separation [4]. In four dimensions the theory reduces to that of a scalar tensor gravity with the Brans-Dicke factor which is a function of the scalar field [5] (for a review on brane cosmologies, see [6]). In view of the present observations and belief that the universe is undergoing an accelerated expansion (for a review see [7]), we investigate if the stabilizing potential provides the cosmological constant contribution to the energy density of the universe. For various scalar field models of dark energy see Refs. [8, 9, 10, 11, 12].

The paper is organized as follows. In Section II we study cosmology of the brane world model. It is shown that with a simple quadratic radion field potential with minimum at zero, we achieve stabilization as well as late time acceleration in the evolution. Section III shows that a quadratic potential with nonzero minimum changes the picture and we have the Hubble parameter oscillating about the average evolution. The main conclusions of this paper are summarized in Section IV.

II. COSMOLOGY IN BRANE WORLD SCENARIO

For cosmological solutions, we assume the modulus r_c to be time dependent [2, 13, 14]. The five-dimensional metric ansatz is [2]

$$ds^2 = e^{-2m_0 b(t)|y|} g_{\mu\nu} dx^\mu dx^\nu + b^2(t) dy^2 \tag{5}$$

with the four-dimensional spacetime being described by the spatially flat Friedmann-Robertson-Walker metric

$$g_{\mu\nu} = \mathrm{diag}(-1, a^2(t), a^2(t), a^2(t)) \tag{6}$$

where $a(t)$ is the scale factor. This scale factor is different from what an observer on the negative tension brane will see and will be discussed later.

Using the above metric ansatz, we integrate over the extra dimension in the action given in Eq. (1). The four dimensional action can be written as

$$\begin{aligned} S_{eff} &= -\frac{3}{k^2 m_0} \int d^4x \left[\left(1 - ø^2\right) \frac{\dot{a}^2}{a^2} \right. \\ &+ \left. m_0 ø^2 \frac{\dot{a}}{a} \dot{b} - \frac{1}{4} m_0^2 ø^2 \dot{b}^2 \right] \end{aligned} \tag{7}$$

where $ø = e^{-m_0 b(t)/2}$ and $k^2 = 1/2M^3$.

In the action, we add a term $V(b(t))$, assuming a potential associated with the radion field $b(t)$ which we will discuss later. The action can be further written as

$$\begin{aligned} S_{eff} &= -\frac{1}{2k^2 m_0} \int d^4x a^3(t) \left[(1 - ø^2) R_4 \right. \\ &- \left. \frac{3}{2} m_0^2 ø^2 \dot{b}^2 + V(b(t)) \right] \end{aligned} \tag{8}$$

The four dimensional Ricci scalar is denoted by R_4. The action can be reduced to the standard Brans-Dicke scalar tensor gravity if we identify scalar field $\phi = 1 - ø^2$, with Brans-Dicke factor $W(\phi)$ given by $W(\phi) = \frac{3}{2}\frac{\phi}{1-\phi}$.

The cosmological equations of motion obtained from this action are

$$\begin{aligned} 3\frac{\dot{a}^2}{a^2} &= \frac{3\dot{ø}^2}{1-ø^2} + 6\frac{\dot{a}}{a}\frac{ø\dot{ø}}{1-ø^2} + \frac{1}{2}\frac{V(ø)}{1-ø^2} \\ 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} &= -\frac{\dot{ø}^2}{1-ø^2} + 4\frac{\dot{a}}{a}\frac{ø\dot{ø}}{1-ø^2} + \frac{2ø\ddot{ø}}{1-ø^2} \\ &+ \frac{1}{2}\frac{V(ø)}{1-ø^2} \\ 6\frac{\ddot{ø}}{ø} &+ 18\frac{\dot{a}}{a}\frac{\dot{ø}}{ø} = 2V + \frac{(1-ø^2)}{2ø}\frac{dV}{dø} \end{aligned} \tag{9}$$

where $H = \frac{\dot{a}(t)}{a(t)}$, $ø(t) = e^{-m_0 b(t)/2}$.

Only two of the above three equations are independent. We transform variables to $\phi = 1 - ø^2$ and rewrite two of the equations

$$\begin{aligned} \frac{\dot{a}^2}{a^2} &= \frac{\phi}{4(1-\phi)}\frac{\dot{\phi}^2}{\phi^2} - \frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{1}{6}\frac{V(\phi)}{\phi} \\ \ddot{\phi} + 3H\frac{\dot{\phi}}{\phi} &= -\frac{1}{2}\frac{1}{(1-\phi)}\dot{\phi}^2 \\ &+ (1-\phi)\frac{1}{3}\left[2V(\phi) - \phi\frac{dV(\phi)}{d\phi}\right] \end{aligned} \tag{10}$$

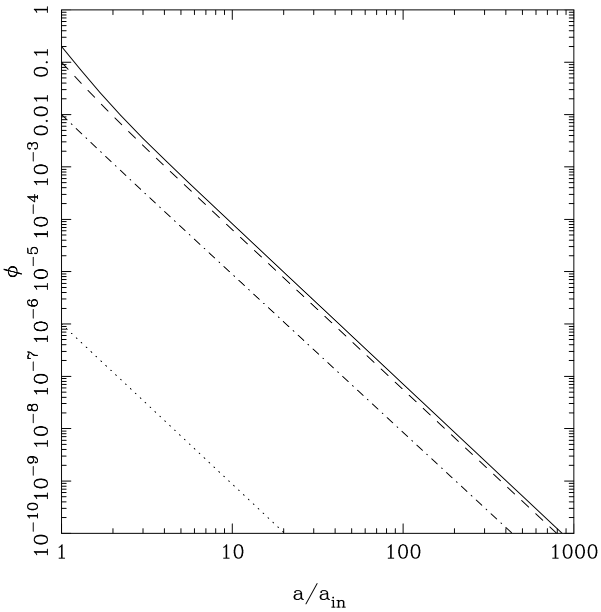

FIG. 1: The plot shows the variation of $\dot{\phi}$ with the scale factor. It is clear in the figure that $\dot{\phi} \propto a(t)^{-3}$. This holds for no potential case and as well as the case for a quadratic potential with a minimum at zero. The slight curve at very small a/a_{in} is due to the $1 - \phi$ term in Eq. 11.

The scalar field equation of motion in Eqs. 10 has a critical point at $\phi = 1$ and $\dot{\phi} = 0$ for a generic potential. The branes can therefore stabilize only at infinite separation for an arbitrary, featureless potential [18]. For a solution to the hierarchy problem, one needs the branes to be at a finite separation.

In the absence of a potential, any value of ϕ is consistent $\dot{\phi} = 0$. Therefore the system converges to a finite separation of branes; the final value of ϕ depends on the initial conditions. For this case, the equation of motion for the scalar field can be reduced to

$$\frac{d}{dt}\left[\frac{\dot{\phi}a^3}{\sqrt{1-\phi}}\right] = 0 \tag{11}$$

It is clear from this expression that in most cases, the brane separation stabilizes at the rate which inverse cube of the scale factor. Therefore, the time taken for stabilization depends on the dominant matter field in the universe. It consequently stabilizes hierarchy between the different scales (for an extensive literature on modulus stabilization and the cosmological implications see [4, 15] and references therein). The derivative of the scalar field $\dot{\phi}$ drops to very small value within a few expansion factors in the scale factor and therefore from cosmological point of view may be considered to be negligible. As ϕ approaches zero, the radion field stops contributing to the cosmological evolution. Even though $\dot{\phi} \neq 0$, and hence strictly speaking the system does not reach a stable point and tends towards it only asymptotically, however, as the rate of change of the brane configuration becomes very small compared to the hubble parameter in a very short time, the system is essentially not changing. This is indeed a remarkable result in that the brane system is stabilized to some value by the cosmological expansion, but the radion field does not affect the cosmological expansion after reaching a stable configuration.

III. QUADRATIC POTENTIAL

A. Potential with minimum at zero

From the scalar field equation we conclude that if we have a quadratic potential, $V(\phi) = V_o\phi^2$ (see also [16]), the term in square brackets in Eqs. 10 vanishes and the branes stabilize at any separation. We get the same evolution behaviour for $\dot{\phi}$ as in Eq. 11. However, unlike the no potential case, the quadratic potential contributes a cosmological constant term in the Friedmann equations. This is a desired feature as the present observations indicate that the expansion of the universe is accelerating [17]. Thus the quadratic potential can solve two problems at the same time.

Once the radion field settles down, the scale factor and time t are scaled by a constant. The scale factor obtained in these coordinates after stabilization has the same interpretation as the scale factor $Y(\tau) = e^{-m_0 b(t)/2}a(t)$ from the point of the view of an observer on the brane. The equations of motion then remain the same with the factor V_0 scaled by $e^{m_0 b_0}$ where b_0 is the value of the stabilized field. We can therefore consider the scale factor $a(t)$ to be describing the 'physical' scale factor.

B. Numerical solutions with a quadratic potential

We solve the equations of motion with potential $V = V_0\phi^2$ numerically to substantiate the points discussed above. We rescale the equations with the initial Hubble parameter by making the following change $x = tH_{in}$ and $y = a/a_{in}$. We consider the presence of nonrelativistic and relativistic matter only on the four dimensional negative tension brane. The equations transform to

$$\begin{aligned} \frac{y'^2}{y^2} &= \frac{\Omega_{M_{in}}}{y^3} + \frac{\Omega_{R_{in}}}{y^4} + \frac{\phi}{4(1-\phi)}\frac{\phi'^2}{\phi^2} \\ &- \frac{y'}{y}\frac{\phi'}{\phi} + \frac{1}{6}\frac{V(\phi)}{\phi H_{in}^2} \\ \ddot{\phi} + 3\frac{y'}{y}\frac{\phi'}{\phi} &= -\frac{1}{2}\frac{1}{(1-\phi)}\phi'^2 \\ &+ (1-\phi)\frac{1}{3H_{in}^2}\left[2V(\phi) - \phi\frac{dV(\phi)}{d\phi}\right] \end{aligned} \tag{12}$$

where $\Omega_{M_{in}}, \Omega_{R_{in}}$ are density parameters for nonrelativistic and relativistic matter respectively. The initial value of x is arbitrary, $y_{in} = \frac{a}{a_{in}} = 1$ and we vary the initial values of ϕ and ϕ' for a given value of Ω_ϕ. As mentioned above, we consider the quadratic potential $V = V_0\phi^2$. From the structure of equations, it is clear that the amplitude V_0 always appears in the combination $\alpha = V_0/H_{in}^2$ and this is how we choose to parameterize its value. We consider the quadratic potential mentioned above and fix the parameter $\alpha = V_0/H_{in}^2$ by

$$\alpha = \frac{6}{\phi}\left[\Omega_{\phi_{in}} - \frac{\phi'^2}{4(1-\phi)\phi^2} + \frac{\phi'}{\phi}\right] \tag{13}$$

Fig. 1 shows the behaviour of $\dot{\phi}$ as a function of the scale factor and it is shown that $\dot{\phi} \propto a(t)^{-3}$. Here we are dealing with a stable attractor at $\dot{\phi} = 0$ for the quadratic potential. The scalar field stabilizes to its asymptotic value within a few expansions factors and it behaves like cosmological constant this point onwards. The effective cosmological constant is proportional to α, therefore accelerating phase sets in early if the value of this parameter is large. After this the rate of expansion continues to accelerate unless one introduces additional coupling with matter to the field. If this parameter is small, then there is late time acceleration and the potential provides the dark energy component to the energy density of the universe. The value of the cosmological constant depends on the initial conditions. Therefore the initial conditions need to be tuned in order to get the desired solutions. This point becomes clear in Fig. 2 where we have shown the phase plot for the scale factor.

C. Coordinate transformation

For a cosmological interpretation of the scale factor from the point of view of an observer on the brane, we transform the variables as (if there is no potential and the scalar field phi is the dominant constituent of the universe) [14]

$$d\tau^2 = e^{-m_0 b(t)}dt^2; \qquad Y(\tau) = e^{-m_0 b(t)/2}a(t) \tag{14}$$

Again for a quadratic potential and for no potential case, the scalar field equation of motion leads us to

$$\frac{d}{dt}\left[\frac{\varphi' a^3}{\sqrt{1+\alpha\varphi}}\right] = 0,$$

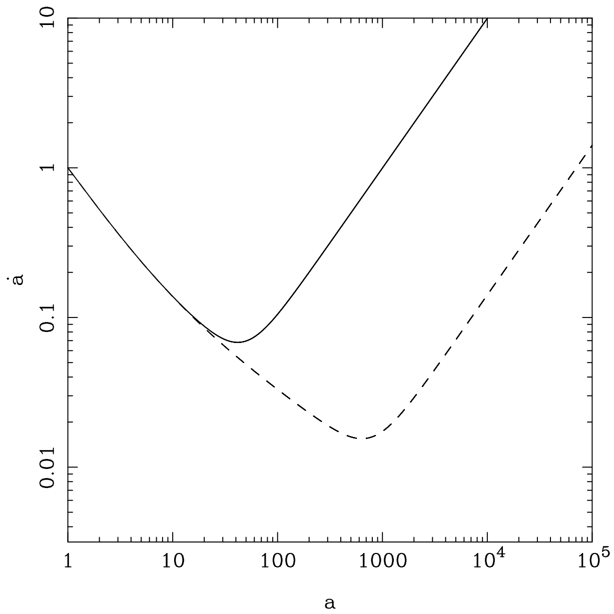

FIG. 2: Phase plot for the scale factor. Dashed lines are for $\alpha_{in} = 10^{-10}$, $\dot{\phi} = 10^{-10}$ and $\Omega_{M_{in}} = 0.0001$. Solid lines are for $\alpha_{in} = 10^{-6}$, $\dot{\phi} = 10^{-6}$, $\phi = 1 - e^{-10}$ and $\Omega_{M_{in}} = 0.0001$.

where $\varphi = \frac{1-\phi^2}{\alpha\phi^2}$ (see for details, [14]), α being a constant. The field shows the same behaviour with respect to the scale factor as the one in the coordinate system considered earlier. Here, prime represents derivative with respect to the variable τ.

The equation for the Hubble parameter changes to (if there is no potential)

$$\frac{Y'^2}{Y^2} = -\frac{Y'}{Y}\frac{\phi'}{\phi} - \frac{\alpha\phi}{4(1+\alpha\phi)}\frac{\phi'^2}{\phi^2} \qquad (15)$$

This equation implies that for an expanding universe, we need a negative $\dot{\phi}$. The second Friedmann equation can be recast in the form

$$2\frac{Y''}{Y} = \frac{Y'}{Y}\left(\frac{\varphi'}{\varphi} - \frac{Y'}{Y}\right) \qquad (16)$$

For the universe to accelerate, we need $Y''/Y > 0$. This means that $\varphi' > 0$ should also be satisfied for accelerated expansion along with the above condition, and the Hubble parameter must satisfy the constraint $H < \frac{\varphi'}{\varphi}$. An expanding universe and a positive ϕ' are not compatible with each other. Therefore, one cannot get inflationary solutions driven by the brane system in the absence of a potential. Of course these restrictions can be circumvented by invoking a potential.

D. Potential with a nonzero minimum

If we wish to stabilise the branes at a particular finite separation, we can use a potential with a minimum at the relevant value of ϕ. This replaces the need for special initial conditions with a tailored potential to guide the brane system towards the desired asymptotic state. We consider a quadratic potential with a minimum at $\phi_0)^2$. The equations of motion then take the form

$$\begin{aligned} \frac{\dot{a}^2}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} &= \frac{\dot{\phi}^2}{4(1-\phi)\phi} + \frac{V_0(\phi-\phi_0)^2}{6\phi} \qquad (17) \\ \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} &= -\frac{1}{2}\frac{\dot{\phi}^2}{(1-\phi)} \\ &- \frac{2}{3}(1-\phi)V_0\phi_0(\phi-\phi_0) \end{aligned}$$

The scalar field rolls down the potential and undergoes oscillations near the minimum of the potential. To look at the behaviour of Hubble parameter around this point we make the perturbation expansion

$$H = \bar{H} + \delta, \qquad \phi = \phi_0 + \epsilon \qquad (18)$$

which up to the first order in δ and ϵ reduce to

$$\begin{aligned} \bar{H}^2 + 2\delta\bar{H} + \bar{H}\frac{\dot{\epsilon}}{\phi_0} &= \frac{8\pi G}{3}\rho \qquad (19) \\ \ddot{\epsilon} + 3\bar{H}\dot{\epsilon} &= \frac{2}{3}V_0\phi_0(1-\phi_0)\epsilon \end{aligned}$$

where ρ is the energy density of the matter fields. The zeroth order equation describes the evolution of the non-oscillatory component of the Hubble parameter $\bar{H}$. This component is insensitive to the scalar field ϕ and its evolution is governed by the radiation/matter fields on the brane. The first order equations describe the coupled behaviour of the field ϕ and the oscillatory component of the Hubble parameter. The scalar field undergoes damped oscillations about the minimum of the potential with $3\bar{H}$ being the damping coefficient. The oscillatory component of Hubble parameter δ is proportional to the velocity of the field ϕ and is given by $\delta = -\dot{\epsilon}/2\phi_0$. These oscillations are undesirable from a cosmological point of view and this toy model is therefore, less than satisfactory. However, adding a coupling between ϕ and matter fields may lead to damping of oscillations in the Hubble parameter.

IV. SUMMARY

This paper presents some cosmological solutions allowed by the five dimensional Randall-Sundrum two brane scenario. We show that the branes are stabilised to a finite seperation. A stable point for brane separation is of interest if this point is reached sufficiently rapidly. The separation of branes is tied up with fundamental constants in particle physics and there is little evidence to show that these have changed much between the present epoch and very high redshifts ($z \approx 10^{10}$). Thus the separation between branes must reach its stable value at sufficiently high redshifts, long before the effective cosmological constant that it provides becomes the dominant constituent of the universe in terms of energy density. We summarise the main results for the cosmology of the model discussed above

- The stable configuration is achieved fairly quickly, i.e. the derivative of the radion field vanishes rapidly.
- After the stabilization is achieved there is no further cosmological consequence of the radion field.
- If we assume a simple quadratic potential for the radion field, radius stabilization can be achieved and the scalar field evolution equation has the same form as in no potential case.
- In addition to this stabilization, the potential also provides the dark energy component at late times.
- If we want to solve the hierarchy problem as well as the dark energy problem using a quadratic potential then fine tuning of initial conditions is required. The level of fine tuning required is similar to that in other models of dark energy.

The small rate of change of $\dot{\phi}$ may reflect itself in changing fundamental constants but clearly the rate of change will be much smaller than the expansion rate of the universe. The fact that we achieve stabilization and dark energy component from the same potential is an attractive feature and hence this model may be a candidate for further study.

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[1] L. Randall and R. Sundrum, Phys. Rev. Lett., **83**, 3370 (1999).
[2] C. Csaki, M. L. Graesser, L. Randall and J. Terning, Phys. Rev. D, **62**, 045015 (2000).
[3] J. Garriga and T. Tanaka, Phys. Rev. Lett., **84**, 2778 (2002).
[4] W. D. Goldberger and M. B. Wise, Phys.Rev.Lett.**83**, 4922 (1999).
[5] T. Chiba, Phys. Rev. D, **62**, 021502 (2000); S. Kobayashi and K. Koyama, JHEP **0212**, 056 (2002);A. O. Barvinsky, Phys.Rev. D **65**, 062003 (2002); P. Brax, C. van de Bruck, A. C. Davis and C. S. Rhodes, Phys. Lett. B, **531**, 135 (2002); P. Brax, C. van de Bruck, A. C. Davis and C. S. Rhodes, Phys. Rev. D, **65**, 12501 (2002);P. Brax, C. van de Bruck, A. C. Davis and C. S. Rhodes, Phys. Rev. D, **67**, 135 (2003).
[6] P. Brax and C. van de Bruck, Class. Quant. Grav. **20**, R201 (2003).
[7] T. Padmanabhan, Phy. Rep., **80**, 235 (2003).
[8] B. Ratra, and P. J. E. Peebles, Phys.Rev. D **37**, 3406 (1988); C. Wetterich, Nucl. Phys. B **302**, 668 (1988); P. G. Ferreira and M. Joyce, Phys. Rev. D **58**, 023503 (1998); J. Frieman, C. T. Hill, A. Stebbins, and I. Waga, (1995) Phys. Rev. Lett. **75**, 2077; P. Brax and J. Martin, Phys. Rev. D **61**, 103502 (2000); L. A. Ureña-López and T. Matos, Phys. Rev. D **62**, 081302 (2000); T. Barreiro, E. J. Copeland and N. J. Nunes, Phys. Rev. D **61**, 127301 (2000); I. Zlatev, L. Wang and P. J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999); A. Albrecht and C. Skordis, Phys. Rev. Lett. **84**, 2076 (2000).
[9] T. Padmanabhan, Phys.Rev. D **66**, 021301 (2002); T. Padmanabhan and T. Roy Choudhury, Phys.Rev. D **66**, 081301 (2002); J. S. Bagla, H. K. Jassal and T. Padmanabhan, Phys. Rev. D **67**, 063504 (2003); H. K. Jassal, astro-ph/0303406.
[10] M. Malquarti, E. J. Copeland, A. R. Liddle and M. Trodden, Phys. Rev. D **67**, 123503 (2003); T. Chiba, Phys. Rev. D **66**, 063514 (2002).
[11] G. W. Gibbons, hep-th/0302199; P. Singh, M. Sami, N. Dadhich, Phys. Rev. D **68**, 023522 (2003).
[12] S. Mukohyama, Phys. Lett. **B473**, 241 (2000); V. Sahni and Y. Shtanov, JCAP, 0311, 014 (2003).
[13] P . Binetruy, C. Deffayet and D. Langlois, Nucl. Phys. B565, 269 (2000), P . Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. **B477**, 285 (2000), J. Cline, C. Grojean and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999), G. Shiu and H. Tye, Phys. Lett. **B516**, 421 (2001), A. O. Barvinsky, Phys. Rev. D,**65**, 062003 (2002). T. Wiseman, Class.Quant.Grav., **19**, 3083 (2002).
[14] S. Bhattacharya, D. Choudhury, D. P. Jatkar, A. A. Sen, Class.Quant.Grav.,**19**, 5025 (2002).
[15] P. Brax, C. van de Bruck and A. C. Davis, JHEP **0110**, 026 (2001); H. K. Jassal, hep-th/0203224; A. Flachi, J. Garriga, O. Pujolas and T. Tanaka, JHEP, **0308**, 053 (2003); J. Martin, G. N. Felder, A. V. Frolov, M. Peloso and L. A. Kofman, hep-th/0309001; A. V. Frolov and L. Kofman; hep-th/0309002, T. Bringmann and M. Eriksson, JCAP, **0310**, 006 (2003); J. P. Hsu, R. Kallosh and S. Prokushkin, hep-th/0311077; G. von Gersdorff, M. Quiros, A. Riotto, hep-th/0310190, S. Mukohyama, Alan Coley, hep-th/0310140; L. N. Granda, hep-th/0309235; K. Kowalska, K. Turzynski, Acta Phys. Polon., **B34**, 3947 (2003); S. Watson, R. Brandenberger, JCAP, **0311**, 008 (2003).
[16] J. M. Cline and H. Firouzjahi, Phys.Lett. **B495**, 271 (2000).
[17] Perlmutter, S., G. Aldering et al., *Astroph. J.*, **517** (1999), Perlmutter, S. , M. Della et al., *Nature*, **391** (1998).
[18] Potentials with minima are discussed below