PRAYAS
Students' Journal
of Physics

TURNING POINTS

Cosmological Redshift

Jayant V. Narlikar
Inter-University Centre for Astronomy and Astrophysics Pune 411007

Communicated by D.P. Roy

1 INTRODUCTION

In this article we will narrate the story of an astronomical discovery that launched a whole new branch of the subject, namely *Cosmology*, which deals with the large scale structure of the universe as a whole. The story has not ended yet since there are a number of unanswered questions still left [1].

As with many things in physics, the beginning of the development can be traced back to Isaac Newton. Thus spectroscopy started when Newton held a glass prism against sunlight and observed that the light entering the prism came out split in several colours, very similar to the seven colours of a rainbow: Violet, Indigo, Blue, Green, Yellow, Orange and Red, often grouped under the acronym VIBGYOR. This light gets bent (refracted) by the prism and the violet colour is bent the most, red the least. Newton argued, rightly, that light of all these colours is present in sunlight. With its property of bending light of different colours by different amounts, the prism is able to act as a separator of those different colour components.

Today we know that light travels as a wave and the wavelength of each colour is different. The violet waves have the shortest wavelengths while the red waves have the longest. The entire range of the wavelengths covering the seven colours is from about 400 nanometres to 800 nanometres. A nanometre is a very tiny unit of length; a billion nanometres make a metre. So the basic rule we arrive at from the prism experiment is that light of different wavelengths is bent differently, the bending being larger for the shorter waves.

Astronomers use this effect to split the light coming from a distant source like a star or a galaxy. They have built instruments called spectrometers that produce a band of colours, these colours being the components of the light from the source. The amount of energy being carried by the different bands depends on the physical mechanism that powers the source. A given source has a spectrum, and if we can interpret the spectrum we can learn something about the source.

The spectrum of the Sun obtained by Newton showed the typical rainbow colours. When we look at the same spectrum taken with a modern spectroscope, we not only see the seven colours, but also some dark lines spread over it. Prima facie, dark lines denote absorption of light. Why is light absorbed selectively at a few wavelengths? The dark lines were discovered by Joseph von Fraunhofer in 1814, and their origin was a mystery until the dawn of quantum theory.

The light coming from a star to us, passes through the outer layers of the star (its atmosphere) and the atoms in that outer part are cooler, and absorb some of the light. The absorption takes place not smoothly over a band of colours but over discrete sets of lines at specific wavelengths. These wavelengths are tuned to the specific structure of the absorbing atoms. Applying the principles of quantum theory, one can relate the wavelengths to specific atoms.

The lines are called *absorption lines*. The information they bring is of great help to the astronomer in many ways. For example, they tell us what material is present in the absorbing material, its temperature and also the motion of the source carrying it. This last named property is known as the *Doppler Effect*. It relates the increase or decrease of the wavelength of a dark absorption line to the motion of the source.

The Doppler effect tells us that the shift of the dark lines, implying an increase or decrease of the original wavelength will be related to the recession or approach of the source. Suppose for example, that in a spectrum a dark line should appear at the wavelength of 500 nanometres. However, in the actual spectrum of a source it is seen at the wavelength of 550 nanometres. The increase is by 10% and, according to the Doppler Effect, it equals the ratio of the speed of recession of the source to the speed of light. Knowing that light travels at a speed of 300,000 km per second, we conclude that the source is moving away from us at 10% of the speed of light, that is at 30,000 km per second.

With this technique, astronomers have studied spectra of many stars beyond the sun and have measured their speeds towards or away from us. Stars moving towards us have their spectral lines shifted towards the short wavelength side, that is the violet or blue end of the spectrum. For stars moving away from us these lines shift towards the longer wavelength side, that is, the red end of the spectrum. For this reason such shifts are referred to as *blueshift* and *redshift* respectively. Thus quantitatively, a redshift z measures the fractional increase in wavelength. In the example of the previous para, $z = 10\%$ or 0.1. From where we are situated in the Galaxy, we see some stars moving towards us and some moving away from us. These speeds are of the order of 100 kilometres per second or less. These are all parts of our own Galaxy - the Milky Way. But for external galaxies, that is, for galaxies other than ours, we encounter a very different situation. In fact, astronomers opened a Pandora's box when they started taking spectra of galaxies.

2 HUBBLE'S LAW

Early in the last century, in 1913 -1914, Vesto Melvin Slipher at the Lowell Observatory in Northern Arizona found that the spectrum of Andromeda nebula shows a blueshift, corresponding to a speed of 300 kilometres per second towards us. This was a high speed by stellar standards. However, as Slipher continued to study the spectra of nearby spiral nebulae he found speeds of the same order; but gradually as measurements covered more nebulae, the numbers of redshifted nebulae began to dominate. Slipher's paper in 1925 reports that initially there were 11 redshifted nebulae to 4 blueshifted nebulae, but by 1917, this ratio had increased to 21 to 4. The trend continued and after a few blueshifted cases, the vast majority of nebulae showed redshifts. In fact, apart from the predominance of redshifts, there was another feature of the data which attracted the highly perceptive astronomer Edwin Hubble.

While he was working on the properties of the nebulae at Mount Wilson, Hubble was aware of Slipher's results. Hubble however, looked at the images of the sources along with their spectra. Typically he saw a pattern emerge which we can understand with the help of the series of photographs of such sources side by side with their spectra shown in Figure 1.

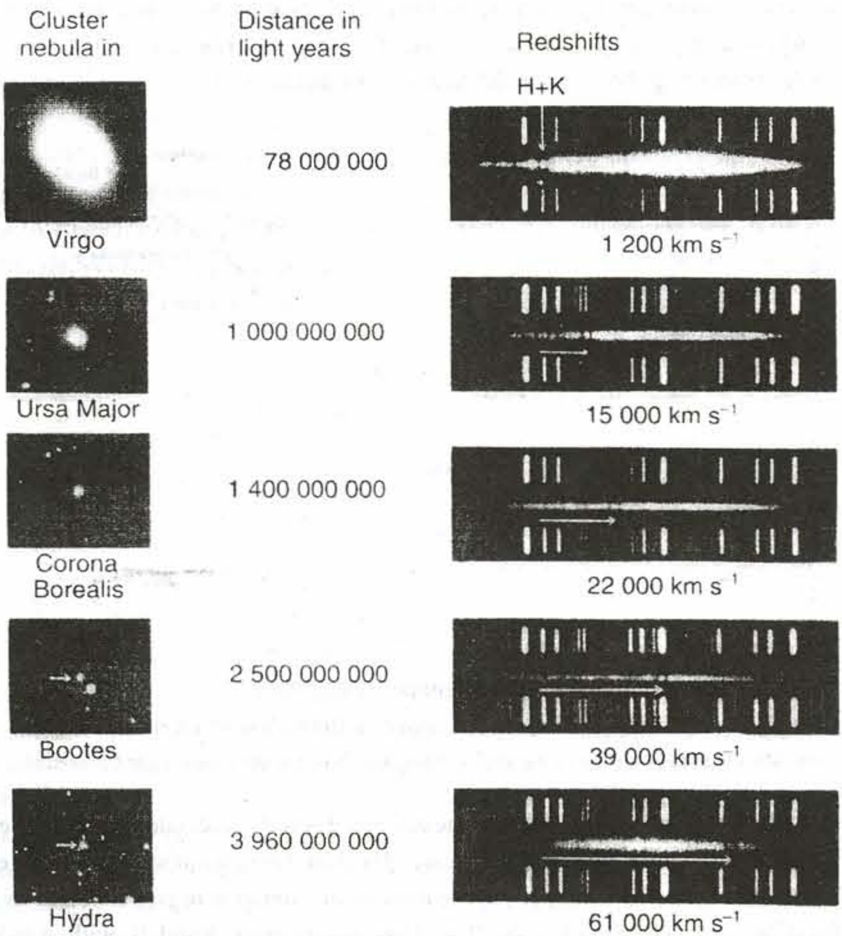

Figure 1. Early data used by Hubble showing the images of galaxies (left) and their spectra (right). Photo from Palomar Observatory, Caltech.

Notice that the images shown on the left get smaller and smaller as we go down the list. The clarity of the image and its brightness are also less and less as we go down. Prima facie we can argue that as we proceed down the list, we are looking at more and more remote sources of light.

[Imagine that we are looking at a townscape from the top of a high tower, with houses dotted round at different distances: as we look farther and farther away, the houses look smaller and smaller and less sharply defined.] On the right side we have the spectra of these sources, each with a couple of dark lines which have shifted towards the red end. The shift is larger and larger as we go down the list. If we appeal to the Doppler effect, we can argue that the farther down the list we go we find more remote galaxies and ones which are moving away from us with greater speeds. In short, speed goes with distance.

What Hubble had deduced from this work was that the velocity of recession - that is, the redshift - was proportional to the distance inferred from the apparent brightness. The further away the galaxies were, the faster they were moving. With his younger colleague Milton Humason, Hubble made a systematic study of a large number of galaxies and clusters of galaxies. In 1929 Hubble published this result in the Proceedings of the National Academy of Sciences .

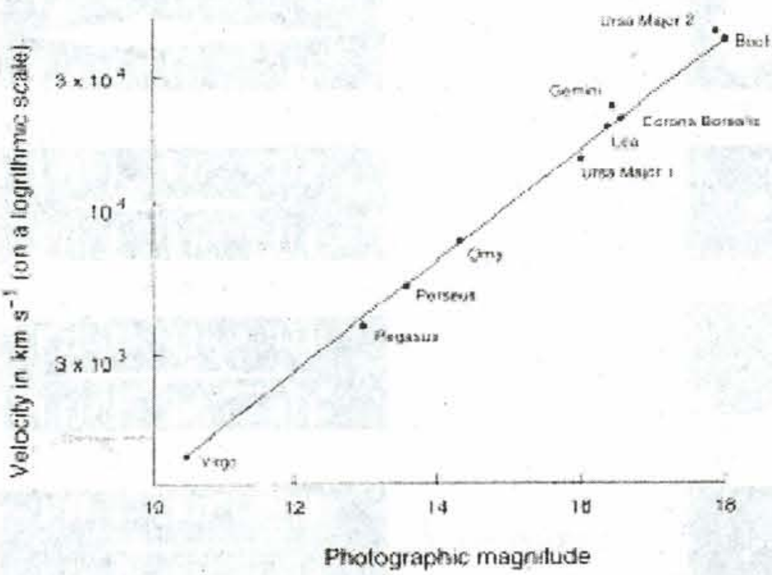

Figure 2. The velocity distance relation based on Hubble's work on clusters of galaxies. The magnitude measures the apparent brightness of a source on a logarithmic scale.

In a graph of nearby galaxies Hubble plotted the estimated speed of recession against the estimated distance. The graph is shown in the Figure 2. The trend is clear, bearing out what the above examples of images and spectra indicated. *The speed of a galaxy away from us is in proportion to its distance from us. This is known as Hubble's law.* So if we have two galaxies A and B, with A twice as far away as B, then we should find that A is moving away from us with speed twice as large as the speed of B. If we denote the distance of a typical galaxy from us by D and its radial velocity of recession by V, then we write the Hubble law as the relation

$$V = HD.$$

The constant of proportionality H is called the *Hubble constant.* It is usually expressed in units of $km\ s^{-1}\ Mpc^{-1}$.

To a physicist, the velocity unit in the above is easy to recognize. The distance unit Mpc or the $megaparsec$ is not generally encountered in physics. It arises naturally when the astronomer seeks to mesure the diastance of a nearby star using the parallax method. We will not go into the details of how this is done; we will simply state that the *parsec is approximately 3.26 light years* (1 light year is the distance traveled by light in one year). A megaparsec is a million parsecs. Hubble found a value of approximately 530 for H in these units but that turned out to be a gross over-estimate. The currently favoured value is approximately 72.

This seems at first sight to indicate that Hubble's observation has finally restored to us a privileged status in the universe! We appear to be located at a vantage point from where all other galaxies are moving away in a highly symmetrical manner. Alas! This is not the case. In fact the Hubble law further strengthens the highly democratic set up of the universe. For, if we imagine transporting ourselves to another galaxy (any one of those we see moving away from us) we can work out and convince ourselves that we would see exactly the same Hubble law operating with our new vantage point as the centre of the universe.

Exercise: A student reading this article should try to prove this statement. As seen by an observer at the origin, the velocity distance relation is $\mathbf{V} = H \times \mathbf{D}$. (The boldface letters denote vectors.) Show that the same holds for any other observer sitting on one of these galaxies.

In fact, if we wish to visualize what is going on, the analogy of resins in dough of cake will help! As the dough rises, it expands and all the embedded resins in it move apart from one another, *without any particular resin having a special status.* In the case of galaxies they are embedded in space which is expanding. Thus was born the concept of the *Expanding Universe.* It changed the existing view of a static universe.

3 ENTER GENERAL RELATIVITY

All this turmoil taking place on the observational front necessarily had an impact on the theorists. In fact theorists had not been idle! After his proposed equations of general relativity, in 1917, Einstein had tried to apply them in order to get a static solution as a model for the universe. This had led him to postulate an extra force basic to nature which opposed gravity. Known as the *cosmological force*, it had the form λr, r being the distance between two particles and λ a constant so small that only at cosmological distances would this force of repulsion be significant.

However, in 1922-24 Alexander Friedmann generated dynamic models from Einstein's equations without the λ-term. These expanding universe models were exactly what Hubble later found in 1929. Unaware of Friedmann's work, Georges Lemaitre also produced the same set of models in 1927. After Hubble's discovery, Einstein felt that the extra force added by him to his equations was unnecessary and he formally gave it up. Nevertheless it survived and has gone through periods of popularity and rejection! Today it is taken very seriously.

How do we understand the phenomenon of redshift in the expanding universe? Imagine that the overall scale of the universe is growing with time and we may indicate it by a *scale factor* $S(t)$. Thus we may scale the distance between any two galaxies at time t by S(t) and write it as the quantity $d \times S(t)$. Now suppose that we receive a light signal from a galaxy at the present epoch t_0 and that the signal had been emitted at the earlier epoch t_1. Then the redshift, known as the *cosmological redshift* is given by the formula

$$1 + z = S(t_0)/S(t_1).$$

Since the scale factor is increasing with time, we expect z to be positive. The Hubble constant at any epoch is simply $(dS/dt)/S$. Thus a positive Hubble constant means the universe is expanding.

Exercise : Another challenge for the reader. What is the fallacy in the following argument? A galaxy following Hubble's law is at a distance $2c/H$ from us. So it is moving with twice the speed of light and hence violates special relativity.

4 TIRED LIGHT HYPOTHESIS

Although this was the interpretation that most astronomers subscribed to, there was an alternative explanation advocated by the astronomer Fritz Zwicky in 1929. Zwicky suggested that light particles, the photons, lose energy through absorption, scattering etc. in the intergalactic space and as a result their wavelength increases. This explanation is known as the 'tired light hypothesis'. This has been proposed from time to time to understand the redshifts of galaxies and quasars; but by and large it has very few adherents because it appears to be in conflict with the laws of atomic physics.

For example, if a photon is to lose energy, it should interact with the ambient matter and in the process will be scattered. So light coming from a distant object will be scattered enroute and will blurr the image of the source. No such blurring effect has been found.

5 WHY IS THE SKY DARK AT NIGHT?

We interrupt our narrative concerning Hubble's work on galaxies to ask the question raised above. The reader may wonder why we raise this simple question about a local observation from the Earth, in the midst of a discussion about far-away galaxies. The connection will soon become clear.

At first sight the answer seems simple enough. The Earth spins about its axis with a period of twenty four hours and the part of its surface facing away from the Sun experiences darkness which is nightfall. Is this not a sufficient answer to the question?

Heinrich Olbers, a German physician and astronomer was not satisfied with this answer. In 1826 he carried out a simple calculation and arrived at an answer so startling, that it kept astronomers busy for a century and a half trying to find where Olbers had gone wrong. For if he were right, then his conclusion was that the sky should not have been dark at all, but extremely bright all of the time, irrespective of which side of the Earth the Sun was on. Known as the *Olbers paradox*, the argument used by Olbers is essentially as follows.

Besides the Sun, the sky contains many other stars which are also emitting light, some of which will reach the Earth. Of course, the amount of light from a typical star will be quite minuscule compared to what we receive from the Sun, because the star is very far away. However, Olbers argued that although the amount of light from distant stars will be very little there are so many stars in the universe, that their combined light might not be negligible. And, he set out to compute it using a simple argument.

Imagine that the universe is infinite in extent and is uniformly filled with stars all of them like the Sun. Suppose we draw a sphere of radius R and consider a thin shell on its surface. Our geometry textbook tells us that the surface area of this sphere is $4\pi R^2$ and so if the shell has small thickness a, its volume will be this area multiplied by thickness, that is, $4\pi R^2 a$. Further, if the universe is uniformly filled with stars and has N of them in a unit volume, then the total number of stars in the shell will be given by simply multiplying the shell volume by N, that is by $4\pi R^2 aN$. Now imagine a typical star in this shell has a luminosity L. Then, the amount of its radiation coming our way per unit area per second would be L divided by the factor $4\pi R^2$. So we see, on multiplying this quantity by the number of stars in our shell, that these stars contribute a total flux of

$$[L/4\pi R^2] \times [4\pi R^2 aN] = LaN.$$

Notice that all terms relating to distance have cancelled out in the arithmetic used by Olbers. So we should get the same amount of light from a shell of the specified thickness a, *no matter how far away it is*.

The last part of Olbers's argument is now straightforward. Divide the entire universe into such thin concentric spherical shells all of the same thickness. Each shell contributes the same flux at the observer. *But the number of such shells is obviously infinite*. Therefore it follows that the total flux from *all* stars in the universe is also infinite! This was the logical conclusion that Olbers came to with his rather simple basic assumptions. Thus it is immaterial whether we are facing the Sun or not. Either way the night sky will be infinitely bright. But, when we face away from the Sun, the night sky *is* dark. So there is something wrong with the arithmetic. But where is the mistake?

Careful consideration of all the Olbers's arguments shows one loop-hole. The stars are not point sources: they have a finite size. So when we start putting stars in successive shells around us a stage will come when they will block the entire sky visible to us. An analogy may help here. If you look through a gap of trees in a park you can see the buildings in the background. However, if you are in a thick forest of trees, you simply can't see beyond the foreground trees which block the view of trees farther back. So, in the revised Olbers calculation, only the stars in the relatively nearby shells will contribute to the total radiation flux. The total flux is therefore not infinite but finite.

Exercise: Work out an approximate estimate of this finite value.

But we are not out of the woods yet! For this finite total flux can be computed and it still turns out to be very high, as high as on the surface of the typical star like the Sun. That means that the sky should not only be bright but the temperature in our neighbourhood should be as high as it is on the surface of the Sun, in the region of about 5500 degrees Celsius. Again, we seem to have arrived at an impossible conclusion.

Astronomers suggested two other ways out of the Olbers paradox. The first is that the universe may not be infinite as Olbers assumed, but is finite in extent. Which means that when we draw our spherical shells we stop at a certain distance beyond which nothing exists. This distance would have to be at least as large as the range of our best telescopes. For, so far as we can see, there is no end to the sources of light up to the distance of some ten billion light years that we can presently probe. If indeed there are no more sources of light beyond say, ten billion light years, we do get a resolution of the paradox, for the contribution of sources out to this distance is negligible compared to the light we get from the Sun.

The other possible solution is that the stars that we see, or can in principle see, came into existence a finite time ago. Suppose the universe itself came into existence ten billion years ago. Then we can receive light from only those stars that lie within a distance of ten billion light years. Light from stars that exist beyond this limit, has not had time to reach us yet.

Another possible resolution of the paradox takes note of the fact that stars in any shell will only last for a finite time. They cannot go on shining for ever, so we cannot expect to find shining stars in all shells. This also reduces the net contribution to the total flux received.

Thus we can see that what started as a simple question of local interest has forced us to think of cosmological issues such as the extent and age of our universe. A further crucial element that has been ignored, is the discovery by Edwin Hubble that the universe is expanding. We now turn to that piece of evidence which was not available to Olbers. It was the discovery of the expansion of the universe.

What difference will an expanding universe make to the Olbers calculation? A major difference is that if the universe is expanding, then the light from distant galaxies will be highly redshifted. The quantum theory of light tells us that light is not only a wave, it is also a collection of tiny packets of energy called *photons.* The energy of a typical photon is determined by the wavelength of light. The longer the wavelength, the smaller is the energy. The light coming from a far away galaxy will therefore be reduced in energy by the redshift effect. So the contribution to the energy from a remote source of light will be much less than that estimated by Olbers. In the calculation described earlier, we should find that light from a more remote shell of sources will be much less than the light from a nearby shell of the same thickness. It was pointed out by Hermann Bondi in the 1950s that once this effect is taken into account in computing the total energy received from the whole universe, we get a sensible answer. The contribution from the rest of the universe is negligible compared to the light we get from the Sun.

6 COSMOLOGICAL MODELS

We will end this discussion with a brief look at how theoreticians model the universe. To simplify the maths, they assume that galaxies have a systematic motion (as indicated by Hubble's law) and take this to identify a *cosmological reference frame.* In this frame each galaxy is identified by a set of constant *commoving* coordinates (r, θ, φ) and the time measured by each galaxy is denoted by t. This coordinate is called the *cosmic time.* The hypersurfaces t = constant are also endowed with symmetry. They are homogeneous and isotropic. Thus the universe looks the same, seen from any vantage point and in any direction.

With these simplifying assumptions we may write the spacetime geometry as described by the following two quantities. The first is the parameter k, which may take values 0, +1 or -1, depending on whether the geometry of the hypersurface is flat (i.e., Euclidean), or positively curved, or negatively curved. The second is the function $S(t)$, whose dynamical behaviour is determined by Einstein's equations of general relativity. All solutions of these equations exhibit the common property that there was an epoch in the past when the scale factor was zero. We may set this time at $t = 0$. The fact that the scale factor was zero and the Hubble constant infinite at this epoch, implies that the spacetime point is *singular*, that is with weird properties both physical and mathematical. Known as the 'Big Bang' epoch, it marks the beginning of the universe.

The challenges faced by physicists these days are concerned with finding how physics operated close to the singular epoch.

References

[1] **For further reading:** see the author's book *An Introduction to Cosmology* (Cambridge University Press 2002).