Astrophys Space Sci (2014) 353:621–624
DOI 10.1007/s10509-014-2033-5

ORIGINAL ARTICLE

Peculiar motions of galaxy clusters: correlation function approach

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Received: 1 February 2014 / Accepted: 17 June 2014 / Published online: 1 July 2014

Abstract The correlation function theory on the basis of prescribed boundary conditions provides a deeper understanding in studying the dynamical parameters of galaxy clusters. The approach approximates that the moderate dense systems discussed by a two point correlation function is helpful for describing the dynamical nature of galaxy clusters. The projected theory of two point correlation function for point mass and extended mass structures can be used an alternative tool in measuring the average peculiar motion and temperature profile of galaxy clusters.

Keywords Large scale structure of universe · Thermodynamics · Clustering · Correlation functions · Dynamical properties

1 Introduction

Galaxy clusters are important astrophysical laboratories providing us with a well characterized physical environment in which we can understand many interesting astrophysical phenomena's. They allow us to study the properties of galaxy clusters in context to their dynamical behavior. In recent years, most of the detailed knowledge on galaxy clusters have been obtained through X-ray spectroscopy but at the same time theoretical approaches are no more less important. Various theories like Percolation (Bhavasar and Splinter 1996), MST (Kreziwina and Saslaw 1996), Fractals (Martinez and Coles 1994), Voids (Arseth and Saslaw 1982), Distribution functions (Efstathiou 1991) and correlation functions (Peebles 1980; Collins et al. 2000; Lee and Park 2002; Bahcall et al. 2003; Estrada et al. 2009; Iqbal et al. 2012; Valogeas and Clerc 2008) have played an important role in understanding the phenomena. Developing the theory for galaxy clustering have been mainly constructed with all the galaxies as point mass approximations. Actually galaxies have real extended structures and a pure semi analytical approach for extended mass galaxy clusters has been developed which has yielded interesting results (Ahmad et al. 2002; Iqbal et al. 2006, 2012). By using equation of state along with the correlation functions of galaxy clusters provides a description of dynamical parameters related to average velocity dispersion and intra-cluster temperature of a system.

In this work, we calculate the magnitude of the average velocity dispersion and ICM temperature of galaxy clusters using an extended theory of correlation functions. The theoretical basis of the correlation function approach for the estimation of peculiar motions and temperature profile of galaxy clusters is the subject of this paper.

2 Correlation functions in the cosmological many body problem

Correlation functions are one of the standard ways of studying formation of structures in the universe. The visible structures mainly the galaxies arise from the large dark matter haloes, and tend to form hierarchically large structures of clusters and super-clusters. Thus the phenomena of clustering is important to understand the distribution of visible and

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dark matter, formation of different structure profiles and interaction between them, the peculiar velocities etc. Dynamical properties of galaxy clusters are determined by the velocity dispersion in the cluster, the mass density and the size parameter of cluster. The virial theorem provides a relation among these three parameters. Here, we use correlation functions of galaxy clusters and virial theorem over a fixed cell size of cluster which directly relates to the clusters velocity dispersion and study the relation of motion with respect to the virial temperature.

From the general pair of equations of state for internal energy (U_e) and pressure (P_e) for extended structures (Hill 1956; Iqbal et al. 2006), the measuring correlation parameter b_e for extended structures is given by

$$b_e = \frac{2\pi G m^2 \bar{n}}{3T} \int \xi_2(\bar{n}, T, r)\left(1 + \frac{\epsilon^2}{r^2}\right)^{-\frac{1}{2}} r dr \tag{1}$$

Here $\bar{n}$ is the average number density of gravitating particles (galaxies) assumed in a volume V, ϵ is a softening parameter taken between 0.01–0.05 in the units of total radius of a system, T is the virial temperature which arise due to the peculiar motions of particles (galaxies), clustering gravitationally in an expanding universe. The parameter b_e measures the effect of correlation in the clustering of galaxies, r denotes the spatial distance between the two galaxies. For $r \to \infty$, Eq. (1) reduces to two point correlation function. So in the limit $r \gg \epsilon$, the point mass approximation of the cluster is valid and if $r \to 0$, the extended nature of cluster is taken into consideration i.e., for $r \ll \epsilon$.

Following the work of Iqbal et al. (2006), the two particle correlation function for extended mass structures defined by Iqbal et al. (2012) is described as;

$$\xi_2(\bar{n}, T, r) = c\bar{n}^{Z/3} T^{Z_N} r^Z \left(\epsilon^2 + r^2\right)^{\frac{Z}{2}} \tag{2}$$

where c, Z and Z_N are constants to be determined on the basis of required conditions. Equation (2) can be discussed on the basis of the boundary conditions like ξ_2 will be positive for smaller values of T, r and ϵ except for the average number density $\bar{n}$. ξ_2 will increase when $\bar{n}\ T^{-3}$ is very large. These conditions leads the system to a complete virialized so that the correlation energy for extended mass galaxies $b_e \to 1$. From Eqs. (1) and (2), the partial derivatives of b_e and ξ_2 w.r.t. $\bar{n}$ and T and after rearranging these equations leads to

$$\frac{\partial b_e}{\partial \bar{n}} = \left(1 + \frac{Z}{3}\right)\frac{b_e}{\bar{n}} \tag{3}$$

$$\frac{\partial b_e}{\partial T} = \frac{b_e}{T}(Z_N - 1) \tag{4}$$

Equations (3) and (4) are interesting for studying the physical dependence of Z and Z_N which seem to play a vital role in describing the particle correlation function for extended mass galaxies clustering gravitationally. The solutions of Eqs. (3) and (4) can be approximated as:

$$b_e(\bar{n}) = k_1 \bar{n}^{(1+\frac{Z}{3})} \tag{5}$$

$$b_e(T) = k_2 T^{(Z_N - 1)} \tag{6}$$

where k_1 and k_2 are the constants. In combination, the solution can be written in the form as;

$$b_e = f\left[\bar{n}^{(1+\frac{Z}{3})}, T^{(Z_N-1)}\right] \tag{7}$$

It simply represents the departure from a non-interacting ($\xi_2 = 0$ or perfect gas) system. The dependence of b with the two particle correlation function is given by

$$b = -\frac{W}{2K} = \frac{2\pi G m^2 \bar{n}}{3T} \int_0^R \xi_2(\bar{n}, T, r) r dr \tag{8}$$

b is defined as correlation parameter for point mass structures and is the ratio of the correlation potential energy to twice of kinetic energy of the system and R is the size of a cluster. When there is no gravitational force, $b = 0$. For $b = 0$, we recover the expected Poisson distribution and value of b measures the influence of gravitational correlation energy which generally depends on density and temperature (Saslaw and Hamilton 1984). The other (generic form) form of b has been described by, Ahmad et al. (2002, 2006) and Iqbal et al. (2006, 2012) as

$$b = \frac{\beta \bar{n} T^{-3}}{1 + \beta \bar{n} T^{-3}} \tag{9}$$

where β is a positive dimensional constant given by $\beta = 3/2(Gm^2)^3$, b and b_e are related with each other as:

$$b_e = \frac{\beta \bar{n} T^{-3} \alpha(\frac{\epsilon}{R})}{1 + \beta \bar{n} T^{-3} \alpha(\frac{\epsilon}{R})} \tag{10}$$

The functional form of b_e emerges directly from partition function and equation of state and the constant α as (Ahmad et al. 2002)

$$\alpha\left(\frac{\epsilon}{R}\right) = 1 + \frac{\epsilon^2}{R^2} \ln \frac{\epsilon/R}{1 + \sqrt{1 + \epsilon^2/R^2}} \tag{11}$$

When $\alpha(\frac{\epsilon}{R}) = 1$ point mass nature of galaxy clusters is taken into consideration and $\alpha(\frac{\epsilon}{R}) \neq 1$ represents extended nature of structures. Equation (10) reduces to the usual result for point masses for $\epsilon = 0$, implying $\alpha = 1$. Thus, for point masses ($\epsilon = 0, \alpha = 1$), we get same expression originally derived by assuming the point mass form of Eq. (10). Thus it is also clear that softening parameter ϵ and cell size R have non negligible effects on the thermodynamics of gravitating masses in an expanding universe.

With the help of Eqs. (5), (6) and (7), the form of b_e (correlation function for extended mass galaxies) shown in Eq. (10) can be written as:

$$b_e = \frac{\beta' \bar{n}^{(1+Z/3)} T^{(Z_N-1)}}{1+\beta' \bar{n}^{(1+Z/3)} T^{Z_N-1}} \tag{12}$$

with $\beta' = \beta k_1 k_2$. The parameters b and b_e have specific dependence on the combination of $\bar{n}T^{-3}$ and $\bar{n}T^{-3}\epsilon$ (Iqbal et al. 2012) which suggests that the virialized system represent a moderately dense system. Physically significant form of b_e suggests that constants Z and Z_N shown in Eq. (12) must take the following values.

When $Z = 0$ and $Z_N = -2$, $b_e \approx b$ for $\bar{n}T^{-3}$ to be large $b_e \approx b \approx 1$, therefore simple conditions can be used for a quasi-equilibrium system to attain virial equilibrium. The mathematical convenience of Z and Z_N suggests that, they can be related by a simple equation $Z + Z_N = -2$. Using $Z_N = 0$ and $Z = -2$, Eq. (2) becomes as:

$$\xi_2(\bar{n}, T, r) = cn^{-2/3} r^{-2}. \tag{13}$$

For a constant number density $\bar{n}$, the two particle correlation function defined here exhibits the power law behavior. From Eq. (1), we have

$$b_e = \frac{2\pi G m^2 \bar{n}^{1/3}}{3T} c \times \log\left(\frac{R}{\epsilon} + \sqrt{1+\frac{R^2}{\epsilon^2}}\right) \tag{14}$$

Equation (14), shows that correlation parameter increases rapidly as softening parameter (ϵ) tends to zero and decreases exponentially with an increase in the softening parameter ϵ. Also b_e shows dependence on the total radius of the clustering region, increasing 'more or less' logarithmically with increase in R. Thus we point out that point mass particles cluster heavily and in case of extended structures clustering decreases in the given range of softening parameter, but increases with increase in the cluster size.

3 Peculiar motion and correlation functions

Clusters are characterized by a virialized region within which all components (galaxies, ICM and DM) are in dynamical equilibrium where motions are well described by average velocity dispersion profile. Different techniques applicable to galaxy clusters are based on the hydrostatic measure of X-ray emissivity and temperature of hot cluster gas (Ettori et al. 2002; Zappacosta et al. 2006; Schmidt and Allen 2007; Host and Hansen 2011). Also observational studies (Rines and Diaferio 2006), N-body simulations (Mamon et al. 2004; Wojtak et al. 2005; Cuesta et al. 2008) and combination of both (Mahajan et al. 2011) have shown that virialized clusters are surrounded by infall zones from which most galaxies move into the relaxed cluster.

For studying the peculiar motions and temperature profile of galaxy clusters on the basis of correlation functions, we follow Eq. (1) with $\bar{n}$, T and r as independent variables and write Eq. (14) in the form as;

$$b_e = \frac{2\pi G m^2 \bar{n}^{1/3}}{3T} cc' \tag{15}$$

where

$$c' = \log\left(\frac{R}{\epsilon} + \sqrt{1+\frac{R^2}{\epsilon^2}}\right) \tag{16}$$

The comparison of Eqs. (10) and (15) gives as;

$$\frac{\beta \bar{n} T^{-3} \alpha(\frac{\epsilon}{R})}{1+\beta \bar{n} T^{-3} \alpha(\frac{\epsilon}{R})} = \frac{K_1}{T} \tag{17}$$

Here

$$K_1 = \frac{2\pi G m^2 \bar{n}^{1/3}}{3} cc' \tag{18}$$

The cubical form of Eq. (17) in T can be written as,

$$K_1 T^3 - K_2 T + K_3 = 0 \tag{19}$$

Here $K_2 = \beta\bar{n}\alpha(\epsilon/R)$ and $K_3 = \beta\bar{n}\alpha(\epsilon/R)K_1$. For solving this equation in T, we assume that two of the three roots (say T_1 and T_2) are equal and the solution takes the form as;

$$T_1 = T_2 = \left[\frac{K_3}{2K_1}\right]^{1/3} \tag{20}$$

$$T_3 = -2\left[\frac{K_3}{2K_1}\right]^{1/3} \tag{21}$$

For the cluster of 1 Mpc size with $\epsilon = 0.05$, the value of α and c' from Eqs. (11) and (16) comes out to be 0.9 and 0.11 respectively. The value for c emerges from the solution of a cubical equation in T taking basic conditions into consideration and after solving the necessary conditions, we arrive at a condition $cc'\alpha^{-1/3} = 0.2$

The correlation energy for extended masses on the basis of virial theorem can also be written as;

$$b_e = \frac{Gm}{(r^2+\epsilon^2)^{1/2} V^2} \tag{22}$$

Equations (15) and (22) gives

$$V^2 = \frac{3T}{2\pi m \bar{n}^{1/3} cc' (r^2+\epsilon^2)^{1/2}} \tag{23}$$

This equation describes the connection between the average peculiar velocity of galaxy cluster for a given number density $\bar{n}$ and temperature T. To determine the average velocity dispersion of a given cluster by above equation, we consider a cluster with fixed number of galaxies as gaseous particles each of same mass in a spherical volume. The average velocity dispersion of cluster gas is calculated by considering the velocity V as order of magnitude of a mean velocity of the hot gas molecules, m is mean molecular weight which is ≈ atomic weight for H-atom. For the estimation of average velocity dispersion from Eq. (23) for a given sample of galaxy cluster, the temperature value is taken as temperature of hot ICM (10^7 K) and number density $\bar{n} = 10^{-3}/\mathrm{cm}^3$ (Bohringer and Werner 2010; Moretti et al. 2011; Abdullah et al. 2011; Tabasum and Iqbal 2014) and the other calculated values of c, c' and r, we find average velocity dispersion in the range of few 100 $\mathrm{km\,s^{-1}}$ and conversely if velocity dispersion is used in hundreds of $\mathrm{km\,s^{-1}}$, the temperature value comes out to be few millions of degrees of kelvin which is temperature of hot plasma of galaxy clusters.

4 Discussion

We have shown explicitly, the significant use of correlation functions for estimating the dynamical property of galaxy clusters in an expanding universe. By introducing the choiced constants and compiling them with a set of boundary conditions for Z and Z_N in the theory of quasi-equilibrium thermodynamics yields a comparative approach for studying various properties. The theory so discussed is helpful in knowing the various significances of correlation energy b and b_e in relation to the average peculiar velocity dispersion of cosmological many body problem. Our theory provides a new insight in looking the agreeing of various theoretical results studied from correlation functional approach with the observational study.

Acknowledgements The authors are grateful to IUCAA, Pune for providing the necessary facilities in completing this work. One of us (Tabasum M.) is thankful to University Grants Commission for providing the FIP fellowship in carrying out Ph.D. work in the Department of Physics, University of Kashmir, Srinagar, India.

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