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Density Perturbations, Gravity Waves and the Cosmic Microwave Background

by

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Abstract

We asses the contribution to the observed large scale anisotropy of the cosmic microwave background radiation, arising from both gravity waves as well as adiabatic density perturbations, generated by a common inflationary mechanism in the early Universe. We find that for inflationary models predicting power law primordial spectra $|\delta_k|^2 \propto k^n$, the relative contribution to the quadrupole anisotropy from gravity waves and scalar density perturbations, depends crucially upon n. For $n < 0.87$, gravity waves perturb the CMBR by a larger amount than density perturbations, whereas for $n > 0.87$ the reverse is true. Normalising the amplitude of the density perturbation spectrum at large scales, using the observed value of the COBE quadrupole, we determine $(\delta M/M)_{16}$ – the rms density contrast on scales $\sim 16 h_{50}^{-1}$ Mpc, for cosmological models with cold dark matter. We find that for $n < 0.75$, a large amount of biasing is required in order to reconcile theory with observations.

keywords : structure formation, density fluctuations, biased galaxy formation, gravity waves, inflation, microwave anisotropy, CMBR

Both gravity waves as well as adiabatic density perturbations arise as natural consequences of the inflationary scenario, due to the superadiabatic amplification of zero-point quantum fluctuations occuring during inflation (Starobinsky 1979, 1982; Hawking 1982; Guth & Pi 1982). As gravity waves and scalar density perturbations enter the horizon during matter domination, they induce distortions in the cosmic microwave background radiation (CMBR) through the Sach-Wolfe effect. In this paper we shall assess the relative contribution to the CMBR anisotropy, arising from both scalar density perturbations as well as from gravity waves. In doing so we shall consider waves created during exponential ($a \propto \exp Ht$), quasi-exponential ($a \propto \exp\int H(t)dt$), and power law ($a \propto t^p, p > 1$) inflation, thus considerably extending previous work on the subject.

The close similarity between gravity waves and massless scalars, in a Friedman - Robertson - Walker (FRW) background (Grishchuk 1974), makes it possible for each polarisation state of the graviton to be expressed in terms of solutions to the massless, minimally coupled Klein-Gordon equation: $h_{\times(+)} = \sqrt{8\pi G}\,(\chi_k/a(\eta))\,\mathrm{e}^{-i\mathbf{k}.\mathbf{x}}$

$$\ddot{\chi}_k + [k^2 - \frac{\ddot{a}}{a}]\,\chi_k = 0 \tag{1a}$$

where $a(\eta)$ is the scale factor of the Universe: $ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$, and k is the comoving wavenumber $k = 2\pi a/\lambda$. Equation (1a) closely resembles the Schroedinger equation in quantum mechanics, with $\ddot{a}/a$, playing the role of the potential barrier 'V'. (The form of $V \equiv (\ddot{a}/a)$, is shown in Figure 1, for inflation followed by a matter dominated epoch.) Solutions of equation (1a) have some interesting properties, for instance, small wavelength solutions of (1a) are adiabatically damped:

$$\phi_k^+(\eta) \equiv \frac{\chi_k}{a(\eta)} \underset{k\eta\gg 2\pi}{=} \frac{1}{\sqrt{2k}a(\eta)}\,\exp(-ik\eta), \tag{1b}$$

whereas long wavelength modes asymptotically approach the form :

$$\phi_k(\eta) \equiv \frac{\chi_k}{a(\eta)} \underset{k\eta\ll 2\pi}{=} A(k) + B(k)\int\frac{d(\frac{\eta}{\eta_0})}{a^2}\;, \tag{1c}$$

where A and B are constants. The form of (1c) implies that the amplitude of a fluctuation (equivalently – gravity wave), freezes to a constant value when its wavelength (during inflation), becomes larger than the corresponding Hubble radius. Since modes are continuously being pushed outside the Hubble radius during inflation, a mode with a fixed wavenumber "k", which left the Hubble radius during inflation ($k = 2\pi\ \eta_1^{-1}$), will re-enter it later during the epoch of matter domination ($k = 2\pi\ \eta_0^{-1}$). The amplitude of the mode between these two epochs is super-adiabatically amplified, which in field-theoretic language corresponds to the quantum creation of gravitons (Grishchuk 1974). Graviton production in an inflationary Universe has been studied in some detail by Starobinsky (1979), Abbott & Wise (1984), Allen (1988), and Sahni (1990). In particular, was demonstrated that the amplitude of gravity waves on scales greater than the Hubble radius, has the time independent form (Sahni 1990)

$$h_{+,\times} = \sqrt{16\pi G}\frac{k^{\frac{3}{2}}}{\sqrt{2}\pi}|\phi_k|\Big|_{k|\eta_1|=2\pi} = \sqrt{16\pi}A(\nu)\left(\frac{H_1}{m_{\rm p}}\right), \qquad A(\nu) = \Gamma(\nu)\pi^{-\nu}(\nu - \frac{1}{2})^{-1}, \tag{2a}$$

where we have summed over both polarisation states of the graviton. H_1 is the value of the Hubble parameter at a time $k|\eta_1| = 2\pi$, when the given mode left the Hubble radius. For exponential inflation $\nu = \frac{3}{2}$ and $h_{+,\times} = (2/\sqrt{\pi})\ (H_1/m_{\rm p})$, in agreement with Abbott and Wise (1984). ($m_{\rm p}$ is the planck mass, $m_{\rm p} = 1.2 \times 10^{19} GeV$.)

Long wavelength gravity waves, entering the Horizon today, perturb the cosmic microwave background through the Sachs - Wolfe effect (Sachs & Wolfe 1967; Rubakov, Sazhin & Veryaskin 1982; Fabbri & Pollock 1983; Abbott & Wise 1984). Writing $(\delta T/T)$ in terms of a multipole expansion:

$$\frac{\delta T}{T} = \sum_{l,m} {\rm a}_{lm} Y_l^m(\alpha, \delta), \tag{3}$$

it can be shown that (Abbott and Wise 1984)

$${\rm a}_2^2 \equiv \langle |{\rm a}_{2m}|^2 \rangle = 0.145\ h_{+,\times}^2. \tag{4a}$$

The corresponding value of the *rms* quadrupole amplitude is

$$Q_T^2 = \frac{5}{4\pi}\ {\rm a}_2^2 = 2.9\ A^2(\nu) \left(\frac{H_1}{m_{\rm p}}\right)^2. \tag{4b}$$

(For exponential inflation $\nu = \frac{3}{2}$, and $Q_T^2 = (2.9/4\pi^2)(H_1/m_{\rm p})^2$.) (The subscript "T", denotes the fact that the quadrupole anisotropy is induced by *tensor* waves, gravity waves being quadrupolar, do not contribute to a dipole component ${\rm a}_1$.)

In addition to gravity waves, scalar density perturbations generated during inflation, also perturb the CMBR via the Sachs-Wolfe effect, caused by potential fluctuations at the surface of last scattering, $(\delta T/T) \sim \frac{1}{3}\ (\delta\varphi/c^2)$. Inflationary density perturbations are characterised by a spectrum $|\delta_k|^2 = A\ k^n$, the scale invariant spectrum $n = 1$, being predicted by inflationary models with exponential expansion. Models with power law inflation ($a(t) \propto t^p, p > 1$), which arise in a number of theories including extended inflation (La & Steinhardt 1989), predict a more general value for the spectral index , $n = (p-3)/(p-1)$ (Lucchin and Matarrese 1985a,b).

The amplitude of density perturbations at horizon crossing ($k = H_0/c$) is given by (Kolb and Turner 1990)

$$\left(\frac{\delta\rho}{\rho}\right)_H^2 \equiv \frac{k^3|\delta_k|^2}{2\pi^2} = \frac{A}{2\pi^2} H_0^{n+3}. \tag{5}$$

(we assume $c = 1$, for simplicity.) The associated fluctuations in the microwave background are given by

$${\rm a}_l^2 \equiv \langle |{\rm a}_{lm}|^2 \rangle = \frac{H_0^4}{2\pi} \int_0^\infty \frac{dk}{k^2} |\delta_k|^2 j_l^2(kx) \tag{6a}$$

where $x = (2/H_0)$, is the present day horizon size. For power law spectra $|\delta_k|^2 = A\ k^n$, (6a) translates into (Gradshteyn & Ryzhik 1980)

$${\rm a}_2^2 = \frac{AH_0^{n+3}}{16}\ f(n) = \frac{\pi^2}{8} f(n) \left(\frac{\delta\rho}{\rho}\right)_H^2, \qquad f(n) = \frac{\Gamma(3-n)\Gamma(\frac{3+n}{2})}{\Gamma^2(\frac{4-n}{2})\Gamma(\frac{9-n}{2})}. \tag{6b}$$

Values of n in the range $0.5 \le n \le 1.7$, are consistent with the recent COBE results (Smoot et al. 1992). For $n = 1$, (6b) reduces to ${\rm a}_2^2 = (\pi/12)\ (\delta\rho/\rho)_H^2$.

The related value of the *rms* quadrupole amplitude is given by

$$Q_S^2 = \frac{5}{4\pi} \mathrm{a}_2^2 = \frac{5\pi}{32} f(n) \left(\frac{\delta\rho}{\rho}\right)_H^2 \tag{7}$$

(the subscript "S" refers to *scalar* perturbations.)

As shown by a number of authors, scalar field fluctuations which left the Hubble radius during inflation, upon re-entering the horizon during radiation (matter) domination, give rise to a density fluctuation, whose *rms* value is given by (Bardeen, Steinhardt & Turner 1983)

$$\left(\frac{\delta\rho}{\rho}\right)_H = b\ \frac{H_1}{\dot{\phi}}\ \delta\phi \tag{8}$$

where H_1 is the Hubble parameter at a time t_1, when a scale which reenters the horizon at t_0, left the Hubble radius during the inflationary epoch (see Figure 1). For modes entering the horizon during radiation (matter) domination, $b = 4(\frac{2}{5})$. (The main contribution to the observed quadrupole anisotropy however, comes from modes entering the horizon during matter domination, we shall therefore assume $b = \frac{2}{5}$ in the ensuing discussion.) $\delta\phi$ arises because of quantum fluctuations in the scalar field during inflation. For modes entering the horizon today $\delta\phi$ is given by $\delta\phi = A(\nu) H_1$ (see (2)), from which we recover the standard result $\delta\phi = \frac{H_1}{2\pi}$, for exponential inflation (Bunch & Davies 1978; Starobinsky 1982; Linde 1982 and Vilenkin & Ford 1982).

For power law inflation, the inflaton potential has the form $V(\phi) = V_0 \exp(-\sqrt{16\pi/p}\ (\phi/m_\mathrm{p}))$ which results in the following exact solution (Burd & Barrow 1988) to the field equations: $H = (p/t)$, $(H/\dot{\phi}) = (\sqrt{4\pi p}/m_\mathrm{p})$, as a result

$$\left(\frac{\delta\rho}{\rho}\right)_H = A(\nu)\ \frac{\sqrt{16\pi p}}{5}\ \frac{H_1}{m_\mathrm{p}}. \tag{9}$$

For models with quasi-exponential inflation ($a \propto \exp \int H(t) dt$), the slow-roll condition $\ddot{\phi} \simeq 0$, when incorporated into the equation of motion of the inflaton field $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$, allows (8) to be recast as

$$\left(\frac{\delta\rho}{\rho}\right)_H = \frac{16}{5}\sqrt{\frac{2\pi}{3}} \frac{V^{\frac{3}{2}}}{m_\mathrm{p}^3 |V'|} \tag{10}$$

where $V(\phi)$ is the inflaton potential. As the inflaton field *slow-rolls* down its potential, the Universe expands quasi-exponentially $a \propto \exp \int H(t) dt$, where $\int H(t) dt \simeq H(t)\Delta t$ must be greater than or equal to 64, in order to explain the observed spatial flatness of the Universe. Taking this into account we get (Hawking 1985; Olive 1990)

$$\frac{V'}{V} m_\mathrm{p} = (\frac{\pi}{8\bar{n}})^{\frac{1}{2}}, \tag{11}$$

where $\bar{n} = (\bar{N}/64)$, and, $\bar{N} = H\Delta t$ is the approximate number of e-foldings. Substituting (11) in (10) we finally obtain

$$\left(\frac{\delta\rho}{\rho}\right)_H = \frac{\sqrt{\bar{n}}}{0.39} \frac{H_1}{m_\mathrm{p}}. \tag{12}$$

Both (9) as well as (12) express the density perturbation at horizon crossing in terms of the Hubble parameter during inflation H_1. Comparing (9) and (11) we find that for large values of p $(p >> 1)$, an interesting relationship exists between $\bar{N}$ and p, namely $\bar{N} \simeq (p/2)$. This result can be established in a straightforward manner by noting that, for slow-roll (quasi-exponential) inflation, the number of e-foldings is given by

$$N = \int_{\tau_1}^{\tau_2} H dt = \frac{8\pi}{m_{\rm p}^2} \int_{V_1}^{V_2} \left(\frac{V}{V'}\right)^2 \frac{dV}{V} \tag{13a}$$

(we have used the slow-roll condition $\dot{\phi} = -(V'/3H) << V^{\frac{1}{2}}(\phi)$ as well as the Einstein equation $3H^2 \simeq (8\pi/m_{\rm p}^2)V$ in establishing (13a).) Substituting (11) in (13a) we finally obtain

$$N = \bar{N} \ln \left(\frac{V_1}{V_2}\right) \tag{13b}$$

relating the approximate number of e-foldings $\bar{N}$, with the exact number – N. We can establish a similar relationship for power law inflation by noting that since $(H/\dot{\phi}) = (\sqrt{4\pi p}/m_{\rm p})$ (Burd & Barrow 1988), we get

$$N = \int_{\tau_1}^{\tau_2} H dt = \frac{\sqrt{4\pi p}}{m_{\rm p}} \int_{\tau_1}^{\tau_2} \dot{\phi} dt = \frac{\sqrt{4\pi p}}{m_{\rm p}} (\phi_2 - \phi_1) = \frac{p}{2} \ln \left(\frac{V_1}{V_2}\right) \tag{14}$$

Comparing (13b) and (14) we find $\bar{N} = (p/2)$, which is what we set out to establish.

From (7), (9) and (12) we finally obtain

$$Q_S^2 = \frac{5}{48} \left(\frac{\delta\rho}{\rho}\right)_H^2 = \frac{5}{48} \frac{\bar{n}}{(0.39)^2} \left(\frac{H_1}{m_{\rm p}}\right)^2 \tag{15a}$$

for quasi-exponential inflation, and

$$Q_S^2 = \frac{\pi^2}{10}\; p\; f(n) A(\nu)^2 \left(\frac{H_1}{m_{\rm p}}\right)^2 \tag{15b}$$

($\nu - \frac{3}{2} = (p-1)^{-1} = (1-n)/2)$), for power law inflation. (See also Fabbri, Lucchin and Mataresse 1986; Lyth & Stewart 1992.)

Since the coefficients a_{lm} in (3) are Gaussian random variables, the variances Q_T^2 and Q_S^2 given by (4b) and (15a,b) must be added, when considering the combined effect of both gravity waves and adiabatic density perturbations on the anisotropy of the CMBR. As a result, the net quadrupole anisotropy will be given by

$$Q_{\rm COBE-DMR}^2 = Q_S^2 + Q_T^2 \tag{15c}$$

where $Q_{\rm COBE-DMR}$ is the *rms*-quadrupole normalised amplitude $Q_{rms-PS} = 5.86 \times 10^{-6}$ detected by COBE (Smoot et al. 1992). From (4b) and (15a,b), we find that the ratio $(Q_T^2/Q_S^2) = (2.94/pf(n))$ depends sensitively on p – the exponent in power law inflation. We find that for $p \le 16$, the contribution from gravity waves to the CMBR anisotropy dominates the contribution from scalar density perturbations, whereas for $p > 16$ the reverse is true. The ratio (Q_T^2/Q_S^2) has been plotted against the equation of state of the inflaton field w $(P = w\rho)$, in Figure 2 $(p = (2/3(1+w)) > 1)$. Substituting the values of Q_T^2 and Q_S^2 in (15c), we obtain

$$\left(\frac{H_1}{m_{\rm p}}\right)^2 = \frac{Q_{\rm COBE-DMR}^2}{[\frac{\pi^2}{10}\; p\; f(n) + 2.9] A(\nu)^2} \tag{16b}$$

for power law inflation. Setting $p = 2\bar{N}$ in (16b) we recover the slow-roll limit of quasi-exponential inflation. As a result (16b) allows us to determine the Hubble parameter during inflation both for power law, as well as for quasi-exponential inflation, (we set $\bar{n} = (\bar{N}/64) = 1$ in this case). Knowing the value of $(H_1/m_{\rm p})$, we can determine the value of Q_S^2 – the fraction of the quadrupole anisotropy contributed by density perturbations, as well as $(\delta\rho/\rho)_H^2$ – the amplitude of density fluctuations at horizon crossing (see (5), (9), (12)). We follow this procedure and use the COBE-DMR results to predict the value of the *rms* mass fluctuation on a given scale

$$\left(\frac{\Delta M}{M}\right)^2(R) = \frac{1}{2\pi^2}\int k^2 dk|\delta_k|^2 W^2(kR), \qquad (17a)$$

where $W(kR)$ is the *top hat* window function (Peebles 1980).

Our results for cold dark matter models with power law primordial spectra : $|\delta_k|^2 = A\, k^n\, T_{k,\rm cdm}$, (where $T_{k,\rm cdm}$ is the CDM transfer function given in Appendix G of Bardeen et al. (1986)) are shown in Figure 4, for different values of the parameters n and h_{50} ($\Omega_{\rm cdm} \simeq 0.9, \Omega_{\rm baryonic} \simeq 0.1$, is assumed). (Values of n in the range $0.5 \le n \le 1.7$, are consistent with the COBE - DMR results (Smoot et al. 1992).) For a scale-invariant spectrum $n = 1$, our results agree with those of Bond and Efstathiou (1984, 1987). Our results also show that $\frac{\Delta M}{M}(16h_{50}^{-1}Mpc) \le 1$ for $n \le 0.87$. Since mass need not trace light, in models with nonbaryonic dark matter, our results can be taken to mean that a biasing factor ($b_{16} = (\frac{\Delta M}{M})_{16}^{-1}$) mostly greater than unity, is required in order to reconcile theoretical models based on power law inflation , with observations. From Figures 3a,b it follows that the value of b_{16} is sensitive to both n as well as h_{50}, decreasing with increasing n and h_{50}. It may be noted that since gravity waves contribute predominantly to the CMBR anisotropy for $n \le 0.87$, the biasing factor $b_{16h_{50}^{-1}}$ is much larger than it would have been had only the contribution from scalar density perturbations to the CMBR been considered. This makes CDM models with power law inflation less compatible with the excess galaxy clustering observed in the APM survey, than had previously been assumed (Maddox et al. 1990; Liddle, Lyth & Sutherland 1992).

The value of $(H_1/m_{\rm p})$ obtained from equation (16) also determines the amplitude of gravity waves generated during inflation, and normalises the gravity wave spectral energy density – $\epsilon(\omega) = \omega(d\epsilon_g/d\omega)$. The ratio of the spectral energy density to the critical energy density $\Omega_g(\lambda) = \epsilon(\lambda)/\epsilon_{cr}$, has been plotted in Figure 4, for gravity waves created during exponential inflation, quasi-exponential inflation and power law inflation. (For the corresponding analytical expressions describing $\Omega_g(\lambda)$, see Sahni (1990) and Grishchuk & Solokhin (1991).) The amplitude of gravity waves is significantly smaller than the sensitivity of the current generation of terrestrial bar and beam detectors. The best hope for the detection of the stochastic gravity wave background appears to lie with the *space - interferometer*, whose development is still in the conceptual stage (Thorne 1988).

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Figure Captions

Fig. 1

The superadiabatic amplification of gravity waves is shown for a "potential barrier" $V[a(\eta)] = (\ddot{a}/a)$, for inflation $(a = (\eta_o/\eta)$) followed by radiation and matter domination $(a = a_o\eta(\eta + \tilde{\eta}_o)$) (Sahni 1990). At early times $t < t_1$, the scalar field is in its vacuum state $\tilde{\phi}_k^{(+)}$. At late times $t \approx t_0$, the scalar field will in general be described by a linear superposition of positive and negative frequency solutions of eq. (1) : $\tilde{\phi}_k(\eta) = \alpha\tilde{\phi}_k^{(+)} + \beta\tilde{\phi}_k^{(-)}$. A mode with comoving wavenumber $k = (2\pi a/\lambda)$ is shown to leave the Hubble radius during inflation (t_1), and re-enter it during matter domination (t_0). (Figure not drawn to scale.)

Fig. 2

The ratio of the Quadrupole anisotropy from tensor and scalar waves is plotted against both the equation of state w and the spectral index n $(|\delta_k|^2 \propto k^n)$ for power law inflation; $n = (7+9w)/(1+3w)$. The maximum value $n = 0.985$ corresponds to $p = 134$ (in $a \propto t^p$), which is equivalent to ~ 67 e-foldings of quasi-exponential inflation.

Fig. 3

a. The *rms* density contrast on scales $16h_{50}^{-1}Mpc - (\Delta M/M)_{16}$, is shown for different values of the Hubble parameter H_o $(= h_{50} \times 50kmsec^{-1}Mpc^{-1})$, and the spectral index n (($n = 1$) corresponds to the Harrison - Zeldovich spectrum). Some values of the biasing factor $b_{16} = (\Delta M/M)_{16}^{-1}$, are also shown.

b. The biasing factor $b_{16} = (\frac{\Delta M}{M})_{16}^{-1}$, is shown plotted against the spectral index n, for three values of the Hubble parameter: $H = 50$, 75 & 100 $kmsec^{-1}Mpc^{-1}$. The dotted lines correspond to $b_{16} = 1$ and $b_{16} = 2.5$.

Fig. 4

The spectral energy density of gravity waves (in units of the critical density) is shown as a function of the wavelength, for exponential inflation, quasi-exponential inflation (dotted line), and power law inflation $a \propto t^p$ with $p = 21$ and $p = 9$. For comparison, the expected sensitivity of the Laser Interferometer Gravitywave Observatory (LIGO), and of the projected "Beam in space" (space - interferometer) has also been plotted (Thorne 1988).