Are some coronal loop oscillations interference fringes?

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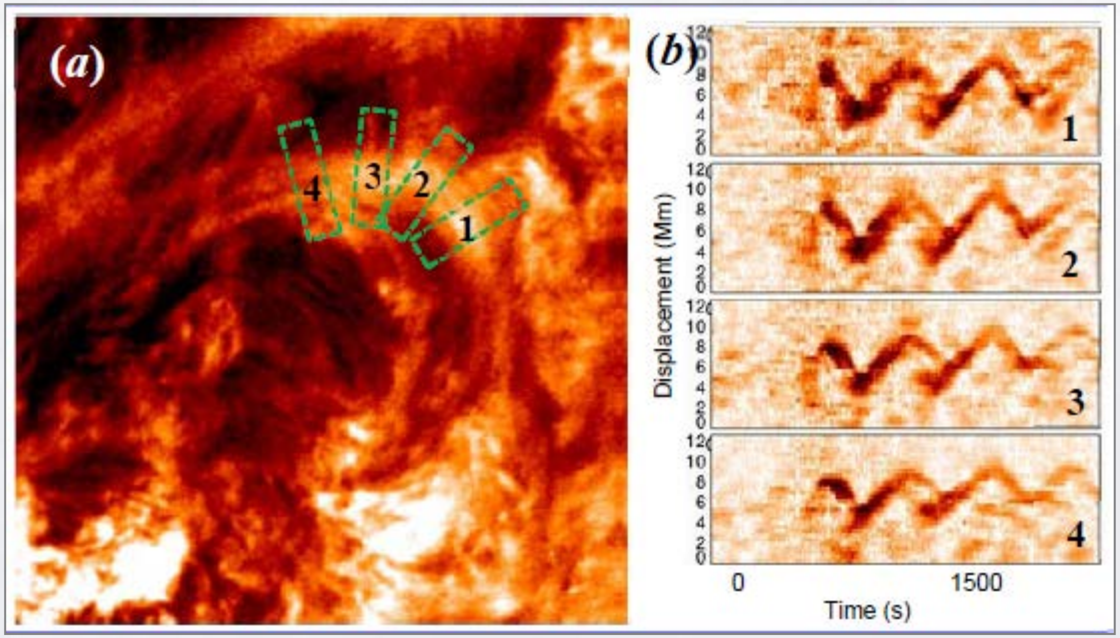
IUCAA (INDIA)
2014, Nov. 12

Oscillations of the coronal loop observed using AIA Fe XI (17.1 nm) images (16 Oct 2010)

Aschwanden & Schrijver (2011)



Time series of the running difference for four cross sections.

The loop sways back & forth without an obvious loss of amplitude, and with the same phase at each location, suggesting standing waves.

Two Varieties of Oscillation

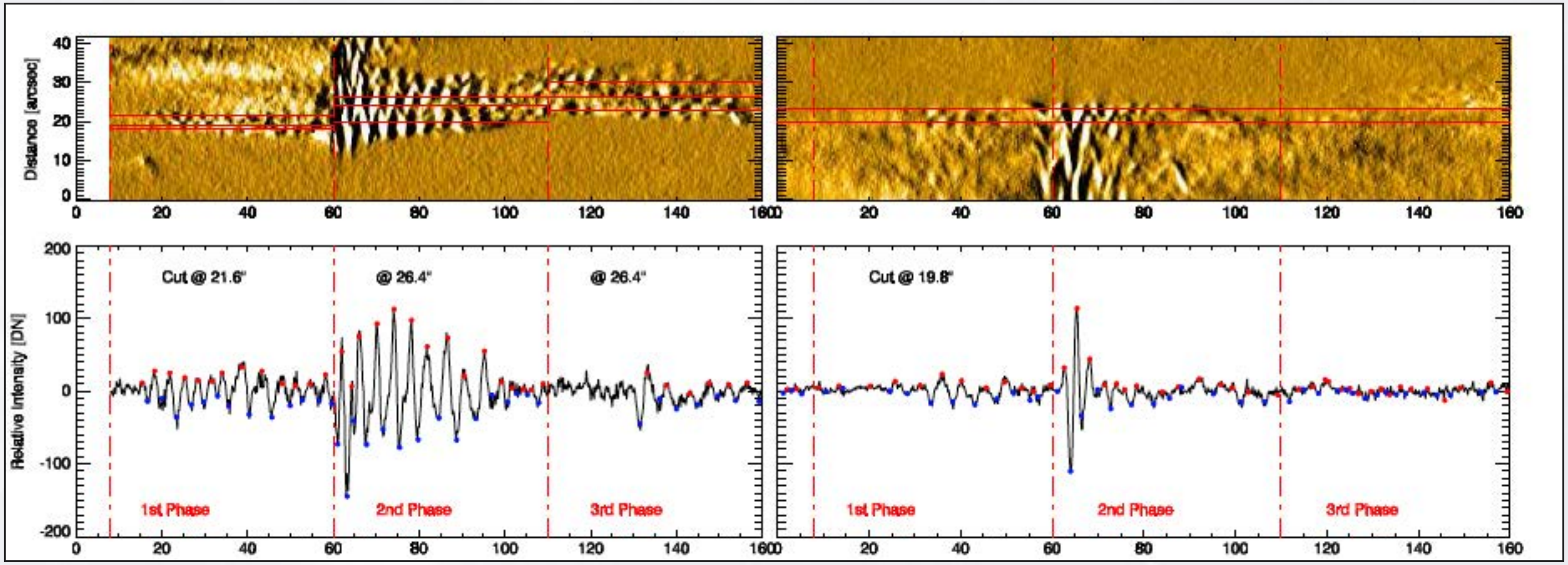
Recent Observations (Nistico et al. 2013)

Time-distance maps: τ = 200-300 sec, τ ~ 500 sec

Flare-initiated, large-amplitude, **decaying** oscillations

+

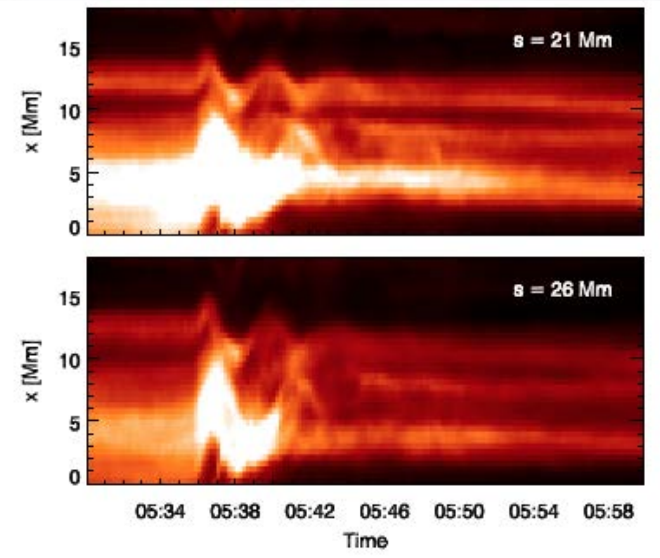
lower-amplitude **decayless** continuous background power oscillating with the same frequency (before and well after the flare)



Time (min)

Rapid Decay of Flare-Induced Waves

Observations: Once initiated, rapidly diminish in 3-4 wave periods
(e.g. White & Verwichte 2012)

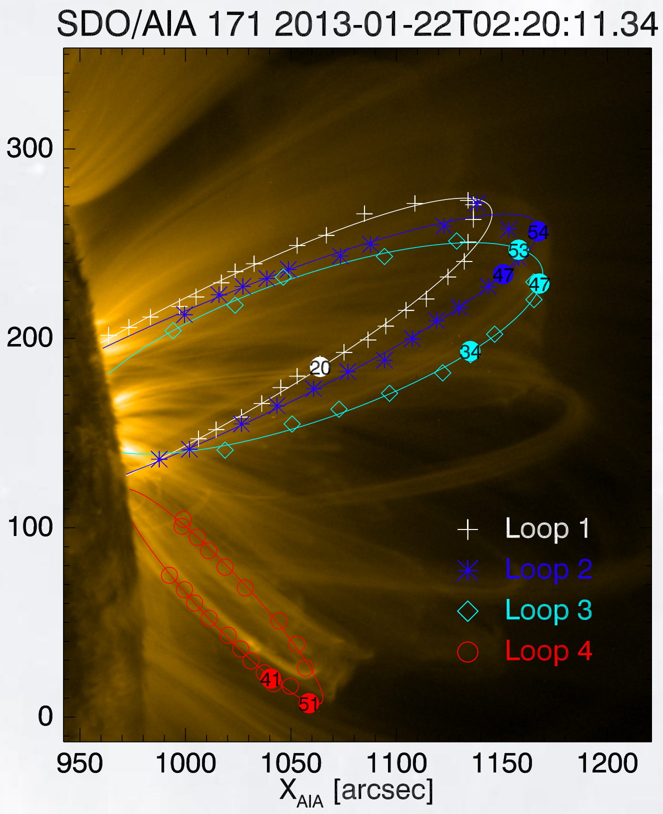


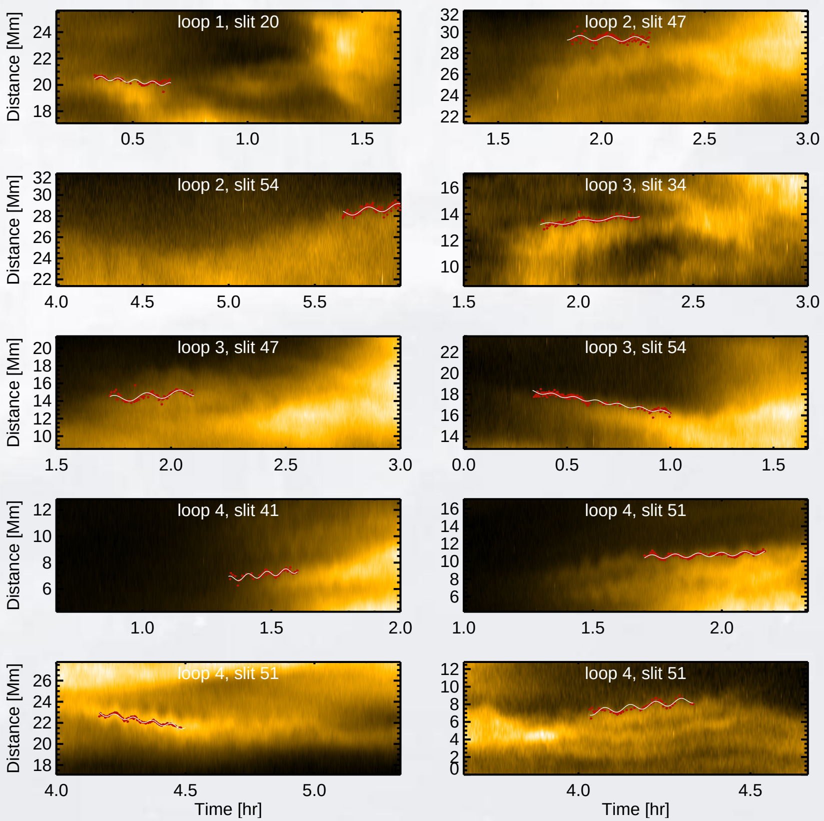
Theory: Damping mechanisms to explain observed diminution

Resonant Absorption (e.g. Ruderman and Roberts, 2002; Goossens et al. 2002)
Phase Mixing between different fibrils (e.g. Ofman and Aschwanden 2002)
Wave Interference (Hindman and Jain 2014): alternative mechanism

Oscillations Not initiated by flares

Anfinogentov et al. (2013)





The oscillatory pattern persists for a long time without significant decay
(some show only 3-4 periods lasting about 20 minutes - other show more than ten periods)

Beginning & end times of oscillations are different for different loops.

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Different Drivers?

Recent Observations of horizontal oscillations
(Nistico et al. 2013; Anfinogentov et al. 2013)

Large-amplitude decaying oscillations (from flares)
+
Low-amplitude oscillations without significant attenuation

Possibility:
background oscillations are a continuous
& perhaps a stochastic driver

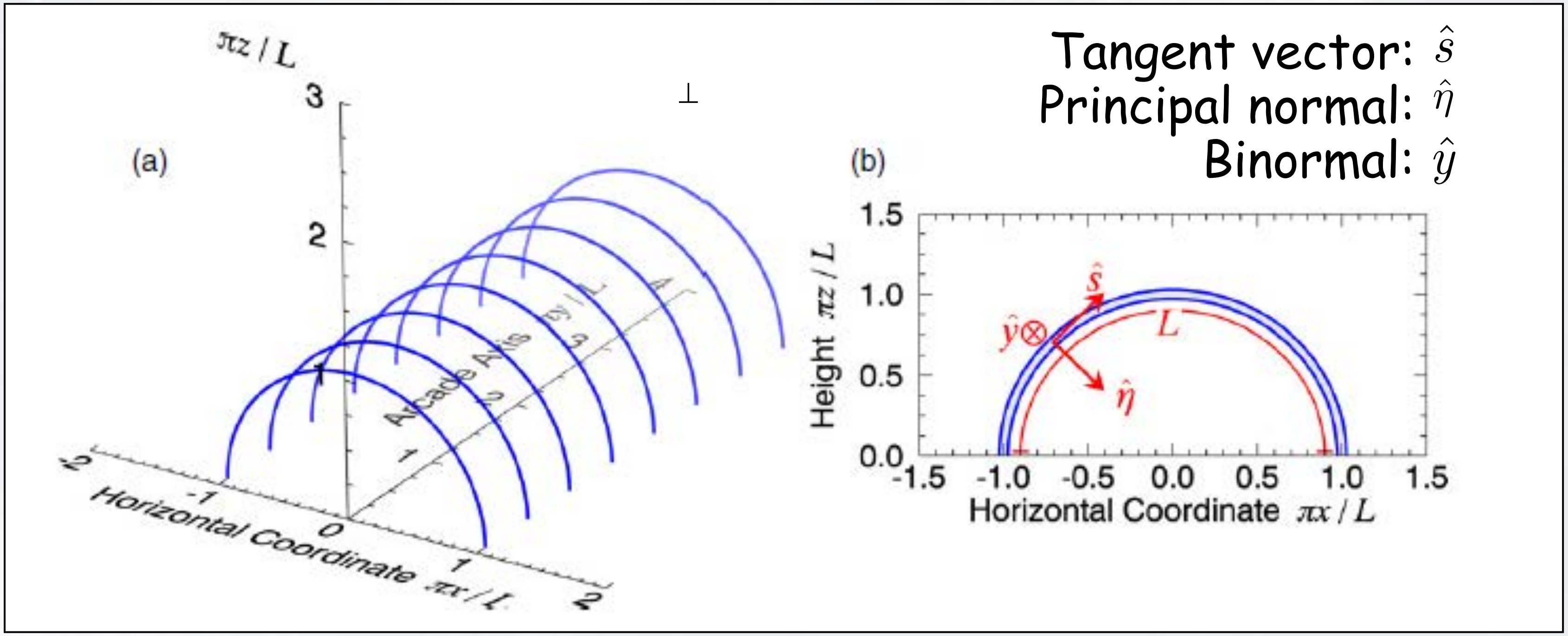
(1) Ongoing
(2) Impulsive

So oscillations come in two flavours with the same resonant nature, but excited by two different sources

Theory: New Interpretation

(Hindman and Jain, 2014)

Idealised coronal arcade with a thin sheet of magnetic field lines



Two-dimensional wave cavity

Trapped standing waves longitudinal to the field lines propagating down the axis of the arcade perp. to the field.

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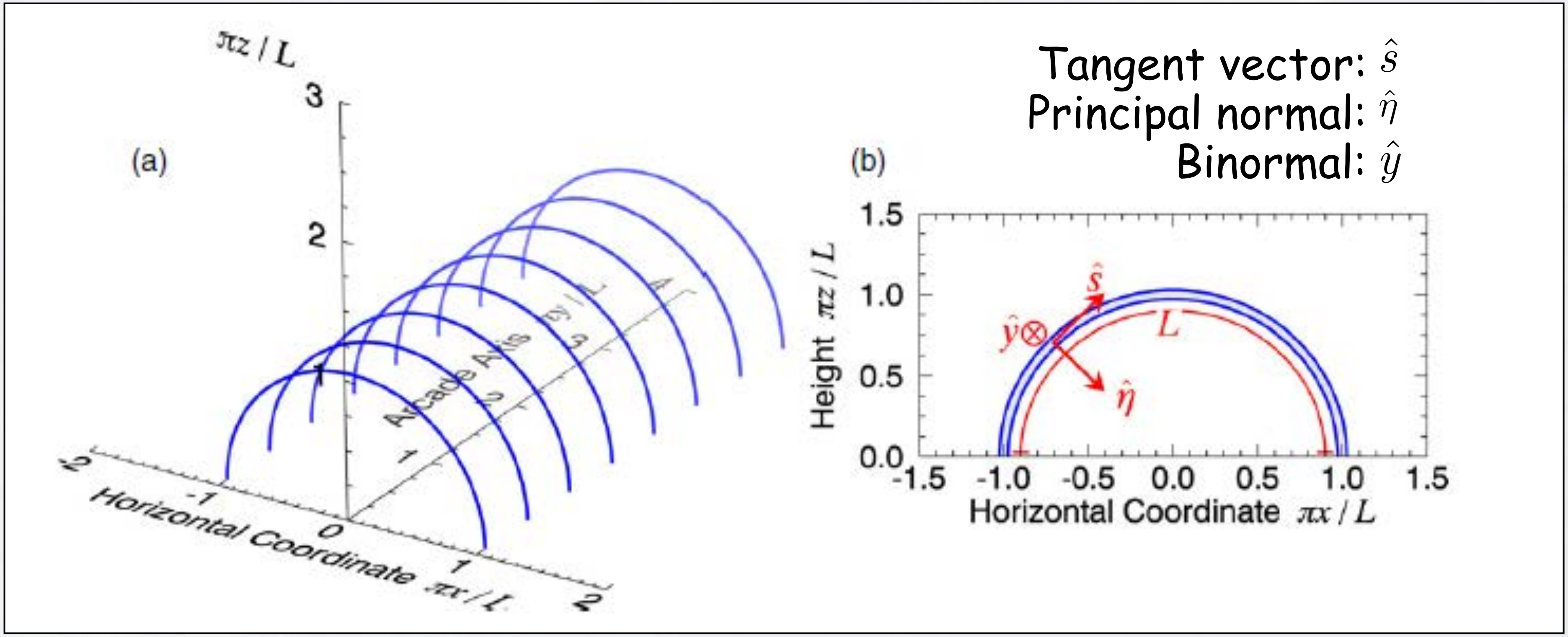
Vertical and Horizontal oscillations



Two-dimensional fluid velocity

$$\boldsymbol{u} = v\hat{\boldsymbol{y}} + w\hat{\boldsymbol{\eta}}$$

Vertical
Horizontal

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Two-dimensional MHD fast wave Equation

No curvature (imp. for vertical oscillations)
Uniform Alfven speed
Magnetically dominated (g = 0, p =0)

$$\left[\frac{\partial^2}{\partial t^2} - V_A^2\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial s^2}\right)\right] v = S(s, y, t).$$

Wave driver

Resonant Modes of the waveguide

Discrete spectrum in the tangential s-direction (line tied: $v = 0$ at $s = 0, L$)
Continuous spectrum in the transverse y-direction (translational invariance)

$$v_n(s, y, t; \kappa) = U_n(s)\, e^{i\kappa y}\, e^{-i\omega_n(\kappa)t},$$

$$U_n(s) \equiv \left(\frac{2}{L}\right)^{1/2} \sin(\lambda_n s),$$

with

allowed parallel wavenumber

$$\lambda_n = \frac{n\pi}{L},$$

and

eigenfrequencies

$$\omega_n^2(\kappa) = \left(\lambda_n^2 + \kappa^2\right) V_A^2.$$

Standing wave parallel to the magnetic field with discrete λ_n and propagating wave in the y direction with continuous wavenumber κ

General strategy for solving the wave equation

(Hindman and Jain, 2014)

- **Fourier transform the equation in the invariant y-direction.**
- **Decompose the source and solution into eigenfunctions of the waveguide.**
- **Solve for the amplitude of each mode in spectral space.**
- **Invert the transform to return to the configuration space.**

Two-component Driver

$$S(s,y,t) = S_{bg}(s,y,t) + S_{imp}(s)\delta(t-t')\delta(y-y')$$

Low amplitude, continuous, broad-band driver (could be due to random movement of the footpoints by convective motions or buffeting from ambient waves in corona outside the waveguide)

Energetic impulsive which has a large initial pulse with subsequent ringing (e.g. flare!)

Each source will independently produce a wave response

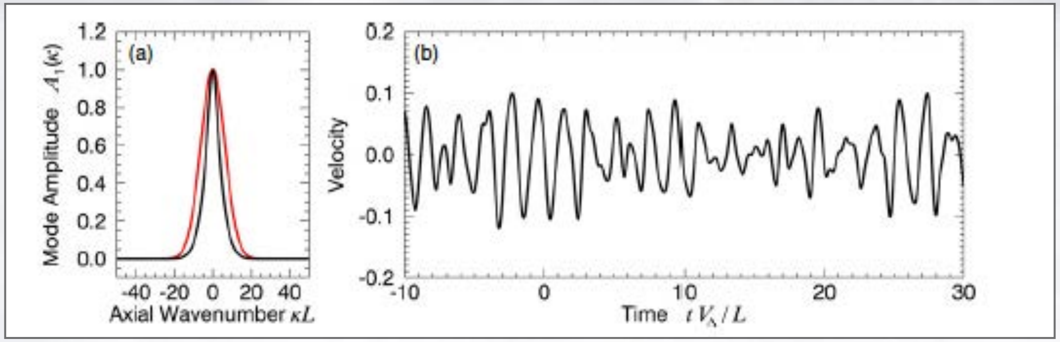
$$v(s,y,t) = v_{bg}(s,y,t) + v_{imp}(s,y,t)$$

The observed ringing is due to a superposition of waveguide modes that form in response to driving by flares+background.

The attenuation of the sinusoidal signal observed at a given field line (or loop) is a fringe pattern resulting from the interference of different portions of an expanding wavefront that has reflected multiple times from the footpoints.

Background oscillations: two examples

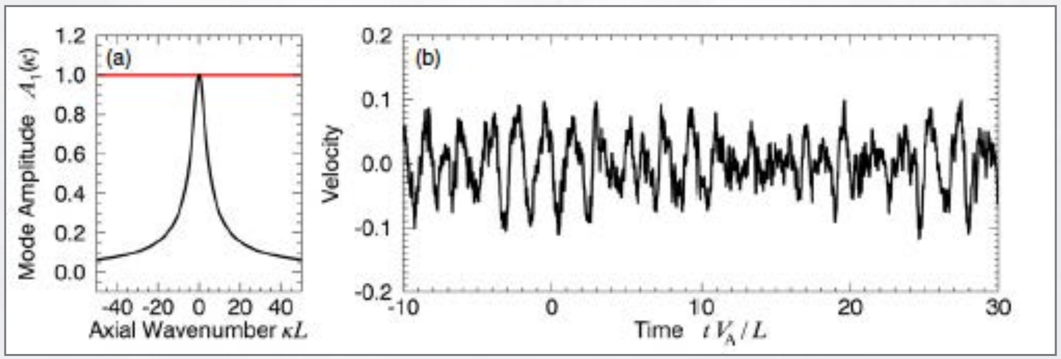
Source strength as a Gaussian function of wavenumber
Amplitude spectrum



$$v_{bg}(s, y, t) = \frac{1}{2\pi} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} d\kappa \, \frac{|\hat{S}_n^{(bg)}(\kappa, \omega_n)|}{\omega_n(\kappa)} U_n(s) \times \sin[\kappa y - \omega_n(\kappa) t + \theta_n(\kappa)],$$

$$\theta_n(\kappa) \equiv \arg\left\{\hat{S}_n^{(bg)}(\kappa, \omega_n)\right\}.$$

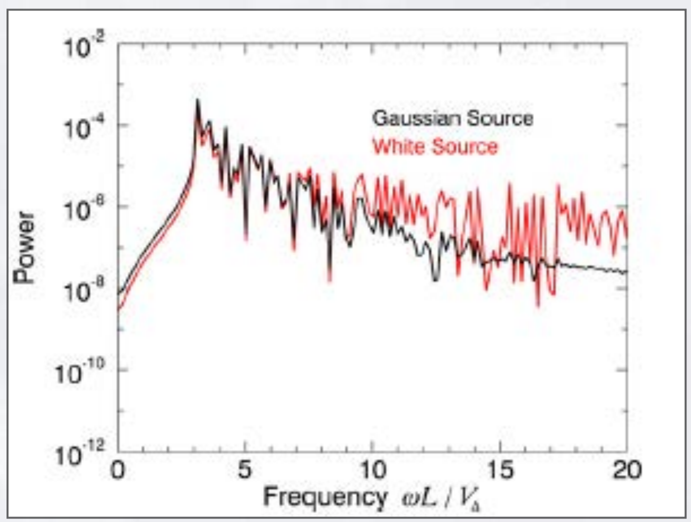
Source strength independent of wavenumber (white source)
Amplitude spectrum



Broader range of wavenumber: time series with richer frequency response

Time series by both sources are highly correlated with very similar low frequency behaviour: same realisation of random phases used to construct both sources.

Temporal power spectra



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Impulsive Driver : single point source

Waves generated are determined by the Green's function.
For a point source of unit amplitude at time t' and at position (s', y').

$$G(s, s', y-y', t-t') = \frac{H(\tau)}{2V_A} \sum_{n=1}^{\infty} U_n(s) U_n(s')\, J_0(\lambda_n V_A T).$$

where

$$\tau \equiv (t-t') - \frac{|y-y'|}{V_A},$$

$$T \equiv \sqrt{(t-t')^2 - \frac{(y-y')^2}{V_A^2}}.$$

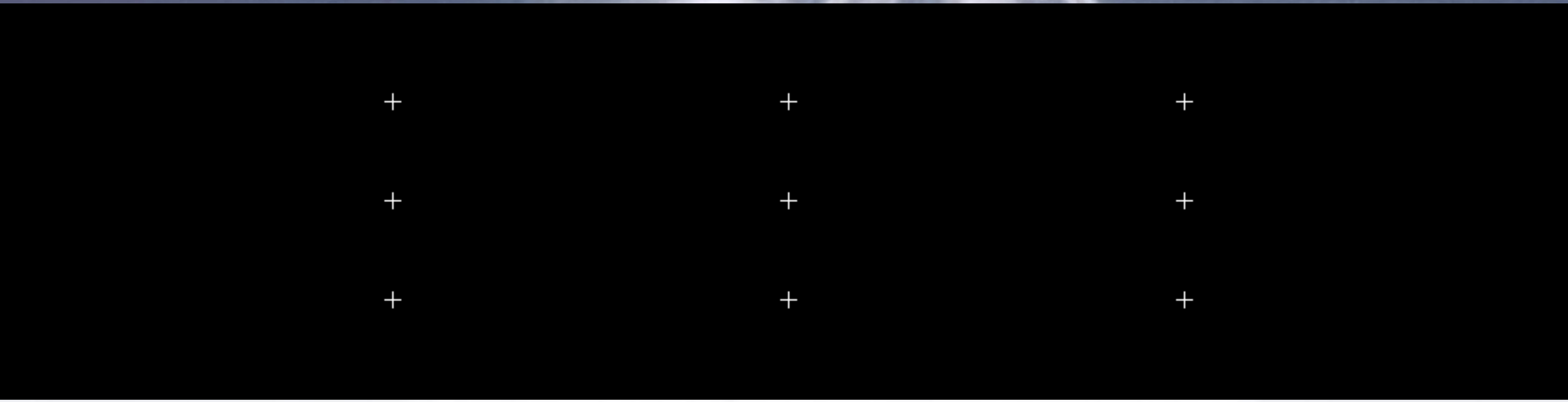
$G \neq 0$ *for* $\tau > 0$

The response of the waveguide to the impulsive source $S_{imp}(s)\,\delta(t-t')\,\delta(y-y')$ is the integral of the product of the Green's function and the source $S_{imp}(s)$ over the point source's location s'

$$v_{imp}(s, y, t) = \frac{H(\tau)}{2V_A} \sum_{n=1}^{\infty} A_n\, U_n(s)\, J_0(\lambda_n V_A T)$$

$$A_n \equiv \int_0^L S_{imp}(s') U_n(s')\, ds'.$$

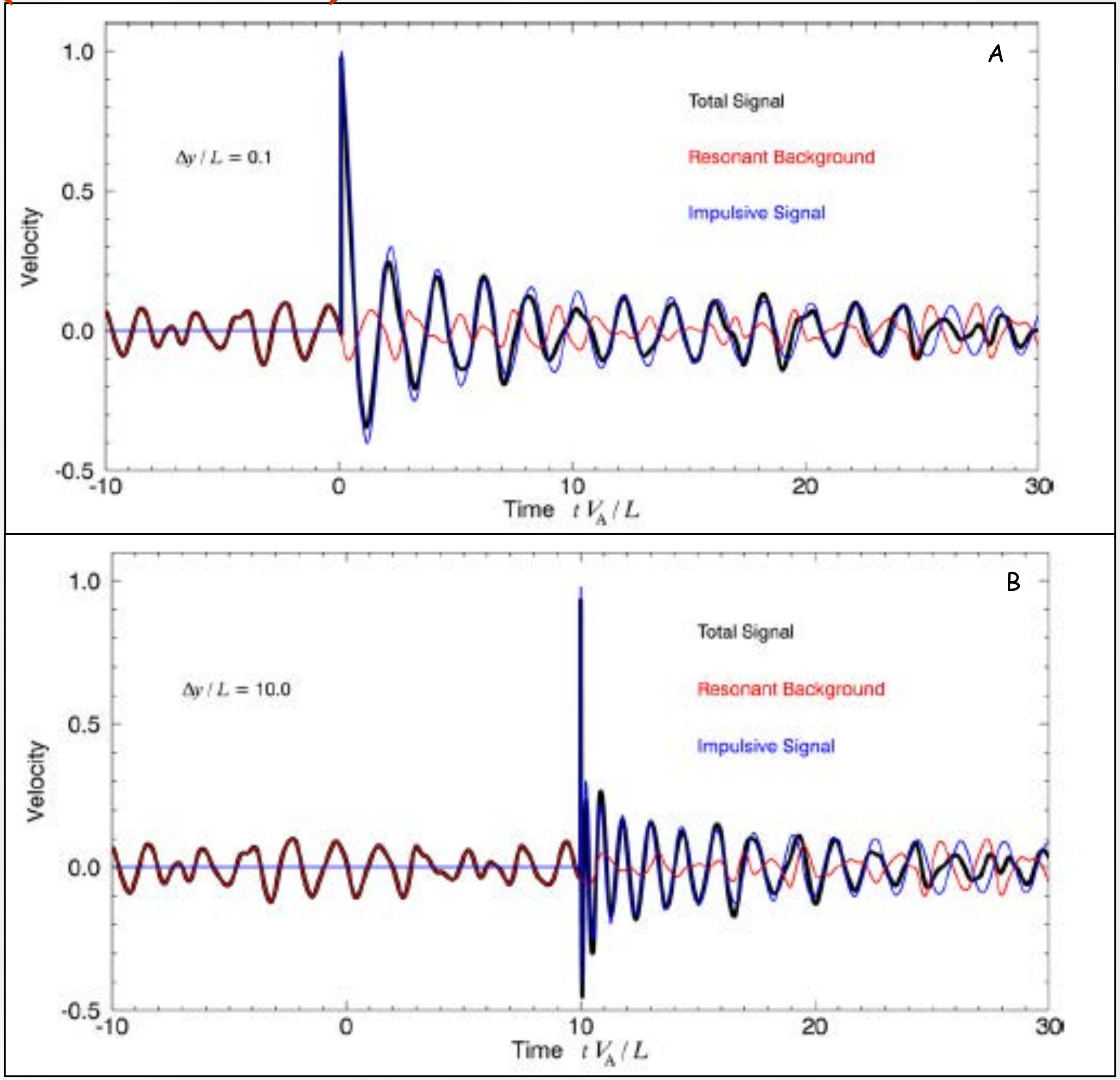
For details, see Hindman and Jain (2014)



- A point source generates a circular wavefront that initially expands isotropically in 2D
- This isotropic expansion stops, when the wavefront impacts the photosphere & reflects
- After many reflections an interference pattern is set up.
- Observations made some distance down the waveguide from the point source will see an oscillatory fringe pattern produced by this interference.
- As time passes the waves that arrive have undergone more and more reflections and therefore have smaller and smaller wavenumber κ . Asymptotically, for very long times all waves contributing to the signal have $\kappa << \lambda$ and thus nearly identical frequencies of $\omega_n = \lambda_n V_A$. Thus, the signal stabilizes to the same frequency that one would obtain for a 1D cavity.

Signal from impulsive driver superimposed on background (Gaussian) oscillation

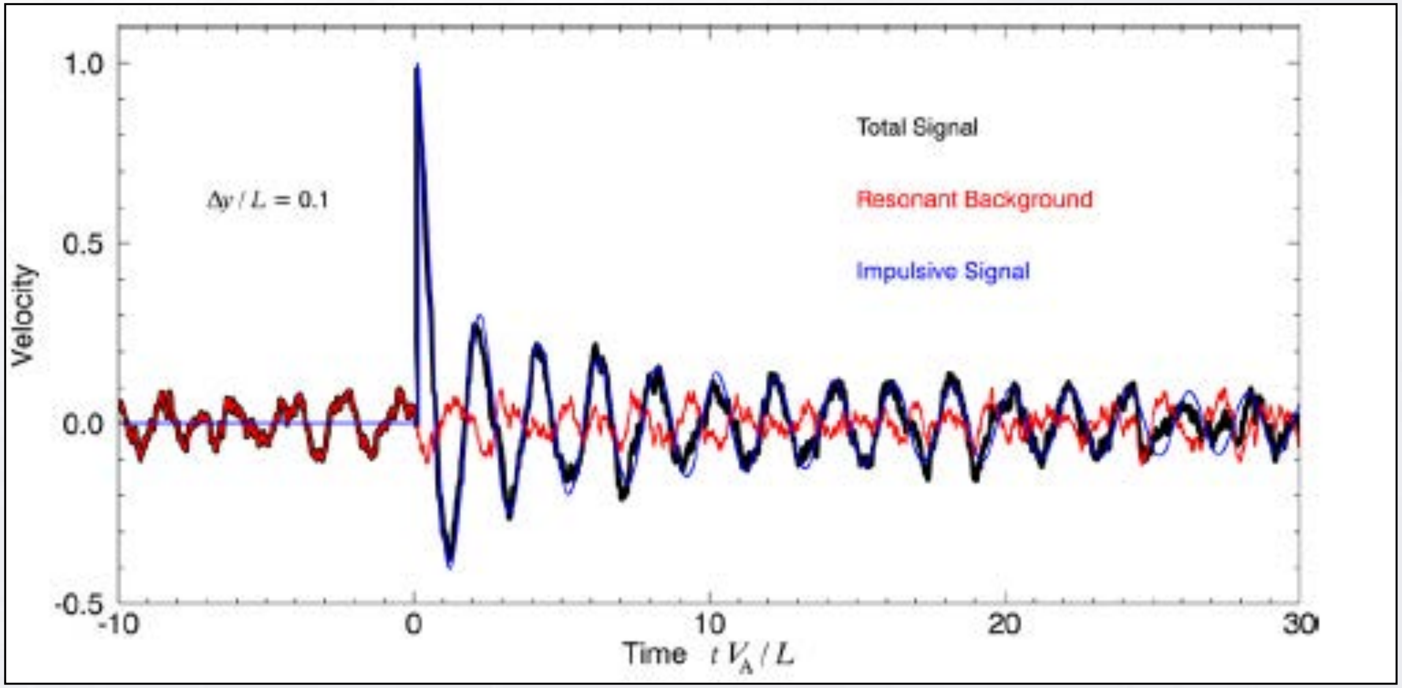
- Impulsive source at t = 0
- Waves observed at the apex but at different distance from the source (hundred times larger in B)



- Time delay at observation point.
- Fringe pattern is compressed near the time of first arrival.
- If the source is extended, the compression at the time of arrival disappears

Signal from impulsive driver superimposed on background (white source) oscillation

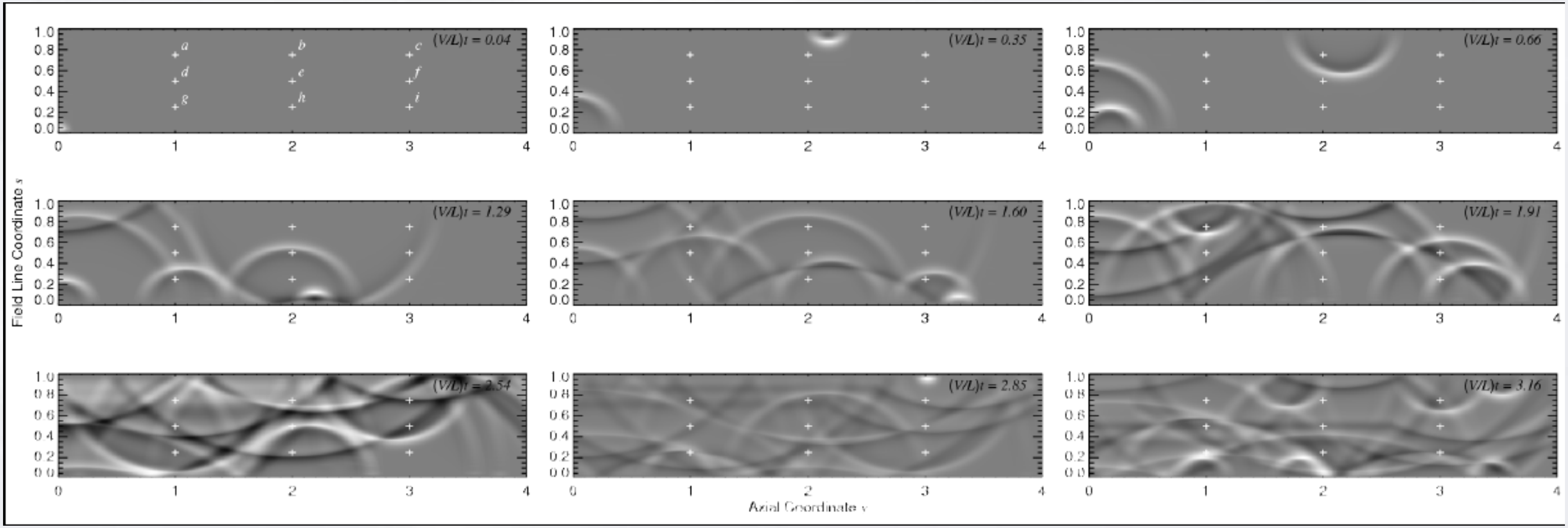
- There is beating and slow modulation of the phase caused by interference between different nearby frequency components.
- The decayless low amplitude and the decaying large amplitude oscillations are very similar to the ones that were observed by Nistico et al.



Other examples of background (stochastic) drivers

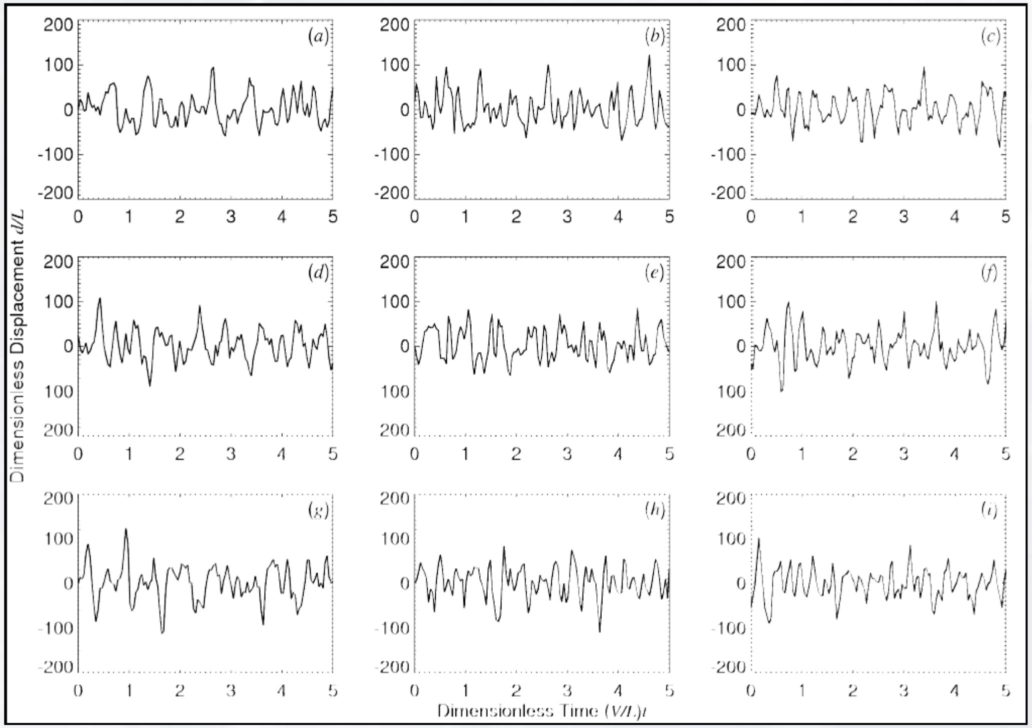
Stochastic wavefield produced by many footpoint sources

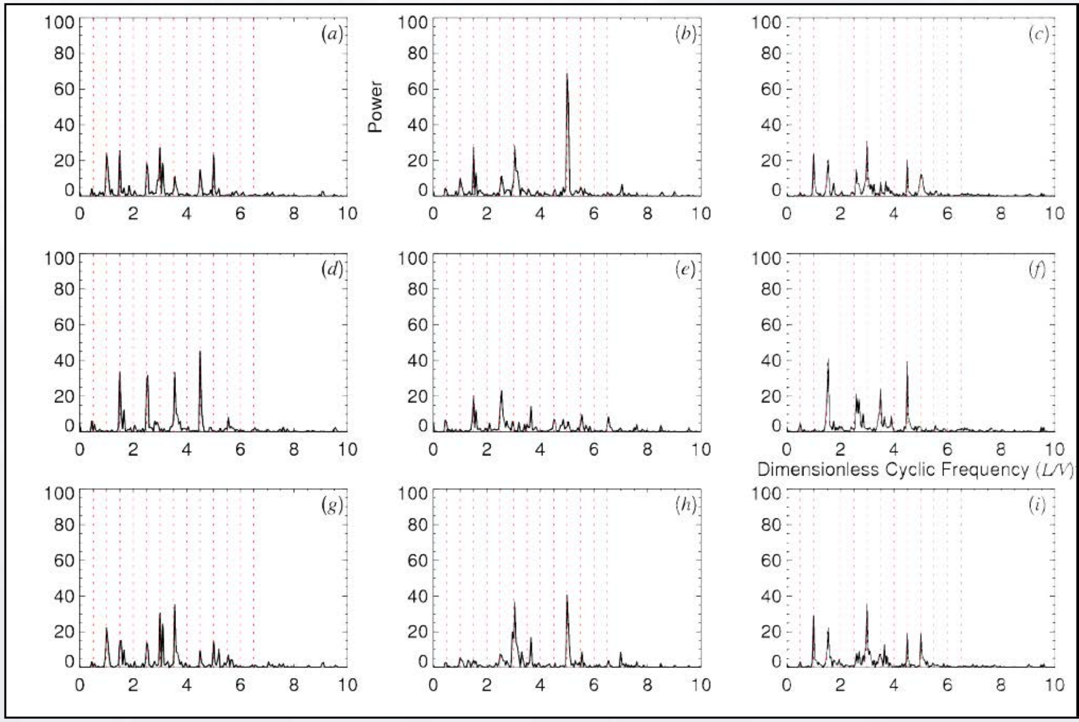
- Each event has the same strength but occurs at random times and places.
- Each event is a gaussian in space and a delta function in time, with a width of 0.05.
- Excitation occurs at the footpoints, so each event occurs (randomly) at either s = 0.001 or 0.999. (100 events used).



Stochastic footpoint-driven wavefield

Time series (left) and power spectra (right)





The fundamental mode is not the dominant mode!
The signal is very broad band with contribution from many high order modes. This is because the source drives at the footpoints.

Conclusions

- We proposed that the observed waves are MHD fast waves that live on the entire arcade which forms a two-dimensional waveguide.

- We showed that the observed **decaying** and **decayless** oscillations can be explained if there are two distinct wave sources: **a continuous, distributed stochastic source** and **a large amplitude impulsive source** which is localised spatially and temporally.

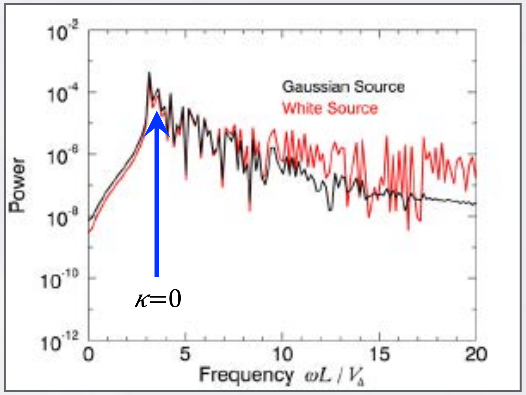
- **The decay in our model is a wave interference effect and the resulting fringe pattern is sensitive to the shape of the waveguide and the position and spatial distribution of the wave source.** The inclusion of a physical damping mechanism is not necessary to reproduce the observed oscillations.

Important Implications

- Aperiodic driving at the footpoints tends to generate a decayless signal with power equally distributed amongst a large number of waveguide modes, contrary to observations. So, the driving must be spatially distributed along the field lines or the driver is periodic.

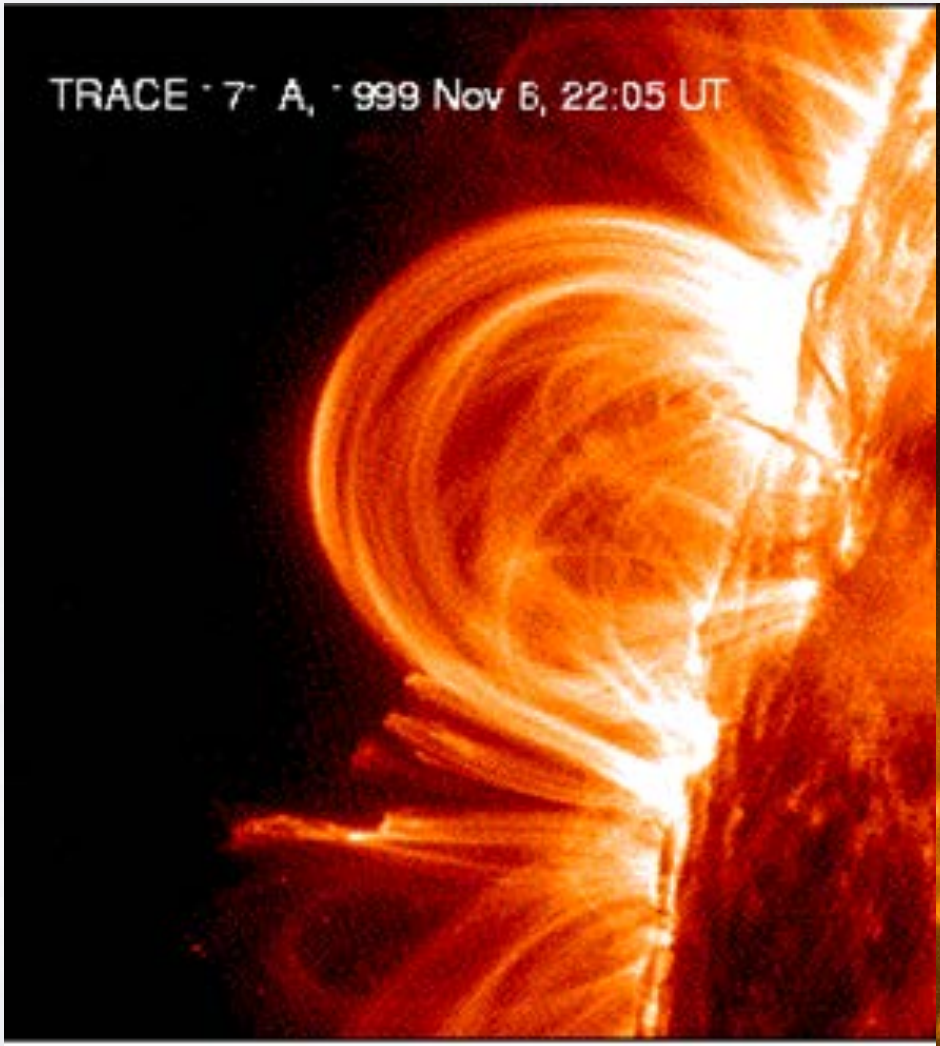
- Each excitation event generates a continuous spectrum of wave frequencies separately for each waveguide mode. Thus, the power is generically broadband and searching for discrete mode frequencies is misguided. However, each spectrum peaks at a frequency corresponding to waves propagating parallel to the field lines. This is dominant frequency that has previously been measured.

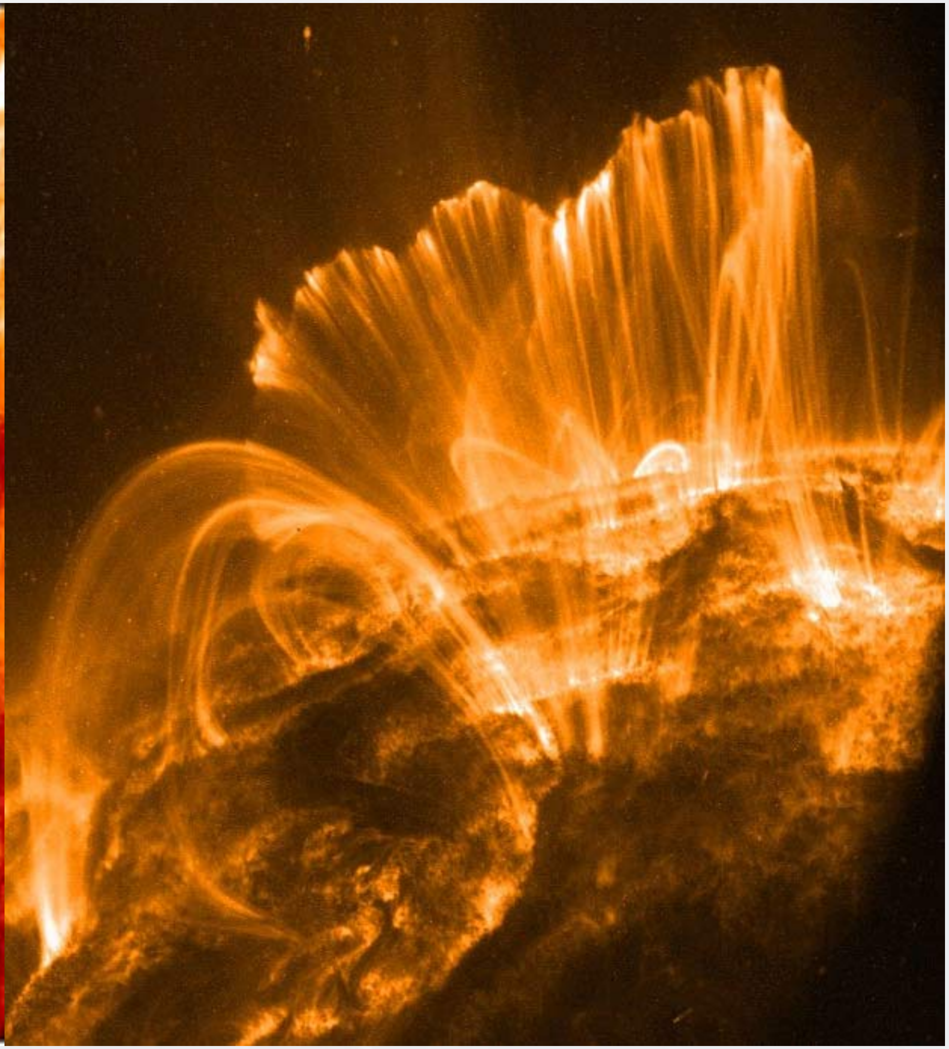
$$\omega_n^2(\kappa) = \left(\lambda_n^2 + \kappa^2\right) V_A^2$$



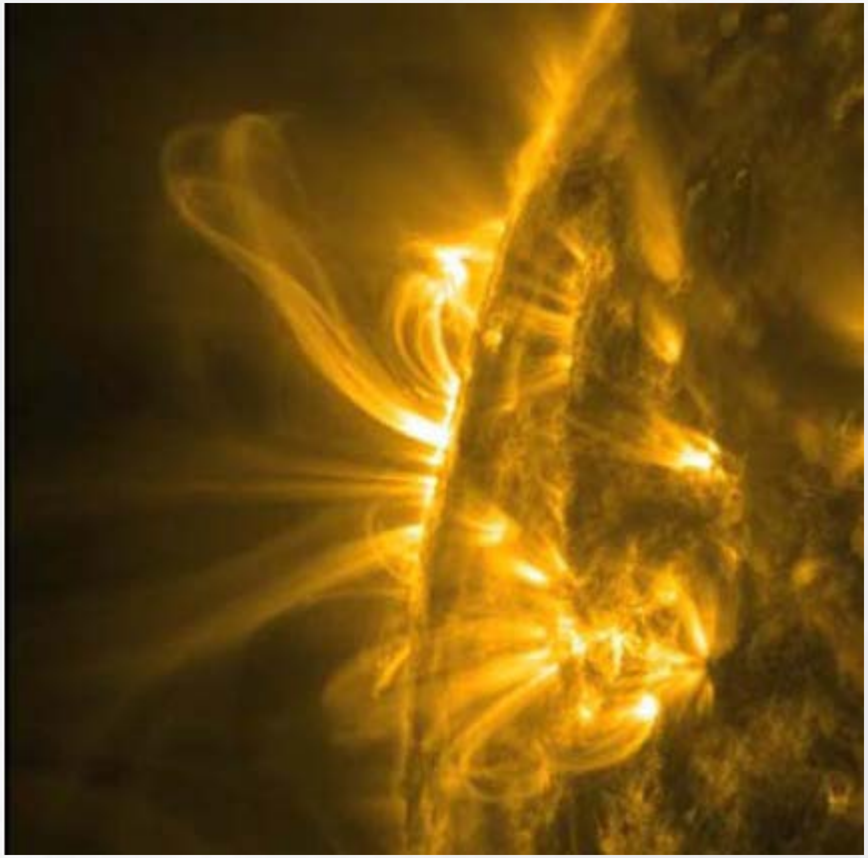
A 2D arcade for which the end effects are not important lacks discrete frequencies. A consequence of this is that a time series can only be fitted with a sinusoid for over short period of time where the phase remains stable. The frequency content of the signal might be better identified in spectral space where the phase variation appears in a more benign fashion.

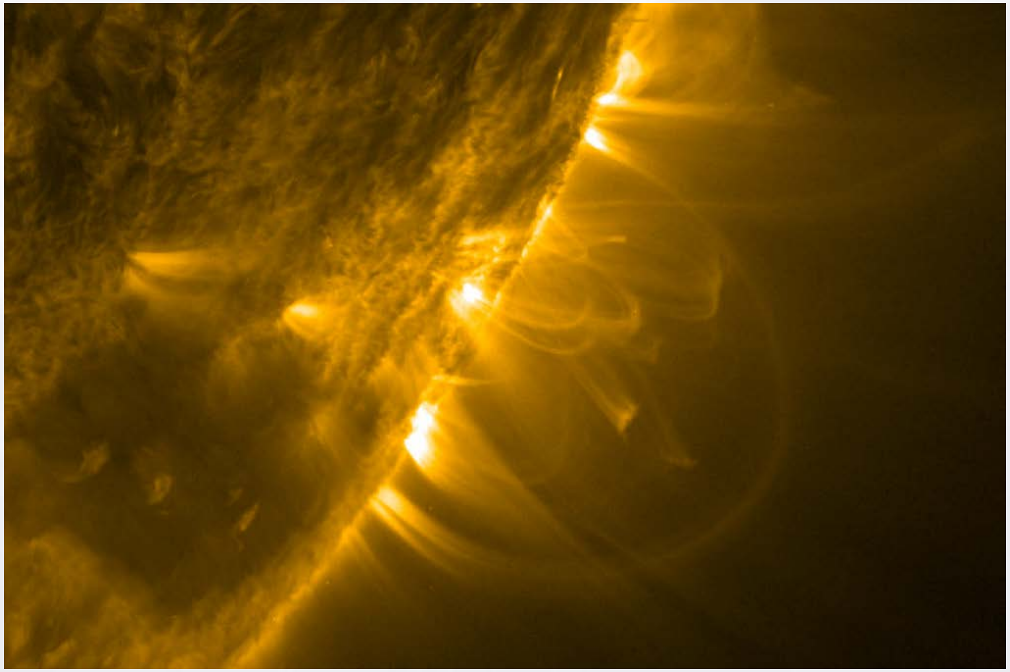
But coronal loops are curved !





… and different radius of curvature!





Source: NASA–SDO/AIA

The gist
(within the framework of our 2D model)

- The waves are trapped in two dimensional waveguide
  (This differs from standard one-dimensional model which treats the wave as the resonant oscillation of just the visible bundle of field lines)

- The two types of observed oscillations, the flare-induced waves and the decay-less oscillations, can both be attributed to MHD fast waves.

- The two components of the signal differ only because of the duration and spatial extent of the source that creates them.

  Flare-induced: strong localised sources of short duration
  Decay-less background oscillations: continuous, stochastic sources