

QUANTUM COSMOLOGY AND THE EARLY UNIVERSE

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ABSTRACT

The classical Friedmann cosmology is known to suffer from three major conceptual problems : (i) spacetime singularity; (ii) particle horizons and (iii) flatness. It is shown that these problems may be resolved during the quantum era of the early universe. Recent attempts in this direction based on the quantization of the conformal degrees of freedom are reviewed here.

1. Introduction

The subject 'quantum cosmology' may appear, at first sight, a contradiction in terms. For, quantum theory deals with microscopic systems while cosmology describes the large scale structure of the universe. Nothing can illustrate the contrast between the two disciplines better than the length scales involved therein : the subatomic distances are $\sim 10^{-12}$ cm while the large scale of the present universe is $\sim 10^{28}$ cm. How then can quantum theory be relevant to cosmology across a gap of forty orders of magnitude?

The answer to this question lies in the 'early universe.' The classical solution of Einstein's equations which was first obtained in 1922 by A. Friedmann describes the expanding universe as given by the line element

$$ds^2 = c^2 dt^2 - Q^2(t) \left[\frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]. \quad (1)$$

Known as the 'Robertson-Walker line element' this describes a homogeneous and isotropic 3-space with a linear expansion given by the scale factor $Q(t)$. The coordinates (r, θ, ϕ) are constant for a fundamental observer who measures the time t .

The parameter k takes values 0, +1 or -1 and the corresponding 3-spaces are flat, of positive curvature or of negative curvature.

The early stages of expansion are described by a function

$$Q \propto t^{1/2} \quad (2)$$

and this solution presupposes that the curvature parameter k is dynamically unimportant at small t . This assumption is, however, nontrivial as the so called 'flatness problem' shows. We will return to this point later.

Thus as $t \rightarrow 0$, $Q \rightarrow 0$ and we have the big bang epoch of the classical cosmological solution. Now as $Q \rightarrow 0$, the classical action for gravity

$$S_g = \frac{c^3}{16\pi G} \int_{\mathcal{V}} R \sqrt{-g} d^4x \quad (3)$$

defined over a characteristic 4-volume \mathcal{V} tends to zero.

The characteristic linear dimension L implied above is of course given by the scalar curvature R through the relation

$$R \sim \frac{1}{L^2} \quad (4)$$

Hence the magnitude of S_g is given by

$$S_g \sim \frac{c^3 L^2}{16\pi G} \quad (5)$$

which tends to zero as $L \rightarrow 0$ at the big bang.

We now receive the answer to the question: "When is quantum cosmology relevant?" That stage is set by the inequality

$$S_g \lesssim \hbar = h/2\pi \quad (6)$$

where h is Planck's constant. Omitting the factor 16π , we find from (5) and (6) that the characteristic scale for quantum cosmology is the Planck length

$$L_P = \sqrt{\frac{G\hbar}{c^3}} \approx 1.6 \times 10^{-33} \text{ cm.} \quad (7)$$

The corresponding time scale is the Planck time t given by

$$t_P = \frac{L_P}{c} = \sqrt{\frac{G\hbar}{c^5}} \approx 5.4 \times 10^{-44} \text{ s.} \quad (8)$$

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Only prior to this epoch do we expect any materially significant inputs from quantum theory. Are these inputs going to be important for the later evolution of the universe?

Experience with electrodynamics shows that quantum theory helps resolve problems of the classical theory. The latter, for example, predicts that the hydrogen atom cannot have a stable structure, that the encircling electron (of mass m) will radiate and spiral onto the proton in a time scale of the order

$$\frac{e^2}{mc^3} \sim 10^{-23} \text{ s.} \quad (9)$$

Quantum theory is needed to explain why the atom retains a stable form. Can quantum cosmology likewise resolve some outstanding problems of classical cosmology?

2. Some Problems of Classical Cosmology

There are three fundamental problems of spacetime geometry that beset classical cosmology : (i) the inevitability of spacetime singularity; (ii) the limitations imposed by a particle horizon and (iii) the apparent fine tuning of the universe to the flat space ($k = 0$) solution. These problems are commonly referred to by 'singularity,' 'horizon' and 'flatness' respectively.

Until about two decades ago the existence of singularity was believed to be due to the extraordinary symmetry of the Robertson-Walker spacetime. Attempts were made to eliminate the singularity by introducing more complicated spacetimes. These modifications (such as rotation) did not succeed and later work demonstrated that provided the usual norms of physics are followed by the matter (and radiation) in the spacetime, it will (or it did) have a singularity (cf [1] for details).

Normally the appearance of a singularity (i.e., the 'blowing up' of physical parameters and a 'breakdown' of mathematical description) is a sign of incompleteness of the underlying physical theory. From this criterion alone, general relativity has to be regarded as an incomplete theory. Can quantum gravity supply the missing ingredient?

The horizon problem began to be noticed in the late 1960s (cf [2]) with the discovery of the remarkable isotropy of the cosmic microwave background radiation (CMBR). The isotropy over large angles (after eliminating the dipole effect due to the Earth's motion) implies large scale homogeneity of the universe. If the CMBR is a relic of the early universe then this homogeneity would have existed at those early epochs.

However, with the expansion factor given by (2) it is easy to verify that the radial coordinate $r_H(t)$ upto which the past light cone of an observer at $r = 0$ extends, tends to zero as $t \rightarrow 0$ according to the relation :

$$r_H \propto t^{1/2}. \quad (10)$$

Clearly, there was no physical way in which particles (or observers) separated by coordinate values exceeding $r_H(t)$ could 'know' one another's physical properties. How then was CMBR made so homogeneous?

It is clear from the above that the expansion factor of the classical solution and the abrupt termination of the past light cone by the singularity at $t = 0$ have been responsible for the horizon effect. Can quantum inputs remove it?

The appreciation of the flatness problem is more recent (cf [3, 4]). It arises because in the early universe scenario there is no time scale characteristic of the present age of the universe $\sim 10^{10}$ yrs. The time scale from grand unified theories (GUTs) is as small as $\sim 10^{-35}$ s. Hence a universe whose physical parameters were fixed in the early GUTs epoch would not normally have exhibited any other time scale.

The abnormality of the situation is reflected in the extraordinary fine tuning that was necessary to keep the universe going to the present times. Since the 'age' of the universe is determined by the overall density of its contents, the fine-tuning is often expressed by the requirement that the fractional range $\Delta\rho$ of densities around a critical value ρ_c had to satisfy the inequality

$$\frac{\Delta\rho}{\rho_c} \lesssim 10^{-50}. \quad (11)$$

Here ρ_c happens to be the density for the flat model $k = 0$. The small number on the right hand side may differ according to the early epoch at which the physical contents of the universe were more or less settled; but its extraordinary smallness cannot be avoided. Can quantum cosmology make this apparent fine tuning look plausible?

3. Approach to Quantum Cosmology

Naturally the only way to answer the above questions is to quantize Einstein's general relativity and to apply the theory to cosmology. This is sooner said than done, as any worker in quantum gravity knows. To make a quantum theory of gravity has not been easy and in spite of several formal approaches the job remains incomplete. (cf [5] for a review of formal approaches.)

What I wish to present here is a relatively naive approach that has the advantage of giving us a glimpse of what a full theory of quantum gravity might be capable of achieving. For details see Ref. [6].

By quantum gravity I do *not* mean the quantization of fields in a curved spacetime. I mean here the quantization of spacetime geometry, which is conceptually much more difficult and has *no* direct analogue in the usual quantum field theory. Here we have to quantize the geometrical parameters that characterize the spacetime. Although there are ten components of the spacetime metric g_{ik} ($i, k = 0, 1, 2, 3$; 0 is timelike) the covariance of general relativity implies that there are only *six* independent quantities.

It is not possible to identify these six quantities explicitly but the following breakup helps in singling out the quantity relevant to our discussion :

$$g_{ik} = (-g)^{1/4} h_{ik} , \quad (12)$$

where g = determinant of the matrix $||g_{ik}||$. The determinant is negative because the signature of the metric g_{ik} is $(+, -, -, -)$.

Note that under a conformal transformation

$$g_{ik} \rightarrow \Omega^2 g_{ik} \quad (13)$$

the h_{ik} remain unchanged while

$$(-g)^{1/8} \rightarrow \Omega (-g)^{1/8} . \quad (14)$$

Thus we may consider $(-g)^{1/8}$ as containing the 'conformal' degree of freedom while the h_{ik} contain the remaining five 'nonconformal' degrees of freedom.

The vanishing of the proper volume bound by fundamental observers of the Robertson-Walker spacetime is due to the vanishing of the conformal factor $(-g)^{1/8}$. In general the spacetime singularities arising from vanishing volumes/infinite densities are identifiable with this factor going to zero. Even the singularities connected with geodesic incompleteness can also be related to the vanishing of this factor [?]. It seems reasonable therefore to identify the conformal degree of freedom as the one most relevant to the singularity problem.

Mathematically speaking, the problem of quantizing only this degree of freedom is completely tractable and allows one to draw general conclusions to which we shall refer later. To what extent may we trust a partial quantization that treats nonconformal degrees classically? Of course a definitive answer can be given only after we have a full quantum gravity theory available (which at present we don't have!). In the meantime the following arguments can be

given in favour of the limited approach.

First, analogy with the hydrogen atom tells us that to resolve the classical difficulty of its stability we have to find a way of preventing r , the radial separation between the electron and the nuclear proton from becoming zero. So, as a first approximation we quantize only the r coordinate, keeping θ , ϕ (the angular variables) classical. With this approach we are able to discover the lowest stationary state of the H-atom wherein it can stay stably. A more complete quantum theory quantizing the angular variables no doubt provides many more stationary states, but the above conclusion remains unaltered. Partial quantization is also used profitably to discuss how atoms (treated quantum mechanically) interact with radiation (treated classically).

Secondly, conformal transformations (13) preserve the global light cone structure of the spacetime and hence they do not alter causal relationships between the contents of the universe. This circumstance ensures a continuity of 'the rest of physics' as the transitions between different conformally related geometries take place. In the general transitions of the nonconformal type the causality is not preserved and the interpretation of physical processes becomes extremely hard.

Finally, the conformal degree of freedom can be identified in a covariant manner (unlike any of the other remaining degrees of freedom). Thus the results of conformal quantization are independent of choice of coordinates.

4. The Basic Formalism

Under a conformal transformation from a metric \bar{g}_{ik} satisfying Einstein's equations, we obtain another metric given by

$$g_{ik} = (1 + \phi)^2 \bar{g}_{ik}. \quad (14)$$

Here ϕ is a general function of space and time, denoting a 'conformal fluctuation' from the classical solution. The gravitational action (3) then takes the form

$$S_g = \frac{1}{16\pi} \int_{\mathcal{V}} [(1 + \phi)^2 \bar{R} - 6\phi_i \phi^i] \sqrt{-g} d^4x. \quad (15)$$

We have put $c = 1$, $\kappa = 1$, $G = 1$ for convenience and the quantities with overhead bar refer to the known classical solution. ϕ_i denote $\partial\phi/\partial x^i$, x^i being the spacetime coordinates. The indices are raised or lowered with the help of the background metric \bar{g}_{ik} .

While applying the formalism to the early universe $t \lesssim t_p$ it is a good approximation to ignore other interactions between material particles, in accordance with asymptotic freedom. Under such circumstances, the matter

part of the action (for free particles a, b, ... with masses m_a, m_b, \dots) transforms as

$$S_m = \sum_a \int m_a ds_a = \sum_a \int m_a (1 + \phi) d\bar{s}_a. \quad (16)$$

The addition of (15) & (16) and the use of classical Einstein equations give

$$S = S_m + S_g = \bar{S} + \frac{1}{16\pi} \int (\bar{R}\phi^2 - 6\phi_i \phi^i) \sqrt{-g} d^4x. \quad (17)$$

The quadratic nature of S with respect to the fluctuation function ϕ tells us that an exact formulation is possible with the Feynman path-integral approach. Accordingly we foliate the spacetime with a series of spacelike hypersurfaces $\{\Sigma\}$ given by $t = \text{constant}$. Let us consider \mathcal{V} to be the 4-volume sandwiched between $t = t_i$ and $t = t_f$, t_i, t_f being constants.

Denote by $K[\phi_f, t_f; \phi_i, t_i]$ the propagator giving the quantum mechanical probability amplitude for the system of conformal geometries to evolve from the initial state $\phi = \phi_i$ at $t = t_i$ to the final state $\phi = \phi_f$ at $t = t_f$. The Feynman formula adapted to 'sum over histories' of ϕ from ϕ_i to ϕ_f gives the functional integral

$$K[\phi_f, t_f; \phi_i, t_i] = \exp(i\bar{S}) \int_{\mathcal{V}} \exp\left\{\frac{i}{16\pi} \int (\bar{R}\phi^2 - 6\phi_i \phi^i) \sqrt{-g} d^4x\right\} \mathcal{D}\phi. \quad (18)$$

Formally, we may apply this formula to study the evolution of the quantum state of the universe from an initial state Ψ_i to a final state Ψ_f , both Ψ_i and Ψ_f being functionals of ϕ :

$$\Psi_f(\phi, t_f) = \int K[\phi_f, t_f; \phi_i, t_i] \Psi_i(\phi_i, t_i) \mathcal{D}\phi_i. \quad (19)$$

The formulae (18) and (19) contain in compact form the results of quantum cosmology as restricted to conformal fluctuations only.

Applications and Results

A pure formalism, however rigorous and beautiful, remains sterile without applications. We now demonstrate how the above formalism might be applied to the early universe.

First we note that it is possible to write down an explicit form for the propagator K by performing the functional integral exactly. For details see Ref [8].

Next consider the initial state of the universe to be empty Minkowski spacetime and suppose that the universe evolved from it by conformal fluctuations. (This happens

because the empty spacetime is unstable to quantum conformal fluctuations, see cf [9]). We can then take \bar{g}_{ik} to be the Minkowski metric η_{ik} with the initial state strongly peaked round it. The final state corresponding to any arbitrary ϕ will have the line element given by

$$ds^2 = (1 + \phi)^2 \eta_{ik} dx^i dx^k. \quad (20)$$

It is possible to calculate the transition probability P that the final geometry is given by (20). The answer (cf [10] for details) is given by

$$P = N \exp \left\{ - \frac{3}{8\pi} \iint \frac{\nabla\phi(\underline{x}) \cdot \nabla\phi(\underline{y})}{|\underline{x} - \underline{y}|^2} d^3\underline{x} d^3\underline{y} \right\}. \quad (21)$$

Here \underline{x} , \underline{y} are spatial coordinates and N is a normalizing constant.

The probability is maximum and relatively very large when the expression in the double integral (which is positive definite) is zero. This condition is satisfied for ϕ a function of t only. But, it is easy to see that this requires the universe to be in the $k = 0$ Robertson-Walker mode. In other words, the universe is in the flat mode because quantum probabilities in the early epochs ensured that it be so. Thus it is not necessary to invoke any fine tuning.

To look at the problem of big bang singularity in this framework, it is convenient to interchange the role of initial and final states in (19). Thus we use the propagator relation to ask the following question: 'Given the near-classical state of the universe at present, what was the state in the past from which it evolved?' A near-classical state may be described by a wavepacket centred on $\phi = 0$ with a small dispersion. The relation (19) then tells us that the initial state was also describable by a wavepacket centred on $\phi = 0$ but with a dispersion that became unbounded at the classical singular epoch.

This result can be obtained under a wide range of conditions and not just for fluctuations around the Friedmann solution [8]. Thus the classical singularity cannot be considered reliable average of the range of quantum solutions. Further it can also be shown that of the full range of quantum solutions those that were singular have a zero measure of probability. In other words, we can assert that it was extremely improbable that the universe evolved from a singular state [11].

The big bang epoch $t = 0$ thus loses most of its formidable nature (as well as metaphysical significance!) and becomes part of a time span of duration $\sim t_p$ in which the universe was governed by quantum rules. We can visualize a 'scattering of states' occurring during this epoch so that a nonsingular contracting phase gives rise to a nonsingular expanding phase across $t = 0$.

Since the universe had existence prior to $t < 0$ it is no longer possible to invoke the particle horizon as before; for, the past light cone of an observer at $t > 0$ can now extend indefinitely out to $t = -\infty$. In other words, the universe is not inhibited from the process of homogenization which it has now ample time to achieve.

Further Work

It is encouraging to see that a relatively straightforward process of quantum conformal quantization can resolve the three outstanding difficulties of classical cosmology. This has prompted further investigations along this direction, two of which I mention briefly below.

First, it is possible to construct a selfconsistent (boot-strap wise) framework in which the background metric is not taken (ad hoc) to be the classical metric. Rather it is obtained as the average c-number type solution of modified field equations of gravity. These field equations are those of general relativity in which the averaged-out influence of quantum fluctuations is built in. The fluctuations themselves are quantized and handled via the path integral method with the c-number metric as the background. Padmanabhan who set up these coupled formalisms has obtained explicit solutions of two kinds. In one there are stationary states of the universe with characteristic length L_p . In the other the universe expands without singularity and horizons with the big bang replaced by a bounce between the contracting and expanding phases. (cf [12]).

Padmanabhan also finds that by introducing quantum conformal fluctuations ϕ in the spacetime describing other field theories (like quantum electrodynamics) and by summing over ϕ it is possible to deduce L_p as the lower bound of length in any physical process. This helps remove the ultraviolet divergences in renormalizable theories by bringing in a natural cut off L_p^{-1} on wave numbers (cf [13]). Conformal quantization has therefore a role to play, well beyond the realm of cosmology alone.

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