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The quasi-steady-state cosmology: a study of angular size against redshift

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ABSTRACT
The data on angular sizes and redshifts of ultracompact radio sources used by Jackson & Dodgson in a recent paper have been applied to the various theoretical models in the framework of the quasi-steady-state cosmology proposed by Hoyle, Burbidge & Narlikar. It is found that although acceptable fits to the data are available for the flat models, those with a negative curvature of spatial sections provide a better fit. These latter models require low densities of matter and as such do not demand too high a proportion of dark matter. A comparison is made with the work of Jackson & Dodgson and theoretical implications of the test are discussed.

Key words: cosmology: theory – dark matter.

1 INTRODUCTION

The angular size (Θ)–redshift (z) relation was proposed as a potential test for cosmological models by Hoyle (1959). The original expectation was that the test would be able to distinguish between the geometries of the various cosmological models. However, this aim was not fulfilled because of the intrinsic scatter and evolution of the sources observed.

Kellermann (1993) argued that the evolutionary effects could be controlled by choosing a sample of ultracompact radio sources of which the angular sizes (of the order of a few marcsec) are measured by very long baseline interferometry (VLBI). Such sources, being short lived and deeply embedded inside the galactic nuclei, are expected to be free from evolution on a cosmological time-scale. He used a sample of 79 such sources and showed that a credible Θ–z relation can emerge. His findings were limited to showing that the Einstein–de Sitter model (with the density parameter $\Omega_0 = 1$) was consistent with the data.

Recently, a more extensive exercise was carried out on a bigger sample of ultracompact sources by Jackson & Dodgson (1997). The original data base for the ultracompact sources that they used was compiled by Gurvits (1994). Jackson & Dodgson have concluded in their work that for the above model, in the optimum case of minimum χ^2, the fit was unacceptable, 'being rejectable at the 98.5 per cent level of confidence'. These authors carried out fits to their data for Friedmann models with various values of Ω_0 and the normalized cosmological constant Λ_0, finding that generally the low-density models with a negative Λ_0 gave a better fit.

The Friedmann models can in principle have any (positive or negative) Λ_0, although, if these models are related to an early inflationary model, then a positive cosmological constant appears less contrived as a remnant of a high positive value, although it was some 108 orders of magnitude above the present one (see Weinberg 1989 for a discussion of the cosmological constant problem).

The quasi-steady-state cosmology (QSSC) on the other hand has a built in negative Λ_0, the magnitude of which can be related to its Machian scale invariant origin (Hoyle, Burbidge & Narlikar 1995). Our preliminary interest was therefore in examining how this cosmology would fare with respect to the same data, in the light of the conclusions drawn by Jackson & Dodgson.

As the QSSC was put forward as an alternative to the standard big bang cosmology, it is expected to satisfy a number of observational tests relating to the large-scale structure of the Universe. These include, apart from the Θ–z relation, the m–z relation, the counts of radio sources and galaxies, the microwave background and the origin of light nuclei, for example. Recent work by Hoyle, Burbidge & Narlikar (1994a,b, 1995), Sachs, Narlikar & Hoyle (1996) and Narlikar et al. (1997) has been concerned with these issues. From these investigations a viable parameter space has emerged within which the QSSC successfully accounts for the present cosmological observations. Thus our main interest is to test if the model operating within such a parameter space can successfully reproduce the observed Θ–z relation.

However, the present work takes a more holistic approach in which we look at a wider parameter space of the QSSC to see how the goodness of fit depends on the parameters of the theory and how the preferred model parameters fare in this test. Our method can therefore be easily applied to the more extensive data sets that may become available in the future.

Hence we undertake here a study of the Jackson–Dodgson data against the background of the QSSC. We begin by outlining the theoretical model used for our analysis and then follow with results of its application to the actual data. To facilitate a comparison with standard cosmology, we recast the QSSC models

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in terms of parameters that resemble the more familiar parameters of standard cosmology.

2 THE QUASI-STEADY-STATE COSMOLOGY

The quasi-steady-state cosmology was developed by Hoyle, Burbidge & Narlikar (1993), and in a series of subsequent papers mentioned in Section 1. The basic structure of this theory is inspired by Mach's principle and it leads to a set of field equations containing the cosmological constant and a scalar field to describe the creation of matter (Hoyle et al. 1995). For the astrophysical and observational features of the QSSC see Hoyle et al. (1994a, b) and Narlikar et al. (1997). Recently Sachs et al. (1996) obtained exact solutions of the field equations describing a homogeneous and isotropic universe that expands exponentially on a long time-scale P, and also oscillates on a shorter time-scale Q. We are concerned here with these models, which are based on the usual Robertson–Walker space–time given by (with the speed of light = 1)

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{1}$$

(r, θ, ϕ) being comoving coordinates of the fundamental Weyl observer. The time coordinate t measures the cosmic time and $k = 0, \pm 1$ represent three possible curvature signatures for the spaces t = constant. The field equations to be solved are

$$R^{ik} - \frac{1}{2}Rg^{ik} + \lambda g^{ik} = -8\pi G[\boldsymbol{T^{ik}} - f(c^i c^k - \frac{1}{4}g^{ik}c^l c_l)], \tag{2}$$

λ being a cosmological constant (negative) and c a scalar field reprsenting creation of matter. x^i $(i = 0, 1, 2, 3)$ stand for the space–time coordinates, with $c_i = \partial c/\partial x^i$. $\boldsymbol{T^{ik}}$ is the matter tensor. In a dust medium the field equations for the Robertson–Walker metric take the form

$$\frac{2\ddot{S}}{S} + \frac{\dot{S}^2 S^2 + k}{S^2} = \lambda + 2\pi G f \dot{c}^2, \tag{3}$$

$$\frac{3(\dot{S}^2 + k)}{S^2} = \lambda + 8\pi G\rho - 6\pi G f \dot{c}^2. \tag{4}$$

The general conservation equation in this case is given by

$$\boldsymbol{T^{ik}_{jk}} = f\{c^i c^k_{;k} + \frac{1}{2}c^i_{;k}c^k\}. \tag{5}$$

The above equation represents two different modes of evolution: the *non-creative* mode, when both sides of the equation have zero value, and the *creative* mode, when both sides are equal and *non-zero*. Sachs, Narlikar & Hoyle (1996) have discussed both the modes separately. The long-term expansion with the characteristic time-scale P is driven by the creative mode, while the non-creative mode is responsible for oscillations with period Q around this solution. The time-scale P is large compared with Q, implying that there may be a large number (20–25, say) of oscillations during one e-folding time of the long-term expansion. Here we are concerned with only the non-creative mode of the model, as the data are expected to relate to the past history of the present oscillatory cycle alone. In this case we have

$$\boldsymbol{T^{ik}_{jk}} = 0 \tag{6}$$

and

$$c^i c^k_{;k} + \frac{1}{2}c^i_{;k}c^k = 0. \tag{7}$$

The above equations with the help of the field equations (3) and (4) lead to

$$\frac{\dot{S}^2 + k}{S^2} = \frac{\lambda}{3} + \frac{A}{S^3} - \frac{B}{S^4}. \tag{8}$$

The constants A and B are positive as the matter density is positive and the c-field energy density is negative. Notice that with a negative λ, the expansion turns to contraction at a large enough S, while a bounce occurs at a small enough S as the constant B is positive. Thus for all values of k, the solution is oscillatory.

For an appropriate choice of the constants A and B, the above equation yields an exact solution as

$$S = \bar{S}[1 + \eta\cos\psi(t)], \tag{9}$$

$\bar{S}$ being constant and η a parameter varying between 0 and 1. $\psi(t)$ is given by the following expression

$$\dot{\psi}^2 = (1 + \eta\cos\psi)^{-2}\left\{\frac{k}{\bar{S}^2} - \frac{\lambda}{3}[6 + 4\eta\cos\psi + \eta^2(1 + \cos^2\psi)]\right\}, \tag{10}$$

with

$$A = 2k\bar{S} - \frac{4\lambda}{3}\bar{S}^3(1 + \eta^2), \tag{11}$$

$$B = k\bar{S}^2(1 - \eta^2) - \frac{\lambda}{3}\bar{S}^4(1 - \eta^2)(3 + \eta^2). \tag{12}$$

The model oscillates between the finite scale limits

$$S_{\rm min} \equiv \bar{S}(1 - \eta) \le S \le \bar{S}(1 + \eta) \equiv S_{\rm max}. \tag{13}$$

The period of oscillation is given by

$$Q = \int_0^{2\pi} \frac{(1 + \eta\cos\psi)d\psi}{\left\{\frac{k}{\bar{S}^2} - \frac{\lambda}{3}[6 + 4\eta\cos\psi + \eta^2(1 + \cos^2\psi)]\right\}^{1/2}}. \tag{14}$$

Notice that the cosmological constant λ has to be negative for the above solution to be possible. The basic theory of QSSC guarantees this. The present epoch in the oscillation may be denoted by t_0, with the last minimum occuring at $t_{\rm min}$, when $\psi = \pi$. The form (9) for expressing the oscillatory solution was chosen for reasons of simplicity. First, the function $\psi(t)$ is almost linear in t for most of the period and differs significantly from it only near the minima of the function S (see Sachs et al. 1996). Secondly, the parameter η (which lies between 0 and 1) indicates how close to zero the minimum scalefactor can be, which in turn can be related to the maximum redshift of any object belonging to the present cycle.

In an earlier paper we had examined the stability of the solution given by (9) with respect to the small perturbations imposed on the metric (g_{ij}), matter density (ρ) and the c-field (c_i) (Banerjee & Narlikar 1997). The solution was found to be stable and thus may be considered physically robust enough to serve as a model of the Universe.

In this paper we will, however, extend our studies to the case $k = -1$ models also, to compare their relative performance vis-à-vis the flat models, in the present cosmological test.

We will now derive the theoretical Θ–z relation in the above models. A typical model has basically three parameters: η, Q and t_0, the present epoch in the oscillatory cycle. However, there are restrictions on these in the form of the measured value of Hubble's constant. We have also to ensure that the maximum redshift in the present cycle is at least ~ 5 to accommodate the largest known redshifts to date.

3 ANGULAR SIZE–REDSHIFT RELATION IN THE QSSC

As stated in the Introduction, we have tried to relate our conclusions to the earlier work of Jackson & Dodgson (1996, 1997), as well as to the standard big bang models. To this end it is necessary to recast some of the above relations into the format of these models. Accordingly we define the following parameters for the c-field:

$$\rho_c = -\frac{3}{4} f \dot{c}^2, \quad p_c = -\frac{1}{4} f \dot{c}^2. \tag{15}$$

Note that although the pressure and energy density are both negative, they follow the equation of state for disordered radiation, viz. $p = \rho/3$. This is hardly surprising when we note that the trace of the energy–momentum tensor of the c-field is zero. For this reason, we also find that the dependence of ρ_c on S is the same as for radiation, namely $\rho_c \propto S^{-4}$. In the QSSC, the Universe is never radiation-dominated, and so the radiation term is dominated by the c-field term. Thus, although in principle it is possible to imagine a universe in which the radiation term dominates over the c-field term, thereby producing a space–time singularity as in the standard models, there is no such possibility here.

We further define the dimensionless parameters by the following formulae:

$$\Omega_0 = \frac{8\pi G \rho_0}{3H_0^2} \text{ density parameter,}$$
$$\Lambda_0 = \frac{\lambda}{3H_0^2} \text{ cosmological constant parameter,}$$
$$\Omega_{c0} = \frac{8\pi G \rho_{c0}}{3H_0^2} \text{ creation density parameter,}$$
$$q_0 = -\left[\frac{S\ddot{S}}{\dot{S}^2}\right]_0 \text{ deceleration parameter,}$$
$$K_0 = \frac{k}{H_0^2 S_0^2} \text{ curvature parameter,} \tag{16}$$

where, to avoid confusion, we have set the velocity of light equal to unity. The suffix zero indicates that the quantity is evaluated at the present epoch. Note that the present value of the scale factor S_0 need not be equal to the scale parameter $\bar{S}$. We define the ratio

$$x_0 = S_0/\bar{S}. \tag{17}$$

In view of the field equations (3) and (4) we have the following relations between these parameters:

$$\Omega_0 = 2K_0 x_0^{-1} - 4\Lambda_0 x_0^{-3}(1+\eta^2),$$
$$\Omega_{c0} = -K_0 x_0^{-2}(1-\eta^2) + \Lambda_0 x_0^{-4}(1-\eta^2)(3+\eta^2). \tag{18}$$

An observational constraint on the QSSC model is provided by the maximum redshift observable in the present cycle. Denoting it by $z_{\max}$ we may use equations (13) and (17) to derive the following relation:

$$x_0 = (1-\eta)(1+z_{\max}). \tag{19}$$

These relations show that the parameter η, which describes the oscillatory part of the solution, is related to the relative physical magnitudes of the three controlling agencies, matter, the c-field and the cosmological constant. In particular, if $\eta \rightarrow 1$, the model tends to have a singular state as in the big bang. The above relation shows that in this limit the c-field term ceases to be effective in causing a bounce.

Corresponding to the relations in the standard cosmology, those connecting these dimensionless quantities in the QSSC are

$$1 + K_0 = \Lambda_0 + \Omega_0 + \Omega_{c0} \tag{20}$$

and

$$\Omega_0 = 2(q_0 + \Lambda_0 - \Omega_{c0}). \tag{21}$$

For $k = 0, K_0 = 0$, whereas for, say, $k = -1$ the parameter K_0 will be negative. At the maximum redshift ($z = z_{\max}$) we have the relation

$$0 = \Lambda_0 - K_0(1+z_{\max})^2 + \Omega_0(1+z_{\max})^3 + \Omega_{c0}(1+z_{\max})^4, \tag{22}$$

which is satisfied identically for all values of the parameters η and K_0. We now use these results to derive the Θ–z relation in the QSSC as follows.

Let the observer be at $r = 0, t = t_0$ when the light from a source at radial coordinate r_1, redshift z and linear size d arrives there. Then, Hoyle's formula gives the angular size as

$$\Theta = \frac{d(1+z)}{r_1 S(t_0)}, \tag{23}$$

where z is the red shift and $S(t_0)$ is the value of the length scale at the present epoch. The coordinate radius r_1 is calculated using the expression

$$r_1 = \int_{S(t_0)/(1+z)}^{S(t_0)} \frac{\mathrm{d}S}{S\dot{S}}. \tag{24}$$

In the case of the exact solution of equation (8)

$$r_1 = \int_{S(t_0)/(1+z)}^{S(t_0)} \frac{\mathrm{d}S}{\left(\frac{\lambda}{3}S^4 - kS^2 + AS - B\right)^{1/2}}. \tag{25}$$

Alternatively we can use the dimensionless parameters defined in equation (16) to replace the constants λ, A, B in the above equation. It is convenient to do so if we are to make a comparison with the work of Jackson & Dodgson. We define $y = S_0/S$. Equation (25) then takes the form:

$$r_1 = \frac{1}{H_0 S_0} \int_1^{1+z} \frac{\mathrm{d}y}{\sqrt{\Lambda_0 - K_0 y^2 + \Omega_0 y^3 + \Omega_{c0} y^4}}. \tag{26}$$

With the relations given above we can calculate the value of Θ for any z, provided we specify these parameters of the model. The calculation is straightforward, if a little tedius and requires numerical computations for specific cases. The general feature of a typical Θ–z curve is a decline of the angular size with increasing redshift, initially as z^{-1}, which slows down to give a flatter curve with a barely perceptible minimum Θ. Qualitatively, this resembles the pattern observed by Jackson & Dodgson. However, details of the fit need to be checked statistically and to be interpreted in the dynamical framework of the QSSC. We will carry out this analysis in the following section.

4 COMPARISON OF QSSC MODELS WITH DATA

The full sample taken by Gurvits (1994) included 337 sources, out of which Jackson & Dodgson took sources with z exceeding 0.5, for reasons well discussed in their paper (Jackson & Dodgson 1997) which do not need to be gone into here. These sources, 256 in number, were binned into 16 redshift bins ranging from 0.511 to 3.787, each bin containing 16 sources. Like Jackson & Dodgson we have used the same values of the mean angular sizes in these bins and their error bars as computed by Jackson (private communication). Thus our analysis deals with the same numbers and a direct comparison can be made between their theoretical models and the QSSC models.

In Section 3 we outlined the broad spectrum of the QSSC models. We shall take $z_{\max} = 6.0$, somewhat exceeding the largest redshift ~ 5 known to date. We have verified, however, that the results described here are not sensitive to this value. Thus from equation (19), we have a unique relationship between η and x_0. Next, given η and K_0, the relations (18)–(20) fix the remaining parameters Ω_0, Λ and Ω_{c0}. It is clear from equation (20) that for flat models, $\Omega_0 \geq 1$, thus requiring a substantial component of dark matter. Models with negative curvature on the other hand do not require such high values of Ω_0. We will discuss this aspect later.

Thus the theoretical Θ–z relation can be completely worked out for given values of $d \times H_0$. The Hubble constant provides the distance scale for extragalactic lengths and as such an observer quoting a value for d uses a value for H_0. We will use the value $65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ as a reference standard and quote the values of d in pc. To fix ideas, we will work with $d = 8$ pc.

To see what constraints on theory are provided by the data, we have fitted theoretical curves for a range of parameters as outlined above. In each case we computed the theoretical values for the angular size at the mean bin redshifts, with the above choice for the linear size d. We have the redshift data for 16 bins. The observed value Θ_i and standard error σ_i of the ith redshift bin is the same as used by Jackson & Dodgson. As there are no constraints in the parameter space, the χ^2 computed as per the formula

$$\chi^2 = \sum_{i=1}^{16} \left[\frac{\Theta_i - \Theta(z_i)}{\sigma_i} \right]^2 \tag{27}$$

has 16 degrees of freedom.

Fig. 1 illustrates four typical cases of the curvature parameter wherein for each K_0 the value of χ^2 is plotted against the parameter Ω_0. Notice that the 95 per cent level line intersects each curve and the portion of the curve lying *below* this line may be considered as giving a reasonable fit. However, notice that the minimum value of χ^2 becomes lower and lower as we tend towards more negative K_0. The value for $K_0 = -2$ for example is $\chi^2 = 15.6$, which at 14 degrees of freedom (two d.o.f. less as we have minimized with respect to the two parameters η and K_0) gives a probability of being exceeded at around 35 per cent, a very good fit indeed.

The corresponding case for flat sections ($K_0 = 0$) as seen from Fig. 1, shows systematically higher values of χ^2, the minimum value being 21.2, with a ~ 10 per cent probability of being exceeded. Thus, although both curves are acceptable at the 5 per cent level, the case for negative curvature is considerably better. Fig. 2 shows the two fits to the actual data side by side.

It is also interesting to compare the ranges of Ω_0 in Fig. 1. For the flat models the acceptable cases require values of Ω_0 around 1.17–1.45, whereas for the $K_0 = -2$ models the range of Ω_0 is around 0.02–0.53, i.e. considerably lower. For the optimum cases of minimum χ^2 the Ω_0 values are 1.26916 for $K_0 = 0$ and 0.0905544 for $K_0 = -2$. As in the standard Friedmann models we have lower density for the negative curvature models and the present test appears to favour these. Fig. 3 represents the variation in the density parameter (Ω_0) with respect to the curvature parameter (K_0) at specified levels of χ^2. The continuous line in the K_0–Ω_0 plane shows the 95 per cent confidence level ($\chi^2 = 23.685$) for the same. For (K_0, Ω_0) values of points above this curve the fit is better than that at the 95 per cent confidence level. In this Fig. it can be seen that the point corresponding to

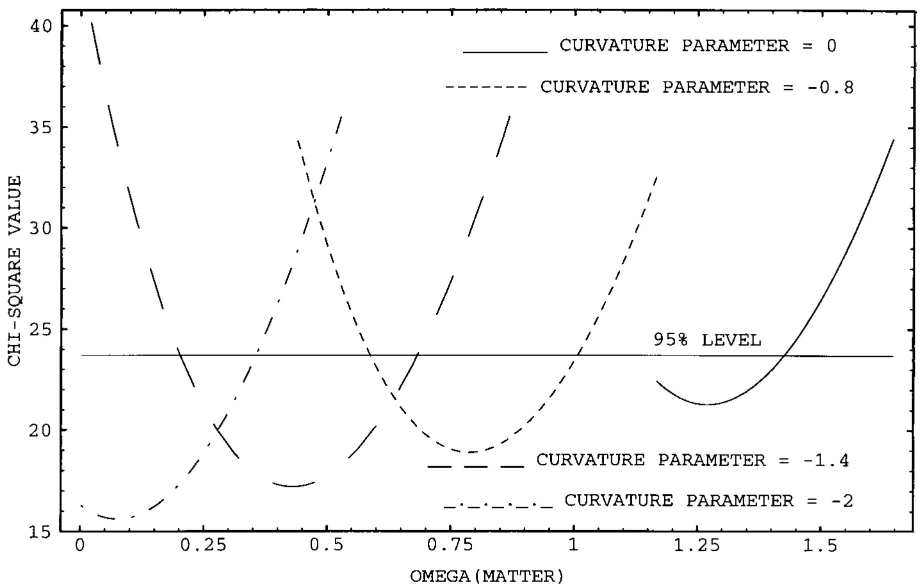

Figure 1. The value of χ^2 plotted against the density parameter for a range of QSSC models with different curvature parameters K_0. The minimum value of χ^2 progressively decreases in the range [−2,0] as the negativity of K_0 increases.

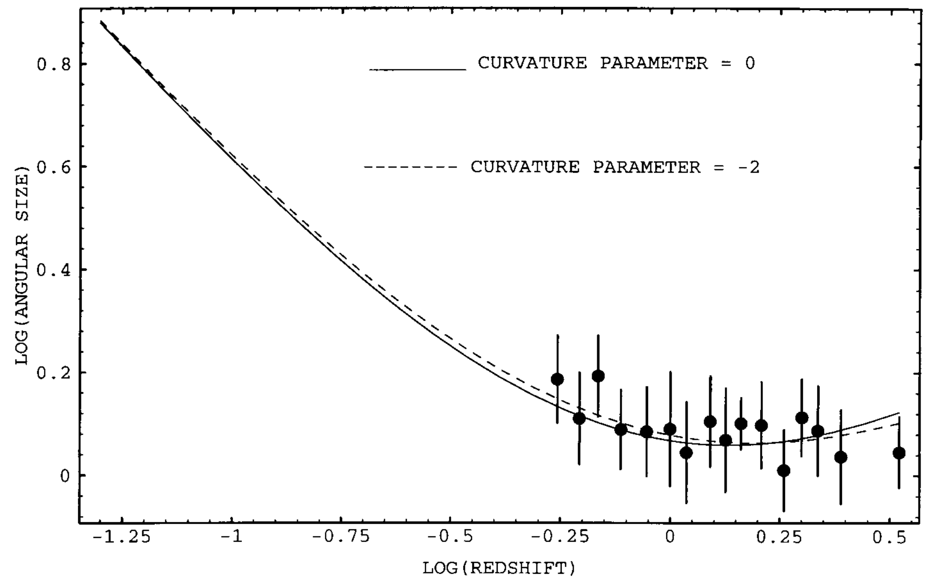

Figure 2. The continuous curve shows the best fit to the data by a flat QSSC model while the dotted curve shows the same for a QSSC model with negative curvature parameter $K_0 = -2$.

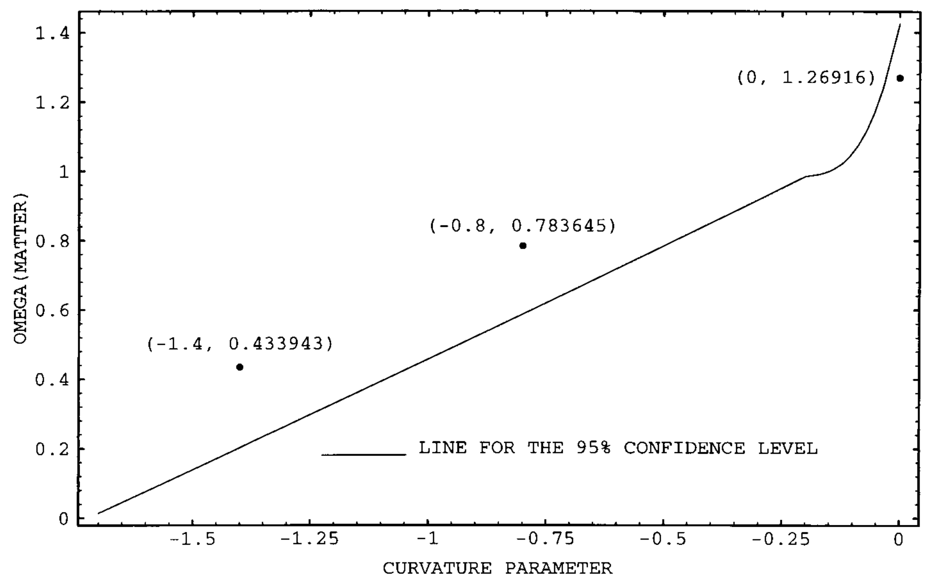

Figure 3. The figure shows how the density parameter (Ω_0) varies with the curvature parameter(K_0) at a specified χ^2 level. The continuous line in the K_0–Ω_0 plane represents the 95 per cent confidence level ($\chi^2 = 23.685$) for these values. The three points corresponding to $K_0 = 0, -0.8$ and -1.4 with their minimum χ^2 values, respectively, 21.257, 18.88 and 17.19 are plotted on the K_0–Ω_0 plane. It is clear from the figure that the sample for $K_0 = 0$ gives us a worse fit than at the 95 per cent confidence level, whereas we can have better fits as the negativity of K_0 continues increasing.

$K_0 = 0$ lies below the 95 per cent confidence line whereas the other two points for $K_0 = -0.8$ and $K_0 = -1.4$ are lying above the line which implies that we cannot have a better fit for $K_0 = 0$. It is also clear from the Fig. that one can get a better fit to the data as the negativity of K_0 goes on increasing.

5 INTERPRETATION AND CONCLUSION

The original test as conceived by Hoyle (1959) visualized the situation in which for most standard models the angular size had a minimum at a finite redshift, usually in the range [1,2] for models of interest. The angular size grew with redshift at large z beyond the minimum and the effect is seen as the result of gravitational bending of light by the smoothly distributed matter in the Universe. The larger the density (Ω_0) the more drastic (i.e. non-Euclidean) the effect.

In the QSSC, the effect may be seen as the resultant of three terms, Ω_0, Λ_0 and Ω_{c0}, of which the last one is of negative energy. The combined effect of the three is therefore dependent on how the three components of the total energy are distributed in space and time; recall that their dependence on the scalefactor is different. The radio-source data discussed here does not show the kind of upturn at high redshift as expected by standard models. In the Jackson–Dodgson case, a negative cosmological constant was needed to achieve a desirable fit, whereas in the QSSC the relative distributions of the three agencies are different. For example, in the best flat case discussed above, $\Omega_0 = 1.26916$, $\Lambda_0 = -0.0878853$, and $\Omega_{c0} = -0.181272$. On the other hand in the best case for a negative curvature parameter -2, the three quantities are $\Omega_0 = 0.0905544, \Lambda_0 = -1.03723$, and $\Omega_{c0} = -0.0533207$. [We are considering more decimal places for these three quantities to justify our results numerically with respect to the equation (22).] Although the *c*-field appears to have a smaller share of the total energy at present, because of the S^{-4} dependence of its energy density, it was more important in the past, i.e. at high redshifts. The behavior of Θ with z is determined by the opposing effects of matter (focusing) and the *c*-field (antifocusing), as the former has positive energy and the latter has negative energy. Thus the effect of matter on focusing at high redshifts is considerably reduced by the *c*-field, which may explain the shape of the Θ–z curve. Notice that, as pointed out by Jackson & Dodgson (1996), the Λ_0 term does not contribute to focusing.

We note that the agreement between theory and observation can be improved still further by minimizing the χ^2 with respect to d and K_0. We have not carried out this exercise as our intention was to see what the key factors are in the QSSC that play a role in determining the shape of the Θ–z curve. Such an exercise can await further accumulation of data for ultracompact sources.

This paper was concerned with Hoyle's test alone, and we have demonstrated that the QSSC meets the requirements of a good fit to the observed Θ–z relation. Perhaps we should mention that the parameter space explored here and found adequate for this test also includes that used for other tests of the theory in previous papers (Hoyle et al. 1994a,b; Sachs et al. 1996; Narlikar et al. 1997). Although Sachs et al. 1996 had discussed the dynamical behaviour of the QSSC models for $k = 0, +1$ and -1, the previous applications to data had been limited to the $k = 0$ case. These include the redshift–magnitude relation, radio-source counts, abundances of light nuclei, the microwave background, ages of galaxies, the nature of dark matter, etc. This is the first time that the $k = -1$ models have been so considered.

On the last count it is worth pointing out that to be consistent with the observed deuterium abundance, the standard cosmology has to respect a stringent upper limit of $\Omega_{0\,\mathrm{baryonic}} \lesssim 0.02$. The evidence of flat rotation curves and cluster dynamics suggests higher values of Ω_0. Therefore, to be consistent with the big bang cosmology, one has to assume the existence of non-baryonic dark matter, considerably in excess of the baryonic component. For the standard $\Omega = 1$ inflationary model, for example, non-baryonic matter is also needed by the various structure-formation scenarios in big bang cosmology.

However, in the QSSC neither the issue of structure formation nor that of production of deuterium requires non-baryonic dark matter. On the other hand, baryonic matter in this cosmology exists also in the form of low-mass stars, old burnt-out stars, white dwarfs from the previous cycles and this is expected to be non-luminous. Its gravitational contribution will nevertheless reveal its existence and in this context, our present findings are of interest.

While this test by itself seems to lean in favour of the low-density negative curvature QSSC models, the current cosmological scenario also seems to favour such models. For example, even accounting for the evidence for dark matter in galaxies and clusters, it is difficult to see how it could lead to a value as high as $\Omega_0 \cong 1.27$ as required by flat models. The structure-formation scenario in the steady-state cosmology is still in an elementary stage and so it is not yet possible to comment on the density criterion from that standpoint.

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REFERENCES

Banerjee S. K., Narlikar J. V., 1997, ApJ, 487, 69
Gurvits L. I., 1994, ApJ, 425, 442
Hoyle F., 1959, in Bracewell R. N., ed., IAU Symp. No. 9, Paris Symp. Radio Astronomy. Standford University Press, Stanford
Hoyle F., Burbidge G., Narlikar J. V., 1993, ApJ, 410, 437
Hoyle F., Burbidge G., Narlikar J. V., 1994a, MNRAS, 267, 1007
Hoyle F., Burbidge G., Narlikar J. V., 1994b, A&A, 289, 729
Hoyle F., Burbidge G., Narlikar J. V., 1995, Proc. Roy. Soc. A, 448, 191
Jackson J. C., Dodgson M., 1996, MNRAS, 278, 603
Jackson J. C., Dodgson M., 1997, MNRAS, 285, 806
Kellermann K. I., 1993, Nat, 361, 134
Narlikar J. V., Wickramasinghe N. C., Sachs R., Hoyle F., 1997, Int. J. Mod. Phys. D, 6, 125
Sachs R., Narlikar J. V., Hoyle F., 1996, A&A, 313, 703
Weinberg S., 1989, Rev. Mod. Phys., 61, 1

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