REFERENCE FRAMES IN COSMOLOGY

J.V. NARLIKAR
Inter-University Centre for Astronomy and Astrophysics,
Post Bag 4, Ganeshkhind, Pune 411 007, India

In this talk a survey is given of the concept of the cosmological rest frame. To begin with one finds that the rising and setting of stars gives us the first indication that we on the Earth are living in a rotating frame of reference. Any frame accelerated relative to it requres inertial forces to be incorporated into the laws of motion. The Foucault pendulum experiment tells us how such forces can be detected on the Earth.

The pendulum experiment also tells us that the local inertial frame is identical with the background of the distant stars in the sky. This led Ernst Mach to postulate his famous principle linking the property of inertia to the stellar background. This background fixes a unique cosmological rest frame. Finally, we explore how the cosmological rest frame can be identified through the somewhat confusing observations of the motions of the large scale structure in the universe and of the microwave background radiation.

Reference Frames, Laws of Motion, Cosmology

1. Introduction :

For a discussion on reference frames in cosmology I can find no better starting point than the verse from the Sanskrit text on astronomy by the fifth century Indian mathematician/astronomer Aryabhata :

Anulomagatirnausthah
Paśyatyacalam vilomagam yadvat,
Acalani bhani todvat
Samapaścimagani Lankayam.

– Aryabhatiya, Chapter 4, Verse 9

Translated into English this means :

Just as an observer on a boat going in a given direction sees the stationary objects moving in the opposite direction so do the fixed stars appear to move towards the West when viewed from the Earth.

Here Aryabhata has captured the essence of how the fixed background of distant stars appears from a rotating frame of reference. I will not go into the historical account of the aftermath of the statement, except to say that in those days the geocentric view prevailed in India too and this perceptive but radical statement of Aryabhata was either ignored by his contemporaries and followers or was misinterpreted.

Today the cosmologists talk of the so called 'cosmological rest frame'. What is that reference frame? To what extent is it the same rest frame of stars that Aryabhata was refering to? And most importantly, from the point of view of physicists, how is that frame related to the local inertial frame?

I will beign with the last question.

2. Newton's Absolute Space

Newton's laws of motion are not only the basic foundations of dynamics, but they are also basic to the whole facade of physics. The first law of motion talks about 'uniform motion in a straight line' by a body on which 'no force acts'. The rest frame of such a body is called an inertial frame. It follows that all frames of reference in uniform motion relative to this frame are also inertial.

The second law of motion relates the acceleration of a body with respect to an inertial frame to the force acting on it. To fix ideas let S be an inertial frame and $\mathbf{r}$ the position vector of a particle at any given time t in it. The time we are using is the absolute even flowing time of Newtonian physics : all observers use the same time t whether moving or at rest relative to one another.

Then the second law of motion becomes

$$\mathbf{F} \propto \ddot{\mathbf{r}}. \tag{1}$$

The constant of proportionality is related to the notion of 'inertia'. The same force will produce diferent accelerations in different bodies depending on their inertia. To quantify inertia we rewrite (1) as

$$\mathbf{F} = m\ddot{\mathbf{r}}. \tag{2}$$

The constant m called the 'inertial mass' of the body is the measure of how much inertia the body processes. An elephant has larger inertia than a rabbit : the same force which hurls a rabbit a long way may hardly budge the elephant.

All this looks fine. But if we examine it carefully we find that these laws involve a deep assumption that has not beer explicitly stated. The first law of motion singles out a special class of reference frames as inertial frames. Why do we have such special frames of reference? We illustrate the question with an example.

Suppose a string is tied round a stone and the stone is whirled round in a circle, holding the other end of the string fixed. In the rest frame of this fixed end, the stone is moving in a circle and therefore accelerated towards the centre. What is the force acting on it? It is the tension of the string also directed towards the centre. So this is in accordance with Newton's second law of motion. But what about the rest frame of the stone? In it the stone is at rest and so according to the first law of motion there should be no force acting on it. But there is! The string still shows tension pulling the stone towards the other end. So an observer at rest on the stone finds a clear violation of the first law of motion.

Newton was aware of this problem. If we have just the stone and the other end of the string, then we cannot tell *a-priori* which end is fixed and which is accelerated. We need an outside fiat to tell us which end is nonaccelerated. Newton provided the fiat by postulating the so called 'absolute space'. The inertial frames are moving with uniform velocity with respect to this space. In the string example, the stone is *not* at rest relative to the absolute space and so we need not worry if the first law appears to break down with respect to its motion.

3. Inertial Forces

Newton described an interesting experiment to illustrate the role of acceleration relative to the absolute space. He suspended a water filled bucket by a rope from the ceiling. After twisting the rope by turning the bucket round and round he let it go. As the rope unwound itself the bucket turned round its axis rapidly and the surface of the water became concave. When the bucket stopped spinning the water surface settled down to its original flat horizontal form.

Newton argued that the latter state of the bucket described it as at rest in absolute space. The state when it is spinning is one of acceleration relative to the absolute space. There is a clear distinction between the two states. An observer at rest in the bucket and not noticing any external background would be able to tell by looking at the surface of water, which state is which. When the water surface is flat the bucket is at rest in the absolute space while when the water surface is curved, the bucket is spinning relative to that space.

How does this observer rationalize the curved water surface? His predicament is similar to that of the observer at rest on the stone in the preceding example. He cannot see any obvious force that made the water surface curved. So, like the observer on the stone, he concludes that the Newtonian laws break down in his case.

Newton salvaged the situation for such 'noninertial' observers by postulating an additional fictitious force. Such a force comes into effect whenever we choose to describe motion in a *noninertial* frame of reference.

Suppose we choose a frame of reference S' accelerated relative to the inertial frame S. Suppose that an observer at $\mathbf{r}$ in S is having coordinates $\mathbf{r}'$ in S' and if he were at rest in S' his acceleration relative to S would be $\mathbf{a}$. Then, in general we have

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}' + \mathbf{a}. \tag{3}$$

The equation (2) then becomes

$$\mathbf{F} = m\ddot{\mathbf{r}}' + \mathbf{a},$$

i.e,

$$\mathbf{F}' = m\ddot{\mathbf{r}}', \tag{4}$$

where

$$\mathbf{F}' = \mathbf{F} - m\mathbf{a}. \tag{5}$$

The equation (4) tells us that we can still have the second law of motion working in S' provided the force is $\mathbf{F}'$, not $\mathbf{F}$. The difference between $\mathbf{F}'$ and $\mathbf{F}$ is the additional inertial force introduced by Newton. The name 'inertial' comes in because, as we see in (5), the additional force $-m\mathbf{a}$ is proportional to the inertial mass of the particle. In the case of rotating frames of reference this is the familiar *centrifugal force*.

4. Mach's Principle

The Earth spinning around an axis is like Newton's bucket. Can we, as inhabitants on its surface detect its spin from the inertial forces? The answer is 'yes' but the effect is more noticeable through the *coriolis force* than through the centrifugal force. To see this result let us briefly look at the mathematical formalism.

Let S be an inertial frame and S' a rotating frame with angular velocity given by the vector ω. Let us denote by $\mathbf{r}$ and $\mathbf{r}'$ the vectors of a point P in S and S' respectively with respect to a common origin. Then we have

$$\dot{\mathbf{r}} = \dot{\mathbf{r}}' + \omega \times \mathbf{r}' \tag{6}$$

and,

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}' + 2\omega \times \dot{\mathbf{r}}' + \omega \times (\omega \times \mathbf{r}'). \tag{7}$$

We have assumed that ω itself does not change with t.

Following (5) therefore we have

$$\mathbf{F}' = \mathbf{F} - 2m\omega \times \dot{\mathbf{r}}' - m\omega \times (\omega \times \mathbf{r}'). \tag{8}$$

The last term on the right hand side is the centrifugal force which goes as ω^2 while the second term, the coriolis force goes as ω. For small ω (as in the Earth's case) the coriolis force dominates.

Although there are natural examples of the effect of coriolis force, e.g., the erosion of right banks of rivers in the norther hemisphere and of the left banks in the southern hemisphere, a laboratory experiment can also demonstrate it. This is the Foucault pendulum whose plane of oscillation rotates around the vertical axis with an angular speed of

$$\Omega = \omega \sin\lambda, \tag{9}$$

where λ is the latitude of the place.

Thus, knowing λ, a sufficiently accurate Foucault pendulum can be used to measure ω. The experiment, of course can be performed within closed walls and one need not be aware of the rising and setting of stars in order to conclude that the reference frame of the Earth is spinning relative to Newton's absolute space.

When this result was derived in the class where I was a student, I accepted it as a matter of course : as a demonstration that Newton's laws of motion work on a system like the Earth. That it carries a deeper significance, I realized much latter.

The fact to be noticed is that the answer for ω one gets from the Foucault pendulum experiment agrees with the answer by astronomical methods. This is not to be expected *a-priori*; for, the former measures the spin of the Earth relative to the absolute space while the latter measures it relative to the distant star background.

It was Ernst Mach who, late in the last century commented on this result. The implication is that the distant star background is the same as Newton's absolute space or the local inertial rest frame. Mach pressed this conclusion further to argue that Newton's absolute space *is determined by* the distant stars. And since this space is essential to write down the laws of motion which give a quantitative expression to inertia, Mach argued that inertia itself owes its origin to this *cosmological reference frame.*

We can rephrase Mach's argument as follows. Suppose that we are talking of only one particle in the entire univere. Such a particle moves under no forces and with no cosmological background. For this particle the second law of motion is

$$m\ddot{\mathbf{r}} = \mathbf{0}. \tag{10}$$

The Newtonian conclusion from (1) is $\ddot{\mathbf{r}} = \mathbf{0}$. But this implies motion with constant velocity which makes no sense in the absence of a background. Instead, the Machian conclusion would be $m = 0$. This makes $\ddot{\mathbf{r}}$ indeterminate which is sensible since there is no reference frame available to measure $\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}$ etc.

So the Machian idea brings the cosmological reference frame in intimate connection with the concept of inertia. Remove the background and there is no inertia. This notion is known as *Mach's principle.* Although Mach gave no direct theory to quantify the effect several authors have attempted this. For a review see [1]. We will not go into those ideas here. The purpose of this talk is mainly to draw attention to the existence of the cosmological reference frame.

5. The Cosmological Rest Frame : Where is it?

Let us turn to the practical/observational aspect of the issue. What exactly is the cosmological rest frame? In Aryabhata's time the distant stars were all confined to the Galaxy as viewed from the Earth. The great revolution brought about by Copernicus demonstrated that the appropriate rest frame for the Solar System is that centred on the Sun. Do inertial forces disappear if we go to the heliocentric frame of reference? Are stars in the Galaxy really at rest?

Not quite! The Sun goes round the galactic centre in a circular orbit of radius $\sim 3 \times 10^{22}$ cm, in a time period of the order of $\sim 7.5 \times 10^{15}$s. This corresponds to a centrifugal acceleration of the order of 2×10^{-8} cm s^{-2}. Although this is small, it is non zero.

Is the galactocentric frame the real cosmological rest frame? Again, not quite! Our Galaxy moves in a Local Group of galaxies of some twenty members. The Local Group itself is falling under gravitational attraction towards the centre of the Virgo Cluster. And, indications are that even the galaxies in the Virgo Cluster are falling under the gravitational force of an enormous mass (not yet seen but inferred from its pull on the neighbourhood galaxies) called the Great Attractor.

So, observational techniques have improved with time but made the interpreter's job difficult! In the 1930s the idea of the expanding universe became widely accepted. It supposed the universe to be homogeneous and isotropic and any galaxy could, in principle, be identified as providing the local standard of rest. In the 1980s it became clear that superimposed on this expansion picture is one of local and large scale peculiar motions that make it very difficult to identify that local standard of rest from the background of nearby galaxies.

To make the problem apparently easier for the cosmologist the local standard of rest is more directly provided by the very smooth background of microwave radiation. As measured from here the background shows a small dipole anisotropy that could be entirely accounted for by a speed of the order of $\sim$ 600 kms^{-1} for our Galaxy in a specified direction relative to an idealized cosmological rest frame in which the radiation background looks completely isotropic.

I used the words 'apparently easier' because although the above observation seems to answer our question about the identity of the cosmological rest frame, it raises another. Why shuld matter distribution be as clumpy and chaotic while the radiation background is so smooth? This question is linked with many more concerning the origin of galaxies and clusters, the origin of the radiation background and the interaction between the two.

Thus from the simple–or rather, the apparently simple observation of the rotating water filled bucket we are led to deeper questions about the origin of the universe, once we start thinking about the various reference frames in the universe.

[1] J.V. Narlikar, INTRODUCTION TO COSMOLOGY, Jones and Bartlett (Boston), 1983.