

Interpretations of the Accelerating Universe

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ABSTRACT. It is generally argued that present cosmological observations support the accelerating models of the universe, as driven by a cosmological constant or “dark energy.” We argue here that an alternative model of the universe is possible, which explains the current observations of the universe. We demonstrate this with a reinterpretation of the magnitude-redshift relation for Type Ia supernovae, since this was the test that gave a spurt to the current trend in favor of the cosmological constant.

1. INTRODUCTION

There are only two instances in modern cosmology in which a genuine theoretical prediction has been directly confirmed by observation. The first of these was the theoretical prediction obtained directly from the solution of Einstein’s equations by Friedmann (1922) and later Lemaître (1927) that the universe is expanding (Hubble 1929). The second was the theoretical prediction made in the framework of the original steady state cosmology (Bondi & Gold 1948; Hoyle 1948) that the universe should be accelerating. This has been demonstrated in modern times by Perlmutter et al. (1999) and by Riess et al. (1998, 2000).

Unfortunately, while the latter was a clear-cut theoretical prediction, the observers almost unanimously have chosen to interpret their detection of acceleration by maintaining the highly popular standard big bang cosmological (SBBC) model explaining the acceleration in terms of a nonzero cosmological constant. Because of this lack of balance in the discussion and the interpretation of the acceleration, we finally published a paper (Banerjee et al. 2000) showing that the observational data available up to that time can very reasonably be interpreted in terms of the quasi-steady state cosmological (QSSC) model. In this model, we had shown that the QSSC model provides a good fit to the above data, provided that one takes account of the absorption of light by intergalactic dust in the form of metallic whiskers. Dust of this kind is required by the QSSC model to explain the cosmic microwave background (Hoyle, Burbidge, & Narlikar 1994a, 2000).

Since that paper was published, new data have been obtained (Riess et al. 2001) involving a Type Ia supernova at a much higher redshift.

In order to give some level of balance to the claim that is now being associated with these discoveries (cf. Filippenko 2001), in this paper we extend our earlier discussion to see

how well (or how badly) the quasi-steady state model fits the new data. However, we begin with a short discussion of why the cosmological constant currently enjoys so much popularity.

2. THE ADVANTAGES OF λ

The recent popularity of the cosmological constant with the proponents of standard cosmology rests on the following aspects:

1. The constraints of “age” are relaxed. By choosing a λ suitably, it is possible to increase the age of the universe. Thus, the age of a standard flat $\lambda = 0$ model is $\sim 6.5 \times 10^9 h_0^{-1}$ yr for $H_0 = 100 h_0 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is too low for accommodating globular cluster ages of $\sim (11-13) \times 10^9$ yr for $h_0 \approx 0.7$. By way of comparison, the age of a flat model with $\Lambda_0 \equiv \lambda/3H_0^2 = 0.7$ is $\approx 13.5 \times 10^9$ yr for $h_0 = 0.7$. At the same time, it should be stressed that the choice $h_0 = 0.7$ is rather arbitrary. While there are those who strongly advocate the model with this value, there is compelling evidence from Saha et al. (2001) that $h_0 \leq 0.6$.

2. The use of λ allows the metric distance to any given redshift to be larger than the corresponding distance (for the same redshift) for the $\lambda = 0$ model. This was demonstrated very clearly by the magnitude-redshift relation for Type Ia supernovae. The excess faintness of supernovae (over the predictions of the $\lambda = 0$ Friedmann models) was interpreted as the proof of a nonzero, positive λ .

3. Indirect checks on $\lambda \neq 0$ are provided by the structure formation scenarios and the observed inhomogeneities of the cosmic microwave background. Here the choice of a nonzero λ helps by expanding the parameter space and thereby relieving pressures on structure formation scenarios from the various observational constraints. For a review of these and other observational constraints, see Bagla, Padmanabhan, & Narlikar (1996) and Narlikar & Padmanabhan (2001).

4. If one insists on a flat model (as is implied by most kinds of inflation), the observed baryonic density of matter is only a few percent of the total cosmological density. In that case,

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the rest ($\geq 90\%$) of all matter has to be dark and probably nonbaryonic. The requirement of locating and identifying so much of the matter in this dark and esoteric form is not so severe if we assume that a substantial part of it is taken over by λ and is called “dark energy” or “quintessence.” Although the problem of bridging the gap between the predicted and the observed still remains, the gap is now to be explained more easily by an “intangible” dark energy than by a more tangible dark-matter particle.

3. THE DISADVANTAGES OF λ

The gifts brought by the cosmological constant come accompanied by a few unwelcome effects too. We name a few below.

1. With the popularity of the inflationary model, the cosmological constant found a new life with its claimed origin in the force generated by transitions of the vacuum. It is this force that is supposed to drive the universe in a transient de Sitter-like mode. It is tempting to suppose that the present λ owes its origin to the inflationary phase. However, Weinberg (1989) has commented on the extremely low value of λ as required by the present cosmological observations compared to the primordial scenarios that drove the universe into an inflationary mode. If the inflation took place at the grand unified theory epoch, the present value of λ is too low by a factor of $\sim 10^{-108}$. If the inflation took place at the quantum gravity epoch, the above factor is lower still at $\sim 10^{-120}$. It is easier to suppose that after inflation was over, the cosmological constant dropped to zero, rather than try to explain why we see its relic at such a low but highly fine-tuned value. One may recall that “fine tuning” was a defect of the standard model that the idea of inflation was invented to correct. To rationalize fine tuning, models with $\lambda \propto H^2$, $\lambda \propto S^{-2}$, or $\lambda \propto t^{-2}$ have been considered by Vishwakarma (2001) to allow for a steady drop of λ from its high primordial value to the presently claimed low value. These models have a somewhat ad-hoc nature and certainly do not share the elegance of the overall structure of general relativity.

2. Although it is tempting to place a larger and larger fraction of the total energy in the λ -basket, beyond a certain level the exercise proves counterproductive. The stretching of space because of a large λ makes gravitational lensing of large redshift objects a much more frequent phenomenon than is observationally allowed. The presently favored value of $\Lambda_0 \cong 0.7$ is already bursting through this limit, and anything in excess may not be tolerated by the lensing data (Falco, Kochanek, & Munoz 1998).

3. If one uses the “best fit to observations” as a criterion for estimating λ , then the angular size-redshift relation for ultra-compact radio sources (Jackson & Dodgson 1997) implies that $\lambda < 0$. There is thus a contradiction between two cosmological tests of the universe going to large redshifts: the m - z relation

gives $\lambda > 0$, whereas the θ - z relation requires $\lambda < 0$ for a best-fit model. Given the uncertainties surrounding the cosmological “standard candle” and “standard rulers,” we feel that the current enthusiasm for a positive cosmological constant may be premature.

4. AN ALTERNATIVE SCENARIO: THE QSSC

If one is willing to move out of the framework of the standard cosmology, the quasi-steady state cosmology (QSSC) provides an already existing framework to understand the above cosmological issues. We briefly describe the salient features of this cosmology before coming to these specific issues.

The QSSC is a nonsingular cosmology that arises from a Machian theory of gravity that gives field equations more general than general relativity (Hoyle, Burbidge, & Narlikar 1995). The equations are

$$R_{ik} - \frac{1}{2} g_{ik} R + \lambda g_{ik} = -8\pi G(T_{ik}^{\text{matter}} + T_{ik}^{\text{creation}}). \quad (1)$$

Here we have taken the speed of light $c = 1$, and the first term on the right-hand side is the usual energy momentum tensor of matter, which, in the case of “dust” (pressure $p = 0$) with density ρ and flow vector u_i , becomes

$$T_{ik}^{\text{matter}} = \rho u_i u_k, \quad (2)$$

while the second term on the right-hand side denotes the contribution from a traceless scalar field C of *negative* energy and stresses with gradient $C_i \equiv \partial C / \partial x^i$:

$$T_{ik}^{\text{creation}} = -f \left(C_i C_k - \frac{1}{4} C_l C^l g_{ik} \right). \quad (3)$$

The coupling constant f is positive, but with its negative coupling to gravity or spacetime geometry, it generates repulsion and *accelerated* expansion of spacetime. We should point out here that the cosmological constant λ in this theory arises from a Machian interaction and *its sign is negative*.

For quantitative details of this model, see Hoyle et al. (1995). General solutions of these equations in the homogeneous-isotropic case were obtained by Sachs, Narlikar, & Hoyle (1996). The QSSC generally uses the simplest of these, which has a $k = 0$ (flat) Robertson-Walker spacetime with a scale factor given by

$$S(t) = \exp(t/P)[1 + \eta \cos(2\pi\tau(t)/Q)]. \quad (4)$$

In this expression, the function $\tau(t)$ is very close to t except near the maxima and minima of the scale factor. The parameters P and Q denote, respectively, the characteristic timescale of the exponential expansion and the time period of 1 cycle, while

the parameter η satisfies the inequality

$$0 < \eta < 1. \quad (5)$$

Evidently for $P \gg Q$, the scale factor describes a universe with a long-term exponential expansion together with short-term oscillations in which $S(t)$ never reaches the value zero.

The details of this cosmology are found in various papers by Hoyle, Burbidge, & Narlikar (1993, 1994a, 1994b, 1995) as well as in their book (Hoyle et al. 2000). The long-term expansion in this model is accelerated, as in the original steady state theory. The repulsive force generated by the C -field is responsible for this feature. However, the C -field plays the crucial role of creating new matter *without violating any conservation law*. Although the scale factor describes a smoothed-out universe, in actuality the creation is in the form of mini-explosions triggered by the strong gravitational fields of highly collapsed (black hole-like) objects. It is claimed that the creation phenomenon occurs at various mass scales, ranging from the stellar to the supercluster scales.

Within the framework of the QSSC, the cosmic microwave background is explained as the thermalized relic starlight from previous cycles. The thermal temperature can indeed be worked out in terms of the known stellar activity, and its present value can be derived as ~ 2.7 K. How is thermalization achieved? The proposed mechanism is via metallic whiskers (Narlikar et al. 1997) that form a component of intergalactic dust. The formation and extinction properties of these whiskers have been discussed in the above paper. Typically the whiskers form from the metallic vapors ejected from supernovae where the metals are synthesized. Experiments have shown that metallic vapors, on cooling, condense into elongated whiskers of ~ 0.5 – 1 mm length and $\sim 10^{-6}$ cm cross-sectional radius (Donn & Sears 1963; Nabarro & Jackson 1958). It is estimated that a dust density of $\sim 10^{-34}$ g cm $^{-3}$ is adequate to achieve the above effect. This model can also explain the observed anisotropy of the cosmic microwave background, including the peak at $l \sim 200$ normally referred to as the Doppler peak in standard cosmology (Narlikar et al. 2002). Indeed, it is this type of dust that helps explain the magnitude-redshift relation for extragalactic Type Ia supernovae.

5. THE MAGNITUDE-REDSHIFT RELATION IN THE QSSC

From the above discussion, it is evident that three new aspects are introduced by the QSSC vis-à-vis standard cosmology when interpreting the m - z relation. There should be (1) a negative cosmological constant; (2) a repulsive force arising from a negative energy C -field; and (3) cosmic dust, which extinguishes radiation traveling over long distances.

Of these, (1) tends to decelerate the universe, while (2) tends to accelerate it. The effect of (3), so far as the Type Ia supernova m - z test is concerned, is to make the distant supernovae pro-

gressively dimmer. The effective magnitude-redshift relation is given by

$$m(z) = \mathcal{M} + 5 \log(H_0 d_L) + \Delta m(z), \quad (6)$$

where $\mathcal{M} \equiv M - 5 \log H_0 + 25$, M is the absolute luminosity of the source, and the luminosity distance d_L is measured in Mpc. Here we have used the notation and definitions of Perlmutter et al. (1999). The quantity \mathcal{M} has been referred to by these authors as a “Hubble constant free” absolute magnitude. The second term on the right-hand side of this equation is the standard luminosity distance (d_L) term, which can be computed for the QSSC. The product $H_0 d_L$ is independent of Hubble’s constant. The last term denotes the dimming produced by the intergalactic dust and may be explicitly written as

$$\Delta m(z) = \frac{\kappa \rho_{g0}}{H_0} \int_1^{1+z} \frac{y^2 dy}{(\Lambda_0 + \Omega_0 y^3 + \Omega_{c,0} y^4)^{1/2}}, \quad (7)$$

where $\Lambda_0 = \lambda/3H_0^2$, $\Omega_0 = 8\pi G\rho_0/3H_0^2$, and $\Omega_{c,0} = 8\pi G\rho_{c,0}/3H_0^2$ are the three parameters of this cosmology corresponding to the cosmological constant, matter density, and the C -field energy density. The present value of Hubble’s constant is $H_0 \equiv 100 h_0$ km s $^{-1}$ Mpc $^{-1}$. The density ρ_{g0} is the present intergalactic dust grain density in the form of metallic whiskers, and κ is the mass-absorption coefficient.

Chitre & Narlikar (1976) were the first to discuss the role of intergalactic dust in the m - z relation. At the time, the general belief about the intergalactic medium did not include dust as a significant component, and so the suggestion in the above paper that the presence of even a small dust component would enhance the apparent magnitudes was largely ignored. Recently, Aquirre (1999) considered the role of dust in the m - z relation, followed by Banerjee et al. (2000).

A comment is necessary on equation (7). In the paper by Banerjee et al. (2000), this formula had appeared with a typographical error, with the factor y^2 missing in the numerator. The factor was nevertheless included in the actual calculation. The calculation redone by Vishwakarma (2002) has slightly lowered the value of χ^2 per degree of freedom from ≈ 1.06 to ≈ 1.00 , thus marginally improving upon the goodness of fit claimed originally by Banerjee et al. (2000).

Here we use this formula to compute the m - z curve as predicted by the QSSC. As reported by Vishwakarma (2002), the calculation is done in two different ways. The first is to use the low-redshift measurements to determine the two free parameters in equation (6), viz., \mathcal{M} and $\kappa \rho_{g0} h_0^{-1}$, and then use these values to calculate χ^2 from the high-redshift measurements including SN 1997ff, the highest redshift supernova observed so far. Note that the model considered here is completely determined by $\Lambda_0 = -0.358$ and $\eta = 0.811$ as considered by Banerjee et al. (2000). The second method is to use SN 1997ff along with the earlier 60 supernovae, taking the high- and low-redshift points together, to get a best-fit curve and then judge

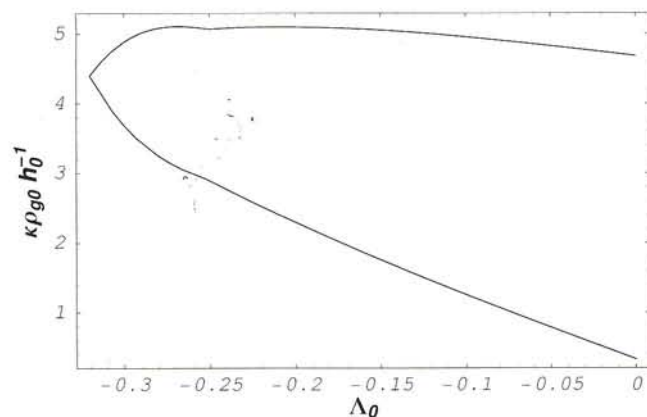


FIG. 1.—Tilted U-shaped contour shows a parameter space of interest in the QSSC. The points of the contour have χ^2 per degree of freedom in the range 1.16–1.26. The coupling constant κ and the present density of the whisker grains ρ_{g0} are measured, respectively, in units of $10^5 \text{ cm}^2 \text{ g}^{-1}$ and $10^{-34} \text{ g cm}^{-3}$. (Adapted from Fig. 6 of Vishwakarma 2002.)

the goodness of fit. If the fit is relatively good, one can argue for the robustness of the theoretical relation.

Method 1.—Using the earlier work of Banerjee et al. (2000), we carry out the calculation of magnitudes at specified redshifts using equation (6). Six of the 60 supernovae were dropped from the calculation as the deviations of the observed magnitudes from the expected ones are somewhat excessive. We will comment on this deviation within the framework of the QSSC later. For the present, we may mention that data on six similar supernovae had been omitted in obtaining the best-fit models of standard cosmology by Perlmutter et al. (1999), who had also considered 54 supernovae in their primary fit C.

With the constraint $\rho_{g0} > 0$, the sample of 16 low-redshift supernovae gives $\mathcal{M} \approx 24$ with a wide range $0 < \kappa\rho_{g0}h_0^{-1} < 10$ as the best-fitting solution, where χ^2 does not change significantly. For example, $\mathcal{M} = 24.03$ with $\kappa\rho_{g0}h_0^{-1} = 6.5$ gives a value χ^2 per degree of freedom of 0.8, which represents an excellent fit. To fix ideas, $\mathcal{M} = 24$ gives the corresponding absolute magnitude at peak luminosity of the supernova as $M = -19.49$ for $h_0 = 0.6$ and $M = -19.16$ for $h_0 = 0.7$. These values are in the right region for supernova peak magnitudes (Saha et al. 2001). When we add the new point SN 1997ff ($z = 1.755 \pm 0.05$, $m = 25.68 \pm 0.34$) to the older list of 38 high-redshift supernovae with $z \lesssim 0.8$, the new sample gives a χ^2 per degree of freedom of 1.49, which, although larger than before, is still acceptable as a reasonable fit. It would be worthwhile to mention that SN 1997ff is claimed to be on the brighter side because of the gravitational lensing effect (Moertsell, Gunnarsson, & Goobar 2000). If this is the case, then an increase in the measured apparent magnitude of this supernova by only as much as 0.5 improves the fit considerably by giving χ^2 per degree of freedom of 1.22. Likewise, a change in the redshift of this farthest supernova from 1.755 to 1.65

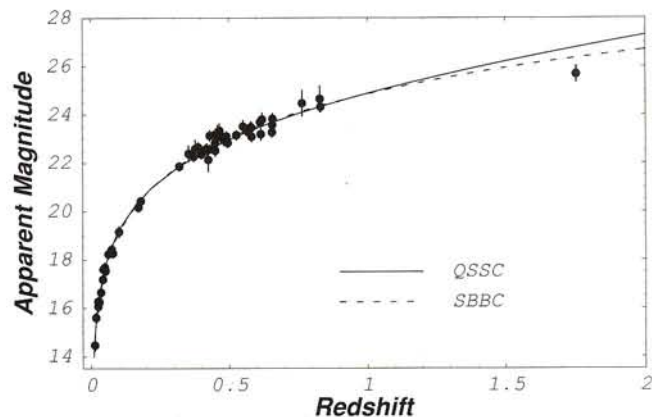


FIG. 2.—Hubble diagram for 55 supernovae is shown. The theoretical curves represent the best-fitting flat QSSC ($\Lambda_0 = -0.358$, $\eta = 0.811$) and flat SBBC ($\Omega_0 = 0.34$) models. The six omitted points are shown by open circles.

improves the χ^2 per degree of freedom to 1.33. We demonstrate this to underscore the need for caution in interpreting data in terms of models until (1) we have data on more supernovae available and (2) we are more certain about their magnitudes and redshifts.

Method 2.—Here we include SN 1997ff along with the 54 supernovae from earlier samples in obtaining the best-fit curve. Naturally, the fit of such a curve to the data will be better than that in the first case. Here we report an exercise carried out by one of us (Vishwakarma 2002) in which the models are fitted in the QSSC parameter space of $0 > \Lambda_0 \geq -0.32$ and ρ_{g0} in the range $(3-5) \times 10^{-34} \text{ g cm}^{-3}$. The region enclosed by the tilted U-shaped curve in Figure 1 has the QSSC models with parameters that yield values of χ^2 per degree of freedom in the range 1.16–1.26, that is, values indicating a satisfactory fit. Even the model chosen by Banerjee et al. (2000), viz., $\Lambda_0 = -0.358$, yields a reasonable fit with the χ^2 value per degree of freedom of 1.27. Moreover, if we assume that SN 1997ff is observed on the brighter side, as we have mentioned earlier, then an increase in its magnitude by 0.5 improves the χ^2 per degree of freedom to 1.12. Likewise, a change in its redshift from 1.755 to 1.65 reduces the χ^2 per degree of freedom to 1.19. In Figure 2, we have shown the fitting of this model ($\Lambda_0 = -0.358$, $\eta = 0.811$ with χ^2 per degree of freedom of 1.27) to the observed data points and compared it with the favored SBBC model ($\Omega_{\text{total}} = 1$). It should be noted that the Perlmutter et al. data, taken together with SN 1997ff, do not give the standard flat model as the best-fitting model; there, the best-fitting model is obtained with $\Omega_0 = 0.87$, $\Lambda_0 = 1.51$ (i.e., $\Omega_{\text{total}} = 2.38$) with a χ^2 value per degree of freedom of 1.09. However, if we concentrate our attention on the flat model ($\Omega_{\text{total}} \equiv \Omega_0 + \Lambda_0 = 1$), which is currently favored, the minimum χ^2 is obtained for the model $\Omega_0 = 0.34$ with a χ^2 value per degree of freedom of 1.17 (Vishwakarma 2002).

Referring back to our opening remarks in this paper, it is

interesting to compare the old steady state model with the classical Einstein–de Sitter model ($\Omega_0 = 1$, $\Lambda_0 = 0$) for the data of Perlmutter et al. (1999). The respective value of χ^2 per degree of freedom for the steady state model is 1.42, compared to the value 1.75 for the Einstein–de Sitter model. This relative performance may be contrasted with the situation prevailing three decades ago when the m - z data on galaxies were claimed to support a *decelerating* universe and were used to rule out the steady state model on the grounds that the model predicts an *accelerating* universe.

6. CONCLUDING REMARKS

As was originally stated by Banerjee et al. (2000), the adjustable parameter in the above exercise is ρ_{g0} , the density of intergalactic dust. By requiring the fit to be “best,” we determine the value of ρ_{g0} . Had the theory been on the wrong track, this value could have come out vastly different from the value needed to thermalize the microwave background. That it comes out in the correct range must be considered a point in favor of this approach.

It will take us too far from the focus of the present paper to elaborate on the details of the microwave background in the QSSC. In a separate paper (Narlikar et al. 2002) devoted to the spectrum and fluctuations of the microwave background radiation (MBR) in the QSSC, we have shown that the MBR anisotropy from rich clusters can be revealed by a peak in the power spectrum around $l = 200$, while smaller peaks are revealed in the $l \sim 600$ – 900 range from groups of galaxies and

perhaps weaker peaks at $l \gtrsim 3000$ from individual galaxies or compact groups.

In the QSSC, the whisker dust is produced and ejected from the vicinity of supernovae, as discussed by Narlikar et al. (1997). Thus, it is quite likely that dust near the parent supernova may show varying degrees of concentration. This can lead to relatively large fluctuations in the supernova magnitudes and may account for the large deviations for the six supernovae omitted from the study.

In principle, despite such fluctuations, it is possible to use the m - z test to distinguish between the standard λ -based approach and the above QSSC approach. As we go to higher redshifts, the effect of acceleration as implied by the cosmological constant will diminish, while that of the intergalactic dust will become greater. However, great care needs to be exercised before drawing any conclusions from such observations. As the example of the m - z test for galaxies showed in the 1970s, uncertain evolutionary effects tend to dominate at high redshifts. In addition, we have seen that even small changes in the measured values of m or z can greatly alter the conclusion. Finally, as we go to higher redshifts, gravitational lensing may selectively amplify the images and thus push the observed brightness to values higher than normal (Lewis & Iбата 2001). Hence, it is prudent to await the accumulation of more Type Ia supernovae in the high- z basket.

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