Cosmic Microwave Background Trispectrum and Primordial Magnetic Field Limits

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Primordial magnetic fields will generate non-Gaussian signals in the cosmic microwave background (CMB) as magnetic stresses and the temperature anisotropy they induce depend quadratically on the magnetic field. We compute a new measure of magnetic non-Gaussianity, the CMB trispectrum, on large angular scales, sourced via the Sachs-Wolfe effect. The trispectra induced by magnetic energy density and by magnetic scalar anisotropic stress are found to have typical magnitudes of approximately a few times 10^{-29} and 10^{-19}, respectively. Observational limits on CMB non-Gaussianity from WMAP data allow us to conservatively set upper limits of a nG, and plausibly sub-nG, on the present value of the primordial cosmic magnetic field. This represents the tightest limit so far on the strength of primordial magnetic fields, on Mpc scales, and is better than limits from the CMB bispectrum and all modes in the CMB power spectrum. Thus, the CMB trispectrum is a new and more sensitive probe of primordial magnetic fields on large scales.

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Magnetic fields are ubiquitous in the Universe from planets and stars to galaxies and galaxy clusters [1,2], yet the origin and evolution of large-scale magnetic fields remains a puzzle. A popular paradigm is that magnetic fields in collapsed structures could arise from dynamo amplification of seed magnetic fields [2]. The seed field could in turn be generated in astrophysical batteries [3] or due to processes in the early Universe [4,5]. Indeed recent γ-ray observations claim to find a lower limit to an all-pervasive intergalactic magnetic field that fills most of the cosmic volume [6], which would perhaps favor a primordial origin. A primordial magnetic field can be generated at inflation [4] or arise out of other phase transitions in the early Universe [5]. As yet there is no compelling mechanism which produces strong coherent primordial fields. Equally, the dynamo paradigm is not without its own challenges in producing sufficiently coherent fields and sufficiently rapidly [2]. Therefore, it is useful to keep open the possibility that primordial magnetic fields originating in the early Universe play a crucial role in explaining the observed cosmic magnetism.

In this context it is important to investigate every observable signature of the putative primordial magnetic fields. Constraints on large-scale primordial magnetic fields have already been derived using the cosmic microwave background (CMB) power spectrum [7,8] and Faraday rotation [9]. However, the effects of a magnetic field on the CMB are relatively more prominent in its non-Gaussian correlations. This is because magnetic fields induce non-Gaussian signals at lowest order as the magnetic energy density and stress are quadratic in the field. On the other hand, the standard inflationary perturbations, dominated by their linear component, can source non-Gaussian correlations only with higher-order perturbations and thus necessarily produce a small amplitude of CMB non-Gaussianity (cf. [10,11]). Primordial magnetic fields can induce appreciable CMB non-Gaussianity when considering the bispectrum [12,13]. Our previous calculation of the magnetic CMB bispectrum sourced by scalar anisotropic stress led to a ~2 nG upper limit on the primordial magnetic field's amplitude on Mpc scales [14]. However, higher-order measures of non-Gaussianity remain unexplored and, as we show here, could be very useful to set further constraints on primordial magnetic fields.

In this Letter, we present the first calculation of the contribution to the CMB trispectrum induced by a primordial magnetic field. In particular, we consider the magnetically induced Sachs-Wolfe effect sourced by a stochastic primordial magnetic field. We show that the trispectrum does significantly better than the bispectrum in constraining the large-scale magnetic field via CMB non-Gaussianity, considering both magnetic energy density and magnetic scalar anisotropic stress as sources. This reveals a new and effective probe to investigate primordial magnetic fields on large scales.

We consider a Gaussian random stochastic magnetic field $\mathbf{B}$ characterized and completely specified by its power spectrum $M(k)$. We further assume the magnetic field to be nonhelical. On galactic and larger scales, any velocity induced by Lorentz forces is generally too small to appreciably distort the initial magnetic field [15]. Hence, the magnetic field simply redshifts away as $\mathbf{B}(\mathbf{x}, t) = \boldsymbol{b}_0(\mathbf{x})/a^2$, where $\boldsymbol{b}_0$ is the magnetic field at the present epoch (i.e., at $z = 0$ or $a = 1$). We define $\boldsymbol{b}(\boldsymbol{k})$ as the Fourier transform of the magnetic field $\boldsymbol{b}_0(\boldsymbol{x})$. The magnetic power spectrum is defined by the relation

$\langle b_i(\boldsymbol{k}) b_j^*(\boldsymbol{q})\rangle = (2\pi)^3 \delta(\boldsymbol{k}-\boldsymbol{q}) P_{ij}(\boldsymbol{k}) M(k)$, where $P_{ij}(\boldsymbol{k}) = (\delta_{ij} - k_i k_j/k^2)$ is the projection operator ensuring $\nabla \cdot \boldsymbol{b}_0 = 0$. This leads to $\langle \boldsymbol{b}_0^2 \rangle = 2\int (dk/k)\Delta_b^2(k)$, where $\Delta_b^2(k) = k^3 M(k)/(2\pi^2)$ is the power per logarithmic interval in k space present in the stochastic magnetic field. We assume a power-law magnetic power spectrum, $M(k) = Ak^n$ that has a cutoff at $k = k_c$, where k_c is the Alfvén-wave damping length scale [15]. We fix A by setting the variance of the magnetic field to be B_0, smoothed using a sharp k-space filter, over a "galactic" scale $k_G = 1h$ Mpc^{-1}. This gives (for $n \gtrsim -3$ and for $k < k_c$)

$$\Delta_b^2(k) = \frac{k^3 M(k)}{2\pi^2} = \frac{B_0^2}{2}(n+3)\left(\frac{k}{k_G}\right)^{3+n}. \tag{1}$$

The spectral index n is restricted to values close to and above -3, i.e., an inflation-generated field, as causal generation mechanisms can only produce much bluer spectra [16]. Further, blue spectral indices are strongly disfavored by many observations like the CMB power spectra [7]. We choose to split the contributions to the CMB trispectrum into that sourced by magnetic energy density Ω_B and by scalar anisotropic stress Π_B rather than the compensated and passive magnetic perturbation modes of Ref. [17]. The subdominant compensated mode is a linear combination of Ω_B and Π_B, whereas the passive mode is the Π_B perturbation considered here.

The Sachs-Wolfe type of contribution to the CMB temperature anisotropy induced by the energy density of magnetic fields [18–20] can be expressed as

$$\frac{\Delta T}{T}(\boldsymbol{n}) = \mathcal{R}\Omega_B(\boldsymbol{x}_0 - \boldsymbol{n}D^*). \tag{2}$$

Here, $\Omega_B(\boldsymbol{x}) = \mathbf{B}^2(\boldsymbol{x},t)/(8\pi\rho_\gamma(t)) = \boldsymbol{b}_0^2(\boldsymbol{x})/(8\pi\rho_0)$, where $\rho_\gamma(t)$ and ρ_0 are, respectively, the CMB energy densities at a time t and at the present epoch. In the same manner as the usual Sachs-Wolfe effect, the $\Delta T/T$ given above is for large angular scales. For numerical estimates we use the most recent estimate of Bonvin and Caprini (Eq. 6.12 of [20]) expressed according to our definitions as $\mathcal{R} = -0.2R_\gamma/3 \sim -0.04$, where $R_\gamma \sim 0.6$ is the fractional contribution of radiation energy density towards the total energy density of the relativistic component. The unit vector $\mathbf{n}$ is along the direction of observation from the observer at position $\boldsymbol{x}_0$ and D^* is the (comoving angular diameter) distance to the surface of last scattering. We have assumed instantaneous recombination which is a good approximation for large angular scales.

The temperature fluctuations of the CMB can be expanded in terms of spherical harmonics to give $\Delta T(\boldsymbol{n})/T = \sum_{lm} a_{lm} Y_{lm}(\boldsymbol{n})$, where

$$a_{lm} = \frac{4\pi}{i^l}\int \frac{d^3k}{(2\pi)^3}\mathcal{R}\Omega_B(\boldsymbol{k}) j_l(kD^*) Y_{lm}^*(\hat{\boldsymbol{k}}). \tag{3}$$

Here, $\Omega_B(\boldsymbol{k})$ is the Fourier transform of $\Omega_B(\boldsymbol{x})$. Since $\Omega_B(\boldsymbol{x})$ is quadratic in $\boldsymbol{b}_0(\boldsymbol{x})$, we have a convolution $\Omega_B(\boldsymbol{k}) = [1/(2\pi)^3]\int d^3s\, b_i(\boldsymbol{k}+\boldsymbol{s}) b_i^*(\boldsymbol{s})/(8\pi\rho_0)$. The trispectrum $T^{m_1m_2m_3m_4}_{l_1l_2l_3l_4}$, or the four-point correlation function of the CMB temperature anisotropy in harmonic space, in terms of the a_{lm}'s, is $T^{m_1m_2m_3m_4}_{l_1l_2l_3l_4} = \langle a_{l_1m_1} a_{l_2m_2} a_{l_3m_3} a_{l_4m_4}\rangle$. From Eq. (3) we can express $T^{m_1m_2m_3m_4}_{l_1l_2l_3l_4}$ as

$$T^{m_1m_2m_3m_4}_{l_1l_2l_3l_4} = \left(\frac{\mathcal{R}}{2\pi^2}\right)^4 \int \left[\prod_{i=1}^{4} \frac{d^3k_i}{i^{l_i}} j_{l_i}(k_i D^*) Y^*_{l_i m_i}(\hat{\boldsymbol{k}}_i)\right]\zeta_{1234}, \tag{4}$$

with $\zeta_{1234} = \langle \Omega_B(\boldsymbol{k}_1)\Omega_B(\boldsymbol{k}_2)\Omega_B(\boldsymbol{k}_3)\Omega_B(\boldsymbol{k}_4)\rangle$. The four-point correlation function of $\Omega_B(\boldsymbol{k})$ involves an eight-point correlation function of the fields. Using Wick's theorem, for Gaussian magnetic fields, we can express the magnetic eight-point correlation as a sum of 105 terms involving the magnetic two-point correlation. Neglecting the 45 terms proportional to $\delta(\boldsymbol{k})$ that vanish and the 12 terms proportional to $\delta(\boldsymbol{k}_i + \boldsymbol{k}_j)$ that represent the unconnected part of the four-point correlation, we are left with 48 terms. A long calculation involving the relevant projection operators gives $\zeta_{1234} = \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4)\psi_{1234}$, where ψ_{1234} is a mode-coupling integral over a variable $\boldsymbol{s}$ and also involves angular terms. The full expression for ψ_{1234} will be presented in our detailed paper [21]. For simplicity we evaluate the mode-coupling integral ψ_{1234} in two cases: (i) considering only $\boldsymbol{s}$-independent angular terms for all equal-sided configurations and (ii) taking all angular terms for the collinear configuration. Considering $\boldsymbol{s}$-independent terms only for a general configuration, we find $\psi_{1234} = -8/(8\pi\rho_0)^4 \mathcal{I}$, where

$$\begin{aligned}\mathcal{I} = &\int d^3s M(s) M(|\boldsymbol{k}_1 + \boldsymbol{s}|)\\ &\times \{M(|\boldsymbol{k}_1 + \boldsymbol{k}_3 + \boldsymbol{s}|)[M(|\boldsymbol{k}_2 - \boldsymbol{s}|) + M(|\boldsymbol{k}_4 - \boldsymbol{s}|)]\\ &+ M(|\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{s}|)[M(|\boldsymbol{k}_3 - \boldsymbol{s}|) + M(|\boldsymbol{k}_4 - \boldsymbol{s}|)]\\ &+ M(|\boldsymbol{k}_1 + \boldsymbol{k}_4 + \boldsymbol{s}|)[M(|\boldsymbol{k}_2 - \boldsymbol{s}|) + M(|\boldsymbol{k}_3 - \boldsymbol{s}|)]\}\\ = &\ \mathcal{I}_{(1)} + \mathcal{I}_{(2)} + \mathcal{I}_{(3)} + \mathcal{I}_{(4)} + \mathcal{I}_{(5)} + \mathcal{I}_{(6)}.\end{aligned} \tag{5}$$

We perform the mode-coupling integral using the technique and approximations discussed in [14,22], while adopting the mean (zero) value of $\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_3$, to find

$$\mathcal{I}_{(1)} \simeq 4\pi A^4 k_1^{2n+3} k_2^n k_3^n \left[\frac{2^{n/2}}{n+3} - \frac{1}{4n+3}\right]. \tag{6}$$

The value of each of the $\mathcal{I}_{(j)}$ integrals for $j = 1$–6 is the same when all the $|\boldsymbol{k}_i| \simeq k$. We perform the $\boldsymbol{s}$-independent [case (i)] trispectrum evaluation for such equal-sided quadrilateral configurations. Hence, $\mathcal{I} = \sum_{j=(1)}^{(6)} \mathcal{I}_j = 6\mathcal{I}_{(1)}$, and we obtain

$$\zeta_{1234} \simeq \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4)\frac{-8(24\pi)A^4 k_1^{2n+3} k_2^n k_3^n}{(8\pi\rho_0)^4} \times \left[\frac{(2^{n/2})(4n+3) - (n+3)}{(4n+3)(n+3)}\right]. \tag{7}$$

Inserting this into Eq. (4) for the trispectrum and following the approach of [23], we decompose our delta function as $\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4)=\int d^3K\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{K})\delta(\boldsymbol{k}_3+\boldsymbol{k}_4-\boldsymbol{K})$. Using the integral form of the delta functions and the spherical wave expansion we perform the integrations over the angular parts of $(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4,\boldsymbol{K})$, with algebra similar to [12,14,24], to give

$$T^{m_1m_2m_3m_4}_{l_1l_2l_3l_4}\simeq\left[\frac{(-768)(A\mathcal{R})^4}{\pi^7(8\pi\rho_0)^4}\right]\left\{\frac{(2^{n/2})(4n+3)-(n+3)}{(4n+3)(n+3)}\right\}\int\left[\prod_{i=1}^{4}dk_ik_i^2j_{l_i}(k_iD^*)j_{l_i}(k_i\bar{r}_i)\right]k_1^{2n+3}(k_2k_3)^n\sum_{LM}(-1)^{L-M}$$
$$\times\int dKK^2j_L(Kr_1)j_L(-Kr_2)\int\left[\prod_{i=1}^{2}d^3r_iY_{l_{2i-1}m_{2i-1}}(\hat{\boldsymbol{r}}_i)Y_{l_{2i}m_{2i}}(\hat{\boldsymbol{r}}_i)Y_{L(-1)^{i+1}M}(\hat{\boldsymbol{r}}_i)\right], \quad (8)$$

with $\bar{r}_i$ equal to r_1 for $i=1,2$ and r_2 for $i=3,4$. The approximations involved (with respect to angular terms) in the $\hat{\boldsymbol{k}}_i$ angular integrals can be made more precise by going to the flat-sky limit (elaborated in our detailed paper [21]). Here the K integral gives $\delta(r_1-r_2)(\pi/2r_1^2)$ via the spherical Bessel function closure relation. This delta function enables us to perform the r_2 integral trivially, then r_1 replaces r_2 in the arguments of j_{l_3} and j_{l_4}. The angular $\hat{\boldsymbol{r}}_1$ and $\hat{\boldsymbol{r}}_2$ integrals may be expressed as [e.g. Eq. 5.9.1 (5) of [25]]

$$\int d\Omega_{\hat{\boldsymbol{r}}_1}Y_{l_1m_1}(\hat{\boldsymbol{r}}_1)Y_{l_2m_2}(\hat{\boldsymbol{r}}_1)Y_{LM}(\hat{\boldsymbol{r}}_1)=\sqrt{\frac{(2l_1+1)(2l_2+1)(2L+1)}{4\pi}}\begin{pmatrix}l_1 & l_2 & L\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}l_1 & l_2 & L\\ m_1 & m_2 & M\end{pmatrix}$$
$$\equiv h_{l_1Ll_2}\begin{pmatrix}l_1 & l_2 & L\\ m_1 & m_2 & M\end{pmatrix}, \quad (9)$$

where we have defined $h_{l_1Ll_2}$ above, along the same lines as [23]. We use the relation $(A/8\pi\rho_0)^4=(2/3)^4(\pi/k_G)^8[(n+3)/k_G^{n+1}]^4V_A^8$, where the Alfvén velocity V_A, in the radiation dominated era, is defined as $V_A=B_0/(16\pi\rho_0/3)^{1/2}\approx3.8\times10^{-4}B_{-9}$ [15], with $B_{-9}\equiv(B_0/10^{-9}\text{ G})$. From the definition of the rotationally invariant angle-averaged trispectrum [26]

$$T^{m_1m_2m_3m_4}_{l_1l_2l_3l_4}=\sum_{LM}(-1)^{-M}\begin{pmatrix}l_1 & l_2 & L\\ m_1 & m_2 & -M\end{pmatrix}\times\begin{pmatrix}l_3 & l_4 & L\\ m_3 & m_4 & M\end{pmatrix}T^{l_1l_2}_{l_3l_4}(L), \quad (10)$$

we separate out the reduced trispectrum $T^{l_1l_2}_{l_3l_4}(L)$ (called the angular averaged trispectrum in [26]) from the full trispectrum. We again use the spherical Bessel function closure relation to perform the k_4 integral that yields $\delta(r_1-D^*)\times(\pi/2r_1^2)$. This facilitates the r_1 integral that results in $r_1\rightarrow D^*$ in the arguments of j_{l_1}, j_{l_2}, and j_{l_3}. The k_1, k_2, and k_3 integrals containing a product of a power law and j_l^2 can be evaluated in terms of Gamma functions (e.g., Eq. 6.574.2 of [27]). For a scale-invariant magnetic index $n\rightarrow-3$, we get

$$[T^{l_1l_2}_{l_3l_4}(L)]_\Omega\simeq-5.8\times10^{-29}\left(\frac{n+3}{0.2}\right)^3\left(\frac{B_{-9}}{3}\right)^8\times\frac{h_{l_1Ll_2}h_{l_3Ll_4}}{l_1(l_1+1)l_2(l_2+1)l_3(l_3+1)}. \quad (11)$$

This gives us the amplitude of the magnetic CMB trispectrum sourced by the energy density Ω_B of a primordial magnetic field. A factor of $1/(D^*k_G)^{4(n+3)}$ also appears which approaches unity for the case $n\rightarrow-3$ of a scale-invariant magnetic field index. We evaluate the magnetic trispectrum for a near scale-invariant index $n=-2.8$, for which this factor is $\sim1/1500$. It turns out that this factor is almost entirely cancelled by the increase in value of the k integrals when evaluated for $n=-2.8$ rather than $n=-3$ [21].

We now compare our magnetic trispectrum with the Sachs-Wolfe contribution to the standard CMB trispectrum sourced by nonlinear terms in the inflationary perturbations [23,28]. More specifically, in the Sachs-Wolfe limit, the dominant term of Eq. (64) of Ref. [29] becomes

$$T^{l_1l_2}_{l_3l_4}(L)\approx25\tau_{NL}C^{SW}_{l_2}C^{SW}_{l_4}C^{SW}_{L}h_{l_1Ll_2}h_{l_3Ll_4}$$
$$\approx5.4\times10^{-27}\tau_{NL}\frac{h_{l_1Ll_2}h_{l_3Ll_4}}{l_1(l_1+1)l_2(l_2+1)l_3(l_3+1)}q. \quad (12)$$

Here τ_{NL} and f_{NL} (below) are standard non-Gaussianity parameters and we adopt the standard estimate for the Sachs-Wolfe contribution C_l^{SW} [23]. The factor q which is equal to $[l_1(l_1+1)l_3(l_3+1)]/[l_4(l_4+1)L(L+1)]$ is of order unity for many configurations. Equation (12) is of the same form as Eq. (11) for the magnetic field-induced trispectrum. We use the negative-sided limit on τ_{NL} derived from searching for the CMB trispectrum signal in the WMAP5 data [29], $\tau_{NL}>-6000$. Magnetic field limits are obtained by taking the one-eighth power of the appropriate ratio of trispectra, which gives $B_0\lesssim16$ nG, at a scale of $k_G=1h\text{ Mpc}^{-1}$ for a magnetic spectral index of $n=-2.8$. This limit is approximately 2 times stronger than the $B_0\lesssim30$ nG upper limit for the magnetic energy density bispectrum [12] (taking into account the recent estimate of $\mathcal{R}$ [20]), for the same scale and magnetic index.

We now calculate the trispectrum for the collinear configuration [case (ii)]. The full mode-coupling integral ψ_{1234} [21] is evaluated over all angular terms for the equal-sided collinear configuration $\boldsymbol{k}_1 \simeq \boldsymbol{k}_2 \simeq -\boldsymbol{k}_3 \simeq -\boldsymbol{k}_4$. The four-point correlation of magnetic energy density for the collinear configuration is found to be

$$\zeta_{1234} \simeq \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4) \frac{8(4\pi)A^4 k_1^{2n+3} k_2^n k_3^n}{(8\pi\rho_0)^4} \times \left[\frac{\frac{8}{3}(2^{n/2})(4n+3) - (12)(n+3)}{(4n+3)(n+3)}\right]. \tag{13}$$

Using $n = -2.8$, we compare the collinear configuration four-point correlation ζ, including all angular terms, to ζ for case (i) [Eq. (7)] that included only $\boldsymbol{s}$-independent terms. The collinear ζ is similar in magnitude but of positive sign and one then expects a trispectrum also of similar magnitude to case (i).

In addition to magnetic energy density, the scalar anisotropic stress associated with a primordial magnetic field will also act as a separate source for CMB fluctuations—dominantly in the passive mode [17]. As we saw in our previous work [14], the magnetic scalar anisotropic stress generates $\sim 10^6$ times larger contribution to the CMB bispectrum compared to magnetic energy density. With this motivation and using the magnetic trispectrum technique, developed above for energy density, we carry out a longer calculation for the trispectrum. The temperature anisotropy, sourced via the magnetic Sachs-Wolfe effect by magnetic scalar anisotropic stress Π_B [defined in Eq. (6) of [14], see also [17,20]], is

$$\frac{\Delta T}{T}(\boldsymbol{n}) = \mathcal{R}_p \Pi_B(\boldsymbol{x}_0 - \boldsymbol{n}D^*), \tag{14}$$

where $\mathcal{R}_p = [-R_\gamma/15]\ln(T_B/T_\nu)$ and T_B and T_ν are the temperatures at the epochs of magnetic field generation and of neutrino decoupling, respectively.

For the magnetic scalar anisotropic stress trispectrum, $\mathcal{R}$ in Eq. (4) gets replaced by $\mathcal{R}_p$ and ζ_{1234} becomes $[\zeta_{1234}]_\Pi = \langle \Pi_B(\boldsymbol{k}_1)\Pi_B(\boldsymbol{k}_2)\Pi_B(\boldsymbol{k}_3)\Pi_B(\boldsymbol{k}_4)\rangle$. The full technical details of the calculation of the magnetic scalar anisotropic stress trispectrum will be presented separately [21]. We give below the results considering only the $\boldsymbol{s}$-independent angular mode-coupling terms for equal-sided configurations. In this case

$$[\zeta_{1234}]_\Pi \simeq \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4) 3^4 \xi \frac{8(24\pi)A^4 k_1^{2n+3} k_2^n k_3^n}{(8\pi\rho_0)^4} \times \left[\frac{(2^{n/2})(4n+3) - (n+3)}{(4n+3)(n+3)}\right]. \tag{15}$$

Here, ξ is a configuration-dependent number that is the sum of all $\boldsymbol{s}$-independent angular terms. This sum involves terms like $\theta_{ab} = \hat{\boldsymbol{k}}_a \cdot \hat{\boldsymbol{k}}_b$ that are constant for a given $(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$ configuration. Values for ξ range between 2 and 14 for equal-sided trispectrum configurations: collinear, square, rhombus, and tetrahedron. We adopt a typical value $\xi \simeq 10$. This leads to a reduced trispectrum

$$[T^{l_1 l_2}_{l_3 l_4}(L)]_\Pi \simeq \left(3\frac{\mathcal{R}_p}{\mathcal{R}}\right)^4 \xi [-T^{l_1 l_2}_{l_3 l_4}(L)]_\Omega \simeq 1.1 \times 10^{-19} \left(\frac{\xi}{10}\right)\left(\frac{n+3}{0.2}\right)^3 \left(\frac{B_{-9}}{3}\right)^8 \times \frac{h_{l_1 L l_2} h_{l_3 L l_4}}{l_1(l_1+1) l_2(l_2+1) l_3(l_3+1)}. \tag{16}$$

We have used $T_B \simeq 10^{14}$ GeV (corresponding to the reheating temperature) and $T_\nu \simeq 10^{-3}$ GeV. We see that the amplitude of the trispectrum sourced by Π_B for equal-sided quadrilateral configurations is approximately 10^{10} times larger than that sourced by Ω_B. Comparison with the trispectrum from inflationary perturbations [Eq. (12)] gives a magnetic field constraint of

$$B_0 \lesssim 1.3 \text{ nG}, \tag{17}$$

using the positive-sided limit $\tau_{NL} < 33\,000$ from WMAP5 data [29]. This is approximately twice as strong as the 2.4 nG B_0 limit obtained from the Π_B bispectrum [14] and does not assume any particular model of inflation or any relation between τ_{NL} and f_{NL}. However, for those theories of inflation, which lead to $\tau_{NL} = (6/5 f_{NL})^2$ [10,30], we could perhaps use the relatively tighter limits for f_{NL}. To be conservative we take the two-sigma limits $-10 < f_{NL}^{\text{local}} < 74$ on the best constrained local f_{NL}, obtained from searching for the CMB bispectrum signal in WMAP7 data [31]. This gives primordial magnetic field limits of

$$B_0 \lesssim 0.7 \text{ nG} \quad \text{and} \quad B_0 \lesssim 1.1 \text{ nG}, \tag{18}$$

respectively, for the negative and positive f_{NL}^{local} limits. If one uses the two-sigma limits for f_{NL}^{equil}, then the 0.7 nG limit becomes 0.6 nG and for f_{NL}^{orthog} it becomes 1.5 nG. However, the uncertainties $\sigma_{f_{NL}}$ for equilateral and orthogonal configurations are 7 and 5 times larger compared to the local configuration [31]. Staying with the best determined f_{NL}^{local} limits thus results in sub-nG upper limits on B_0. The expected $\Delta f_{NL} < 5$ [11] from Planck data will imply even tighter sub-nG magnetic field upper limits from the scalar anisotropic stress trispectrum. Future consideration of magnetic vector and tensor modes in the trispectrum is likely to give additional constraints on primordial magnetic fields.

In summary, we have calculated for the first time the CMB trispectrum sourced by primordial magnetic fields. The magnetic energy density trispectrum allows us to place stronger limits on the primordial magnetic field compared to a similar calculation with the magnetic energy density bispectrum [12,13]. Further, the trispectrum due to magnetic scalar anisotropic stress leads to the tightest constraint on large-scale magnetic fields of ~ 0.7 nG, approximately 3 times as strong as the corresponding bispectrum limit (~ 2.4 nG) [14]. The trispectrum's sensitivity is illustrated by the magnetic to inflationary trispectrum ratio, which is $\sim 10^3$ compared to ~ 1 for the bispectrum (taking $f_{NL} \sim 100$ and $B_0 \sim 3$ nG).

The relative contribution of different configurations to the trispectrum is different for magnetic compared to inflationary trispectra and will be useful to distinguish between them. We also note that the magnetic field limit at Mpc scales derived from only the scalar magnetic CMB trispectrum is already better than the limit ($\sim$ 2–6 nG) [7] from the combined scalar, vector, and tensor modes in the magnetic CMB power spectrum. Therefore, the trispectrum turns out to be a new and more powerful probe of large-scale primordial magnetic fields.

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