WHITE HOLES AND HIGH ENERGY ASTROPHYSICS

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Abstract. The role of exploding objects in high-energy astrophysics is discussed. Quantitative results are obtained for the simple case of the canonical white hole which explodes from a singularity and consists of pressure-free matter in the comoving frame of reference. General relativity is used for calculating the dynamical results. Applications to X-ray background, transient X-ray sources, γ-ray bursts and high energy cosmic rays are considered. White holes of more general types are discussed in a qualitative manner.

1. Introduction

The subject of high energy astrophysics deals with the problems of generation and behaviour of high energy particles and quanta in astronomical settings. Attention was first focused on these problems in the context of cosmic rays and the extragalactic radio sources. The cosmic rays contain evidence for particles with energies as high as 10^{21} eV. While the quanta of radiation from the radio sources are of low energy, their generation through the synchrotron process requires the existence of fast moving charged particles. Recent developments in X-ray and γ-ray astronomy have revealed the existence of sources of hard electromagnetic radiation in the Universe.

In his attempts to understand these high energy sources the astrophysicist is usually faced with two difficulties. The first relates to the enormous reservoir of energy needed to account for the total output from a source. For strong extragalactic radio sources the total energy involved is estimated at $\sim 10^{62}$ ergs or more. What is the primary source of this energy? Considerations of this question inevitably lead to the second difficulty. How are the fast particles generated which by the synchrotron process lead to the radio emission? In the case of X-ray sources or γ-ray bursts, even if the radiation is not of primary origin, it must have arisen at the expense of particle energies through either the synchrotron emission or the inverse Compton scattering or some other process. So it becomes necessary at some stage to face the question of the primary origin of high energy particles and quanta.

In this connection it is significant that there is already some evidence for large-scale explosions taking place in the Universe. Optical details of some radio sources show rapid outward motion of matter from a central region. Nuclei of Seyfert galaxies also show explosions in many cases. Indeed violent activity accompanied by ejection of matter is not at all uncommon in extragalactic astronomy, and it is tempting to suggest that such seats of activity play a crucial role in the problem of primary origin raised above. What is the nature of these explosions?

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Many astronomers accept with equanimity the idea that all the matter in the Universe originated in one single explosion or the big-bang. Some theorists (Neeman, 1965; Novikov, 1965; Novikov and Thorne, 1972) have argued that over and above the big-bang there are 'delayed bangs' or 'lagging cores' in which matter explodes into existence over a limited region of space. Others have suggested that the explosions are due to the bounce of a collapsing massive object. Conventional general relativity together with conventional physics of matter precludes such bounces. However, unusual equations of state (cf. Novikov and Zel'dovich, 1973) or the introduction of a negative energy field as required by Hoyle and Narlikar (1964) can convert implosions to explosions. An even more unusual idea involves the gravitational collapse of a massive object to a singular situation and its re-emergence as an exploding object in another topologically connected universe (Hjellming, 1971).

In this paper we will study the observable properties of exploding objects. In contrast to the imploding objects which end up as black holes in conventional general relativity, we will refer to the exploding objects as white holes. We propose to show that a white hole has the natural tendency to uplift the energy of a particle or a quantum emitted outwards from its surface, i.e. to cause an energy blue shift. Under certain simplifying assumptions it becomes possible to study the spectral features, time variation, energy output etc. of the white hole. We shall investigate further how these properties may be looked for in phenomena of high energy astrophysics and how they may be used to understand some of the problems mentioned before.

2. The Canonical White Hole

To begin with we will consider the white hole as a spherical object of uniform density and zero pressure in the comoving frame of the outward moving particles. In the simplest case the object emerges from a singular state and subsequently obeys Einstein's equations of gravitation, which in their usual notation are given by

$$R_{ik} - \tfrac{1}{2}g_{ik}R = -\frac{8\pi G}{c^4} T_{ik}. \tag{1}$$

We will consider this case in some detail. It resembles the big-bang Friedmann model of the Universe. Like the big-bang model it does not explain how the matter was created at the singular event. Later we shall consider a brief modification of this picture where the creation event is also brought within the scope of the discussion.

In the comoving frame of reference the interior of the white hole has the line element

$$ds^2 = c^2\,dt^2 - S^2(t)\left[\frac{dr^2}{1-\alpha r^2} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\right], \quad r \leqslant r_b, \tag{2}$$

where (r, θ, ϕ) are the constant coordinates of a comoving particle and t the proper time of a comoving observer. r_b is the r-coordinate of a particle on the boundary of the object. $S(t)$ is a function of t which represents the linear scale of the exploding object. It satisfies the differential equation

$$\dot{S}^2 = \alpha c^2\left(\frac{1 - S}{S}\right). \tag{3}$$

The parameter α is related to the mass M of the white hole and its radial boundary coordinate by

$$\alpha = \frac{2GM}{c^2 r_b^3}. \tag{4}$$

The proper surface area of the object is

$$A(t) = 4\pi r_b^3 S^2(t). \tag{5}$$

We will measure t from the instant of explosion ($S=0$), so that from (3), the maximum size ($S=1$) is attained by the object at time

$$t_0 = \frac{\pi}{2c\sqrt{\alpha}}. \tag{6}$$

We shall be concerned throughout with the time interval $0 \leqslant t \leqslant t_0$.

The space exterior to the white hole is described by the Schwarzschild line element

$$\mathrm{d}s^2 = \left(c^2 - \frac{2GM}{R}\right)\mathrm{d}T^2 - \frac{\mathrm{d}R^2}{1 - (2GM/c^2R)} - R^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2). \tag{7}$$

A typical exterior Schwarzschild observer has constant R, θ, ϕ and we shall consider such an observer with $R = R_1 \gg 2GM/c^2$. At the boundary of the object the line elements (2) and (7) which use different coordinate systems must be matched in the appropriate way. We shall return to this problem in the next section. It is clear, however, from a comparison of the angular parts of (2) and (7) that on the boundary $R = R_b = r_b S(t)$. It is convenient to define a new variable η by

$$S = \sin^2\eta, \quad 0 \leqslant \eta \leqslant \pi/2, \tag{8}$$

where t is given in terms of η by the relation

$$t = \frac{2t_0}{\pi}(\eta - \sin\eta\cos\eta). \tag{9}$$

At $t = t_s$, $\eta = \eta_s$ the white hole bursts out of the Schwarzschild barrier, at which state

$$S(t_s) \equiv \sin^2\eta_s = \alpha r_b^2. \tag{10}$$

In the very early stages after the explosion, when $s \ll 1$, we can use the approximations

$$S \cong \eta^2, \qquad \eta = \left(\frac{3\pi t}{4t_0}\right)^{1/3}. \tag{11}$$

Thus $S \propto t^{2/3}$, as in the Einstein-de Sitter cosmology. We shall use this approximation when discussing very high-energy radiation, as in γ-ray bursts.

3. Outward Light Propagation

We will now consider light transmission from the surface of the white hole in the radial direction. Specifically, consider the following situation. A light ray sent out at the comoving time t is received by the distant Schwarzschild observer at R_1 at time T_1. The next light ray is emitted at comoving time $t+\Delta t$ and is received by the same observer at $T_1+\Delta T_1$. How is Δt related to ΔT_1? This problem was solved numerically by Faulkner *et al.* (1964) and an analytical approach was given later by Nariai and Nomita (1965). We shall adopt a different analytical approach which will be useful for subsequent work.

First consider a transformation of the interior line element to a Schwarzschild-like form. It can be shown (cf. Hoyle and Narlikar, 1964) that the following transformation

$$R = rS(t), \qquad T = \Phi\left[\int_{t_b}^{t} \frac{c^2\,d\tau}{S(t)\,dS/d\tau} - \int_{r}^{r_b} \frac{x\,dx}{1-\alpha x^2}\right], \tag{12}$$

where t_b is an arbitrary constant and Φ an arbitrary function, reduces the line element (2) to the form

$$ds^2 = e^{\nu}\,dT^2 - e^{\lambda}\,dR^2 - R^2(d\theta^2 - \sin^2\theta\,d\phi^2), \tag{13}$$

with

$$e^{\nu} = \frac{S^2\dot{S}^2(1-\alpha r^2)}{c^4\Phi_1^2(1-\alpha r^2 - r^2\dot{S}^2/c^2)}, \qquad e^{\lambda} = \frac{1}{(1-\alpha r^2 - r^2\dot{S}^2/c^2)}. \tag{14}$$

In (14) Φ_1 denotes the derivative of Φ with respect to its argument. At $r=r_b$, we get from (3), (4) and (14),

$$e^{-\lambda} = 1 - \frac{\alpha r_b^3}{R} = 1 - \frac{2GM}{c^2R}. \tag{15}$$

Thus e^{λ} is continuous at $r=r_b$. The continuity of e^{ν} at $r=r_b$ requires that

$$\frac{S^2\dot{S}^2(1-\alpha r_b^2)}{c^4\Phi_1^2(1-\alpha r_b^2 - r_b^2\dot{S}^2/c^2)} = 1 - \frac{2GM}{c^2R}. \tag{16}$$

However, at $r=r_b$, Equation (12) yields

$$\left[\frac{\partial T}{\partial t}\right]_{r_b} = \Phi_1\frac{c^2}{S\dot{S}}. \tag{17}$$

Comparing (16) and (17) we get

$$\left[\frac{\partial T}{\partial t}\right]_{r_b} = \frac{\sqrt{1-\alpha r_b^2}}{1-\dfrac{2GM}{c^2R}}. \tag{18}$$

This relation will be needed in the calculation of $\Delta T_1/\Delta t$.

Suppose we consider a light ray emitted by the object in the radially outward direction from $R_b = r_b S(t)$ to $R = R_1$. A straightforward calculation shows that

$$c(T_1 - T) = R_1 - r_b S(t) + \frac{2GM}{c^2} \ln \left[\frac{R_1 - \dfrac{2GM}{c^2}}{r_b S(t) - \dfrac{2GM}{c^2}} \right], \tag{19}$$

where T is the time of emission and T_1 the time of reception in Schwarzschild coordinates. For the next signal we similarly replace t, T, T_1 by $t + \Delta t$, $T + \Delta T$, $T_1 + \Delta T_1$ respectively, while R_1 stays constant. Also, from (18) we have

$$\Delta T = \frac{\sqrt{1 - \alpha r_b^2}}{1 - \dfrac{2GM}{c^2 R_b}} \Delta t. \tag{20}$$

Therefore a simple calculation leads to the result

$$\frac{\Delta T_1}{\Delta t} = \frac{\sqrt{1 - \alpha r_b^2} - \dfrac{r_b \dot{S}}{c}}{1 - \dfrac{2GM}{c^2 R_b}}. \tag{21}$$

Before proceeding further we make a few comments. In arriving at (21) we have used the Schwarzschild coordinates in the exterior of the object. This is because of convenience, although we have to pay the price of dealing with divergent quantities when the object bursts out of the Schwarzschild radius – as is evident from (19). These divergent quantities could be avoided by using different coordinate systems, e.g. the comoving one, in the exterior. This was done by Faulkner *et al.* (1964) in their numerical work. However, the intermediate steps then are more complicated. In any case the final result given by (21) is well defined and has no singularities at $R_b = 2GM/c^2$. Thus, (21) may be applied with confidence at all instants. Further (21) is applicable to a collapsing object also, provided we change the sign of $\dot{S}$ to make it negative. In that case as $R_b \to 2GM/c^2$ from outside, $\Delta T_1/\Delta t$ diverges. This is the familiar result that the exterior Schwarzschild observer never sees the collapsing object cross the Schwarzschild barrier. We emphasize here that no such divergence occurs when the white hole crosses the Schwarzschild barrier.

Returning now to (21), what is its physical interpretation? If ν_0 is the frequency of electromagnetic radiation emitted from the surface and ν the frequency of reception at R_1, we have

$$\frac{\nu_0}{\nu} = \frac{\Delta T_1}{\Delta t}. \tag{22}$$

This is because Δt is the proper time interval for the emitter and for $R_1 \gg 2GM/c^2$, ΔT_1 is very nearly the proper time interval for the receiver. In terms of η and η_s,

(21) becomes

$$\frac{\Delta T_1}{\Delta t} = \frac{\nu_0}{\nu} = \frac{\sin \eta}{\sin (\eta + \eta_s)}. \tag{23}$$

Thus signals emitted during the period corresponding to the interval $0 \leqslant \eta \leqslant (\pi - \eta_s)/2$ are blue-shifted while the signals emitted in the subsequent period are red-shifted. The change over from blue shift to red shift occurs, therefore, at the comoving time

$$t_c = \frac{t_0}{\pi} [\pi - \eta_s - \sin \eta_s] = \frac{t_0}{\pi} [\pi - \sqrt{r_b^2 \alpha} - \sin^{-1} \sqrt{\alpha r_b^2}]; \tag{24}$$

$t_c \gtrless t_s$ according as $\eta_s \lessgtr \pi/3$ – i.e., $\alpha r_b^2 \lessgtr 3/4$.

It is easy to calculate the corresponding periods over which blue shift and the red shift occur, as measured by the exterior Schwarzschild observer at $R = R_1$. The two periods are, respectively, given by

$$T_B = t_0 \int_0^{\pi - \eta_s/2} \frac{4 \sin^3 \eta}{\pi \sin (\eta + \eta_s)} \, d\eta, \qquad T_R = t_0 \int_{\pi - \eta_s/2}^{\pi/2} \frac{4 \sin^3 \eta}{\pi \sin (\eta + \eta_s)} \, d\eta. \tag{25}$$

In Table I below the ratios T_B/t_0, T_R/t_0 are given for various values of η_s. We see that, in general, these ratios are comparable to unity.

TABLE I

η_s	T_B/t_0	T_R/t_0
0	1.00	0
$\pi/10$	0.69	0.20
$\pi/9$	0.66	0.22
$\pi/8$	0.63	0.25
$\pi/7$	0.59	0.29
$\pi/6$	0.54	0.38
$\pi/5$	0.47	0.43
$\pi/4$	0.39	0.57
$\pi/3$	0.28	0.89
$\pi/2$	0.12	∞

4. Spectral Features

We now derive the spectral properties of the electromagnetic radiation to be expected from white holes. Several cases of possible interest to astrophysics will be considered. Their application to high-energy astrophysics will be discussed in the following two sections.

A. MONOCHROMATIC RADIATION OF CONSTANT LUMINOSITY

This is the simplest case to begin with. Suppose the white hole is emitting radiation with luminosity L, concentrated around the frequency ν_0. What is the spectrum of radiation as seen by the external observer at R_1? To calculate this consider the radiation emitted in the comoving time interval t, $t+\Delta t$. This will consist of

$$\Delta n = \frac{L\,\Delta t}{h\nu_0} \tag{26}$$

photons, all of which will cross the spherical surface $R=R_1$ in the time interval $(T_1, T_1+\Delta T_1)$, where ΔT_1 and Δt are related by (23). However, these photons will appear with frequencies in the range $(\nu, \nu+\Delta\nu)$ at R_1 where ν is given by (23). To calculate $\Delta\nu$ we make use of (23) and (9) in the relation

$$\Delta\nu = \frac{\mathrm{d}\nu}{\mathrm{d}\eta}\frac{\mathrm{d}\eta}{\mathrm{d}t}\Delta t, \tag{27}$$

leading to

$$\Delta t = \frac{4t_0\nu^3 \sin^3\eta_s\,\Delta\nu}{\pi(\nu^2+\nu_0^2-2\nu\nu_0\cos\eta_s)^2}. \tag{28}$$

Therefore, the number of photons crossing unit area at $R=R_1$ is given by

$$N(\nu)\,\Delta\nu = \frac{Lt_0}{\pi^2R_1^2h}\frac{\nu_0^2\sin^3\eta_s\,\Delta\nu}{(\nu^2+\nu_0^2-2\nu\nu_0\cos\eta_s)^2}. \tag{29}$$

In terms of energies, $E=h\nu$, $E_0=h\nu_0$, the photon number spectrum is given by

$$N(E)\,\mathrm{d}E = \frac{Lt_0}{\pi^2R_1^2}\frac{E_0^2\sin^3\eta_s\,\mathrm{d}E}{(E^2+E_0^2-2EE_0\cos\eta_s)^2}. \tag{30}$$

For $E \gg E_0$, the number spectrum becomes

$$N(E)\,\mathrm{d}E \approx \frac{Lt_0E_0^2\sin^3\eta_s}{\pi^2R_1^2}\frac{\mathrm{d}E}{E^4}. \tag{31}$$

The corresponding energy spectrum is given by

$$I(E)\,\mathrm{d}E = EN(E)\,\mathrm{d}E \propto \frac{\mathrm{d}E}{E^3}. \tag{32}$$

Thus a monochromatic constant emission at source results in a E^{-3} energy spectrum at the receiver.

The total radiation emitted by the white hole is $U=Lt_0$. This, however, is not the total radiation crossing over the sphere $R=R_1$ during the period T_B+T_R. The total blue-shifted radiation is represented by

$$\begin{aligned} U_B &= \frac{4Lt_0}{\pi}\int_{E_0}^{\infty}\frac{E_0^3\sin^3\eta_s\,\mathrm{d}E}{(E^2+E_0^2-2EE_0\cos\eta_s)^2} \\ &= \frac{Lt_0}{\pi}[\pi-\sin\eta_s-\eta_s], \end{aligned} \tag{33}$$

while the total red-shifted radiation is given by

$$U_R = \frac{4Lt_0}{\pi} \int_0^{E_0} \frac{E_0^3 \sin^3 \eta_s \, \mathrm{d}E}{(E_0^2 + E^2 - 2EE_0 \cos \eta_s)^2}$$

$$= \frac{Lt_0}{\pi} [\pi + \sin \eta_s - \eta_s + \sin 2\eta_s]. \tag{34}$$

Thus $U_B + U_R \neq U$. This inequality represents the energy exchanges with the gravitational field.

B. GENERAL TIME-INDEPENDENT SPECTRUM

Suppose the radiation emitted by the white hole surface has the luminosity spectrum $L(\nu)$. Since

$$L(\nu) = \int_0^\infty L(\nu_0)\, \delta(\nu - \nu_0)\, \mathrm{d}\nu_0, \tag{35}$$

we immediately get from (29) the number spectrum at $R = R_1$ of the form

$$N(\nu)\, \mathrm{d}\nu = \frac{t_0\, \mathrm{d}\nu}{\pi^2 R_1^2 h} \int_0^\infty \frac{L(\nu_0)\nu_0^2 \sin^3 \eta_s \, \mathrm{d}\nu_0}{(\nu^2 + \nu_0^2 - 2\nu\nu_0 \cos \eta_s)^2}. \tag{36}$$

If $L(\nu_0)$ is non-zero only over the range $\nu_1 < \nu_0 < \nu_2$, then $\nu \gg \nu_2$, we again recover the power-law spectrum of the type given by (31).

C. TIME-DEPENDENT SPECTRUM IN THE EARLY STAGES

The most general case is one in which the luminosity spectrum from the surface of the white hole changes with time. We shall consider this case in the early stages of the expansion. In the early stages Equation (11) will apply.

Suppose the exterior observer measures time $T = 0$ when the signal sent out by the white hole at $t = 0$ reaches him. Then using (23) with the approximations implied by (11), we get the relations

$$T = \frac{3}{4\eta_s} \left(\frac{3\pi}{4t_0}\right)^{1/3} t^{4/3} \tag{37}$$

and

$$\nu = \nu_0 \left(\frac{T_0}{T}\right)^{1/4}, \qquad T_0 = \frac{t_0 \eta_s^3}{\pi}. \tag{38}$$

If the surface of the object emits $L[\nu_0, t]\, \mathrm{d}\nu_0\, \mathrm{d}t$ energy in the frequency range $(\nu_0, \nu_0 + \mathrm{d}\nu_0)$ and time interval $(t, t + \mathrm{d}t)$, then the flux of energy crossing unit area at $R = R_1$, in the corresponding frequency range $(\nu, \nu + \mathrm{d}\nu)$ and the time range $(T, T + \mathrm{d}T)$

is given by

$$S(\nu, T)\,\mathrm{d}\nu\,\mathrm{d}T = \frac{L(\nu_0, t)\,\mathrm{d}\nu_0\,\mathrm{d}t}{h\nu_0}\,\frac{h\nu}{4\pi R_1^2};$$

i.e.,

$$S(\nu, T) = \frac{L(\nu_0, t)}{4\pi R_1^2}\left(\frac{T_0}{T}\right)^{1/4}. \tag{39}$$

Using (37) and (38) we get

$$S(\nu, T) = \frac{1}{4\pi R_1^2}\left(\frac{T_0}{T}\right)^{1/4} L\left[\nu\left(\frac{T_0}{T}\right)^{-1/4}, \tfrac{4}{3}T_0^{1/4}T^{3/4}\right]. \tag{40}$$

This general formula relates the emission spectrum of the white hole to the reception flux spectrum in the very early stages when the most dramatic effects are expected to arise.

It is worth emphasizing that in all these cases the general rule is that the radiation quanta with stronger blue shift arrive *earlier*. Thus the observed spectrum softens with time.

In the following section we consider the properties of the canonical white hole which are relevant to different astrophysical situations. We will then return to certain theoretical problems connected with white holes in general.

5. White Holes and Astrophysical Phenomena

The two outstanding characteristics of white holes are their explosive and transient nature. It is natural then to see if these can explain some of the explosive and transient phenomena observed in astronomy. Since the nature of the emission spectrum and the luminosity inside the white hole is not known, it becomes necessary to make assumptions about these. The approach here has been to make some reasonable assumptions and see if connection can be found between the deductions and the observed phenomena. We have chosen the following phenomena to see the connection between them and white holes: (a) Nuclei of Seyfert galaxies, (b) Transient X-ray sources, (c) Gamma ray bursts and (d) Production of high energy particles.

A. EMISSION OF RADIATION FROM NUCLEI OF SEYFERT GALAXIES

The nuclei of Seyfert galaxies are observed to emit infrared radiation up to about 10^{45} ergs s^{-1}. The infrared radiation is believed to be emitted by dust heated by ultraviolet radiation emanating from the centre. X-radiation has also been observed from Seyfert galaxies (Burbidge and Stein, 1970). It has also been suggested that Seyfert galaxies can be sources of universal X-ray background (Silk, 1973). We wish to connect the above statements by suggesting that radiation can be emanating from white holes at the centres of these galaxies.

The energy spectrum of radiation emanating from a white hole has a E^{-3} form; the universal background radiation however seems to fall as E^{-1}. We suggest that this change occurs due to absorptive effects of the gas surrounding the white hole. This is certainly possible in the energy range 0.2–1 keV and we show that the Seyfert nuclei can be responsible for the universal background radiation in this range. To see the absorption effect, consider a Seyfert nucleus having hydrogen gas of mass $\sim 10^5\ M_\odot$ and radius $\sim 1.5 \times 10^{20}$ cm. The photo-ionization cross-section (for hydrogen) at 0.2 keV is around 2.5×10^{-21} cm^2 (Brown and Gould, 1970), which implies an attenuation of the radiation incident by a factor $e^{3.3}$, i.e. by ~ 27. At 1 keV, the cross-section (*op. cit.*) is 10^{-23} cm^2 implying no attenuation. The white hole radiation implied by E^{-3} spectrum at 0.2 keV is $5^3 = 125$ times stronger than the radiation at 1 keV. Therefore the emerging radiation at 0.2 keV is only ~ 5 times stronger than the emerging radiation at 1 keV. This is then consistent with the observed E^{-1} spectrum.

The energy in the ultraviolet radiation from the white hole has to be about 10^{45} ergs s^{-1}, if it has to explain the infrared emission of the Seyfert nucleus through the dust grain heating mechanism suggested by Rees *et al.* (1969). If we take a ratio of 10–20 between the ultraviolet and the soft X-rays considered above, the energy emission in X-rays by the white hole implied by the law (32) will be a few times 10^{42} ergs s^{-1}. Absorption processes of the type discussed above will bring it down to $\sim$ a few times 10^{41} ergs s^{-1}. This is of the same order as that required from a typical X-ray source in order to account for the isotropic X-ray background (Silk, 1973). Thus a reasonable case can be made for white holes being responsible for the infrared and soft X-ray emission from Seyfert nuclei.

B. TRANSIENT X-RAY SOURCES

Five transient X-ray sources have been observed to date (see Strong, 1973). The main feature pertaining to these sources is that in a place where there was no source, in a matter of days, a source appears; thereafter the intensity decreases and goes below the background in a few months. The other feature is the softening of the spectrum with time. Apart from these two features, there are no other common characteristics. These features generally agree with the white hole phenomenon.

The decrease in intensity of some of these sources shows a single rate, while some show different rates in two different periods. The spectral index α of the photon energy flux, if expressed in the power law form $E^{-\alpha}$, shows initial values ranging from 1 to 3 and increasing with time. To understand some of these it is instructive to consider the spectral features of white hole emission, when the spectral emission inside the white hole is (i) blackbody type, (ii) free-free emission type and (iii) synchrotron type.

(i) *Blackbody type spectrum*

In this case

$$L(\nu_0) \propto \nu_0^3 (e^{h\nu_0/k\theta} - 1)^{-1}, \tag{41}$$

where h and k are the Planck and Boltzmann constants and θ is the temperature.

Assuming θ is constant, we find the energy spectrum inside, using (40), to be given by

$$S(\nu) \propto T^{-1/4} L[\nu(T_0/T)^{1/4}],$$

i.e.,

$$S(\nu) \propto \nu^3 T^{1/2}(e^{(h/kT_0)(\nu T^{1/4}/\theta)} - 1)^{-1}. \tag{42}$$

If observations outside are made in a narrow spectral range ν_1, ν_2 then the flux in this range ΔS changes with time approximately as shown in Figure 1.

If θ is a function of time, say it decreases with time, then the maximum in the figure is shifted to earlier times and with drastic change of θ with time it may be shifted close to $T=0$.

If a small region of frequency is observed, the energy flux spectrum can be fitted with a power law and the index stays constant for a while and then decreases to $-\infty$ (Figure 1).

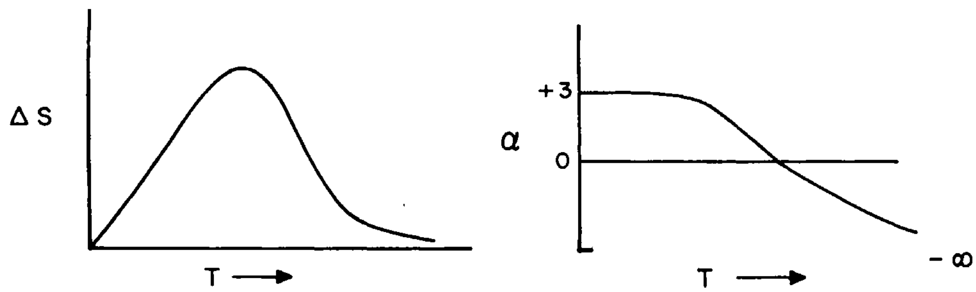

Fig. 1. Rate of change of flux ΔS and spectral index α with time, if the energy spectrum of emission of radiation in the white hole is of the blackbody type.

(ii) *Free-free emission spectrum*

Here

$$\begin{aligned} L(\nu_0) &\propto (a + \ln b/\nu_0) \quad \text{at low frequencies}, \\ &\propto (e^{-h\nu_0/k\theta})/(h\nu_0/k\theta)^{1/2} \quad \text{at high frequencies}, \end{aligned} \tag{43}$$

where a and b are constants. Again if θ is taken constant

$$\begin{aligned} S(\nu) &\propto [a + \ln(b'/\nu T^{1/4})] \quad \text{at early times}, \\ &\propto \frac{e^{-(h/k\theta T_0^{1/4})\nu T^{1/4}}}{\nu^{1/2}T^{3/8}} \quad \text{at later times}, \end{aligned} \tag{44}$$

where b' is a constant. The time variation of ΔS and the spectral index as defined above are given in Figure 2.

(iii) *Power law spectrum*

Here

$$L(\nu_0) \propto \nu_0^{-n},$$

so that (45)

$$S(\nu) \propto \nu^{-n} T^{-(n+1)}/4.$$

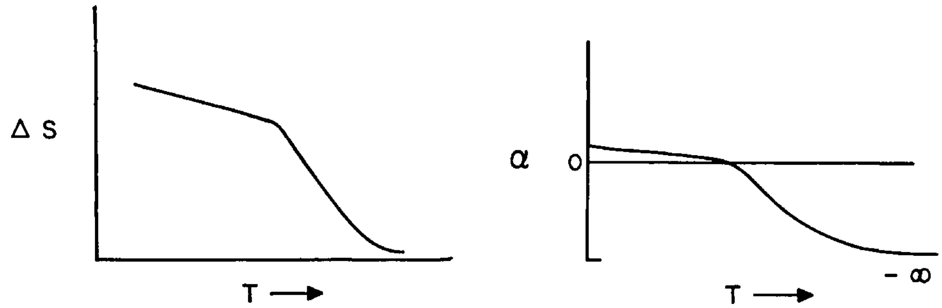

Fig. 2. Rate of change of flux ΔS and spectral index α with time, if the energy spectrum of emission of radiation in the white hole is of the free-free emission type.

The time variation of ΔS and the spectral index are given in Figure 3. If $L(\nu_0)$ is a function time the index of time variation of $S(\nu)$ is changed. It seems the behaviour of the transient X-ray sources can generally be explained by invoking one of the above possibilities.

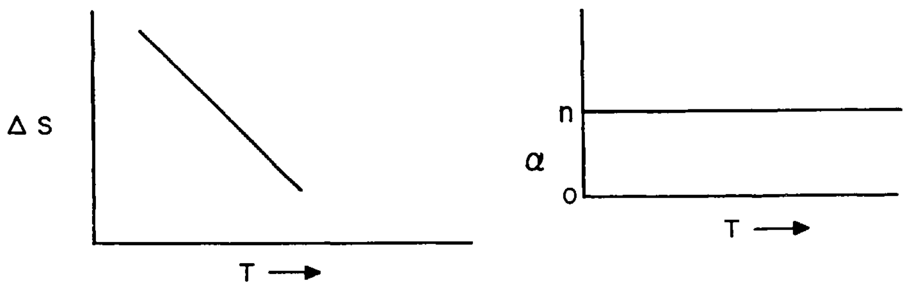

Fig. 3. Rate of change of flux ΔS and spectral index α with time, if the energy spectrum of emission of radiation in the white hole is of the power law type.

C. GAMMA RAY BURSTS

Bursts of radiation from the sky between a few keV and a few MeV have been observed (see Strong, 1973). Individual bursts show a variety of behaviour. The main characteristics are: (a) the bursts lasts from a few seconds to a few tens of seconds, (b) the pulses show structure; some have double structure and some have structure on the time scale of a few milliseconds, (c) they show a fast rise time and a slow decay of intensity, (d) the photon number spectra of several events is of the form E^{-3} or steeper when integrated over the pulse, (e) the photon spectrum softens with time. Again the general behaviour of the bursts suggests radiation from a white hole. The double pulse structure can be explained by an *ad hoc* assumption that in the white hole emission takes place in two energy bands say in the optical and the microwave regions. The radiation from the latter wave band arrives earlier since it is more strongly blue-shifted.

There is an interesting relation between the frequency band of observation and time duration of the pulse. Suppose the spectrum of radiation inside the white hole is between (ν_1, ν_2) and the corresponding observed frequency range is (ν_a, ν_b). Then using (38), the times corresponding to the beginning of the pulse T_i and the end of the pulse T_f are

$$T_i = T_0\left(\frac{\nu_1}{\nu_b}\right)^4, \qquad T_f = T_0\left(\frac{\nu_2}{\nu_a}\right)^4. \tag{46}$$

If, now, an observation is also made in another band (ν_a', ν_b') and T_i', T_f' are the lines corresponding to this band, then

$$\frac{T_i - T_f}{T_i' - T_f'} = \frac{\left(\frac{\nu_1}{\nu_b}\right)^4 - \left(\frac{\nu_2}{\nu_a}\right)^4}{\left(\frac{\nu_1}{\nu_b'}\right)^4 - \left(\frac{\nu_2}{\nu_a'}\right)^4}. \tag{47}$$

This formula in principle can be used with observations to evaluate ν_1 and ν_2. In practice, however, it may be difficult to estimate the absolute duration of the pulse due to the presence of a background. In general the signal is stronger at lower frequencies because of the E^{-3} dependence, while the background has E^{-1} dependence. This difficulty can be overcome if observation is made in two frequency bands which are narrow and close to each other.

D. HIGH ENERGY PARTICLE RADIATION

Just as in the early stages a white hole enhances the energy of a photon emitted from its surface, it will also lift the energies of particles of non-zero rest mass emitted outwards. Calculations to estimate this effect for such particles are more involved. However, since at very high energies the rest mass does not play a significant role, the energy spectrum is expected to have the E^{-3} form. One can think of white holes in the Galaxy on the scale of supernovae, which might yield this radiation. The connection of this radiation with cosmic rays is not obvious, since the cosmic radiation in general has a flatter (E^{-2}) energy spectrum. It is possible, however, to invoke magnetic effects to change the white hole particle radiation energy spectrum.

6. The Remnant of a White Hole

It is interesting to discuss the remnant left by a white hole after the expansion ceases. As seen from Table I the time of expansion seen by the receiver is generally of the same order as t_0.

Thus

$$T = T_B + T_R \sim t_0 \simeq \frac{\pi}{2c\sqrt{\alpha}}. \tag{48}$$

Now

$$\alpha = 8\pi G\varrho/3c^2,$$

so that

$$T \simeq \frac{\pi}{2}\sqrt{\frac{3}{8\pi G\varrho}} \simeq \frac{4 \times 10^4}{\varrho}. \tag{49}$$

Thus knowing T, the time scale of the phenomenon, we can obtain ϱ. The mass of the object M is given by (4)

$$M = \frac{\alpha r_b^3 c^2}{2G}. \tag{50}$$

By use of (10) we find that

$$M = \frac{1}{\sqrt{\alpha}} \frac{c^2}{2G} \sin^3 \eta_s. \tag{51}$$

By use of the expression for α and numerical values of the physical constants G and c,

$$M \simeq T \times 10^5 \, M_\odot \sin^3 \eta_s, \tag{52}$$

where $M_\odot$ is a solar mass.

Consider now the transient X-ray sources where the lifetime T is of the order of 10^6 s. Then $\varrho \sim 10^{-3}$ gm. Since the transient X-ray sources seem to be galactic objects, M should be of the order of a few solar masses. Then the formula for M implies $\eta_s \sim 10^{-3.6}$ radians. This implies, using (25), that $T_R \simeq 0$; or that the white hole emission is blue-shifted throughout its expansion.

In the case of γ-ray burst $T \sim 10$ s giving $\varrho \sim 10^7$ gm. The location of the γ-ray bursts are not known, but if we assume that they are in the Galaxy, again η_s should be small. Using $\eta_s \sim 10^{-2}$ yields $M = M_\odot$, so that the remnant of the burst phenomenon will be a white dwarf star.

7. General Considerations

We have discussed the astrophysical applications in the framework of the canonical white hole. In this concluding section we shall consider more general white holes qualitatively and indicate the scope of future work.

A. NON-SINGULAR WHITE HOLES

It is well known from the work of Penrose and Hawking (1969, see also Hawking and Ellis, 1973) that objects which have collapsed into their event horizons, inevitably shrink towards a singularity – provided the equations of general relativity and the conventional ideas about physics are valid. This tends to suggest that white holes must have a singular origin, as in the canonical case. If, however, we relax some of the restrictions imposed by conventional physics, non-singular white holes may be obtained.

One example of this was discussed by one of us (Narlikar, 1974) in the context of creation of matter in the white hole. The canonical model is unsatisfactory in that it leaves unaswered the question 'where does the matter come from in the first place?' This question has the same status as the question about the origin of the big-bang universe. To explain primary matter creation in the steady state cosmology, Hoyle and

Narlikar (1963) called upon the C-field. Later work (Narlikar, 1974) showed that the same C-field can account for the creation of matter in the big-bang universe or in a compact white hole. The matter created is compensated by the creation of an equa amount of negative energy in the form of the scalar C-field. Because of its negative energy the C-field prevents the singularity from developing at the instant of creation. Thus Equation (3) is modified to an equation of the form

$$\dot{S}^2 = \alpha c^2\left(\frac{1 - S}{S}\right) - \frac{A^2}{S^4}, \tag{53}$$

where A is related to the amount of matter created and the coupling constant of the C-field. This equation does not permit the singular state $S=0$. There is a finite minimum value of $S=S_0$ at which $\dot{S}=0$. However $\dot{S}$ rises rapidly for $S>S_0$ so that the Equation (53) resembles (3) for most practical purposes. Instead of (11) we have, fo early stages (i.e., $t \ll t_0$),

$$S \approx S_0\left(1 + \frac{t^2}{t_1^2}\right)^{1/3}, \tag{54}$$

where t_1 is related to A.

The signal propagation in this case is difficult to discuss because the external space ($r>r_b$) is non-empty with the C-field present. Qualitatively the forms (53) and (54) suggest that the radiation is highly red-shifted for a short interval to start with (when $\dot{S}=0$), it is then highly blue-shifted when $\dot{S}$ becomes large, and finally it is red-shifted again as $\dot{S}$ tends to zero. Also since $\dot{S}$ is bounded, both the red shift and blue shift are bounded.

B. NON-SPHERICAL AND CHARGED WHITE HOLES

Like their black hole counterparts white holes may be non-spherical and (or) charged. The technique given in earlier sections may be applied to discuss the spectral features of charged spherical models. Axially symmetric models may be discussed with the help of the Kerr metric.

The axially symmetric case may have astrophysical applications where non-isotropic radiation, in jet or discs or in the form of searchlights is involved.

C. STABILITY OF A WHITE HOLE

In a recent communication Eardley (1974) has argued that white holes of the lagging core type become unstable through accretion and quickly turn into black holes. It is worth investigating the stability of white holes in general since this will have a bearing on their possible existence in the Universe. It seems to us that we are in a somewhat similar situation with regard to the existence of white holes as existed for black holes about a decade ago. Then there were objections to supermassive objects forming on the grounds of angular momentum and fragmentation difficulties. In spite of these objections gravitational collapse of supermassive objects was studied for various reasons, including the important one that the presence of QSOs indicates the existence of compact massive systems. We feel that the evidence of exploding objects in the Universe (including perhaps the Universe itself if it started with a big-bang) indicates the necessity of studying white holes in the astrophysical context in spite of any possible stability problem seen at present.

D. PARTICLE EJECTION FROM WHITE HOLES

As mentioned in the previous section, the problem of ejection of particles of non-zero rest mass from the white hole surface is of interest to cosmic ray physics and the radio-source problem. What is the energy of a particle at $R=R_1$, if it were ejected from the white hole surface with a relative velocity v? Qualitatively it is easy to see that the energy must depend on the comoving epoch t of ejection. Work is in progress to obtain a quantitative solution to this problem.

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