Spinor driven inflation

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Indo-UK meeting @ IUCAA 11 August 2011

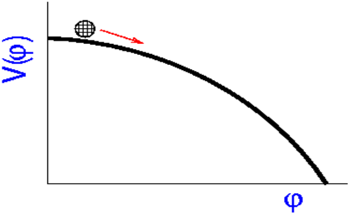
Theoretical problems of inflation

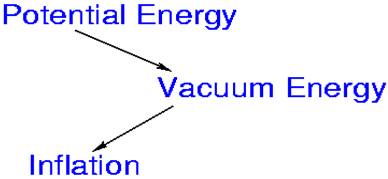
- Inflation has several problems including
  - Reheating problem
  - Hierarchy problem
  - Trans-Planckian problem
  - . . .
- These problems seem to be related to the fundamental question

  What is the nature of the field which drives inflation?

Inflationary models

- Usually assumed to be a scalar field with a potential





Potential Energy

Vacuum Energy

Inflation

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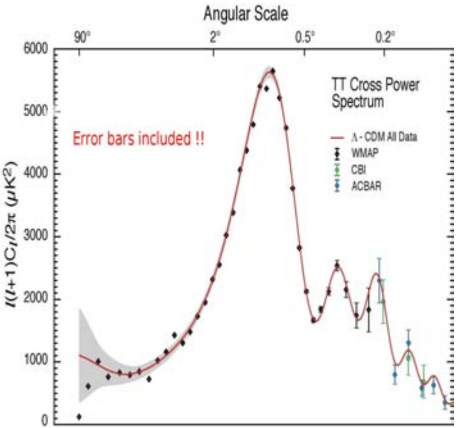
- Higher Ricci scalar curvature terms $R + \alpha R^2$ [Starobinsky, 1980]

  Very specific to Ricci scalar; not if Ricci/Riemann tensors are included do not have a well defined initial value problem
- Vector field [Golovnev et al '08; Dimopoulos et al '09]

  leads to directional asymmetry;
  require infinite of them to restore symmetry; has instabilities

Constraints from current CMB observations



- WMAP-5 data [Komatsu etal '10]
  - $\frac{\delta\rho}{\rho} \simeq 5 \times 10^{-5}$ $[k_1 = 0.002 Mpc^{-1}]$
  - $n_s \simeq 0.96$
  - $\frac{dn_S}{d\ln k} \simeq -0.037$
  - $-9 < f_{NL} < 111$
- Physical consequence
  - perturbation theory is valid
  - Broadly consistent with inflationary paradigm

However, ...

Canonical single scalar field inflation predicts no running and tiny $f_{\rm NL}$.

Need to go beyond and look for other alternatives.

Spinor driven inflation

Condensate of spinor fields

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- The transition from the free fermions to a highly interacting Bosons occurs below the critical temperature:

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- In the early universe, $\rho \sim 10^{98}\mathrm{g/m^3} \sim 10^{74}\mathrm{GeV}^4$, $m \sim 10^{15}\mathrm{GeV}$

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What kind of spinors can form such a condensate?

Elkos

Ahluwalia and Grumiller '05

- Elkos $\equiv$ Eigenspinoren des Ladungskonjugationsoperators
  Eigen spinors of charge conjugation operator.

Elkos

Ahluwalia and Grumiller '05

- In 1928, Dirac formulated wave-equation for charged spin $1/2$ particles.
- essentially Dirac wanted to compute the square root of the Klein-Gordon equation $(\partial^2 + m^2)\phi = 0$; using matrix valued objects $(i\gamma^a\partial_a + m)(i\gamma^a\partial_a - m)\psi = 0$
- What kind of spinors are used in the Dirac equation?

  Eigen spinors of parity operator

Elkos

Ahluwalia and Grumiller '05

- How does one describe a neutral spin 1/2 particle?

  Majorana particle

- Under charge conjugation operator, the usual set of two Majorana spinors have eigenvalue one.
- Ahluwalia & Grumiller showed that there also exists anti self-conjugate set
- Complete set of four spinor (Elko) span the four-dimensional representation space of spin 1/2 and come to par with Dirac spinors
- Elko are the eigen spinors of charge conjugation operator

Elkos

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Dirac spinors (ψ)

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- charge conjugation operator

$$\hat{C}|\lambda\rangle = c|\lambda\rangle$$

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- Form of spinor and conjugate

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} \quad \psi^\dagger = \gamma^0 \overline{\psi}$$

8 real, independent functions

Elkos (λ)

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- Form of Elko and conjugate

$$\lambda = \begin{bmatrix} \sigma_2 \phi_1^* \\ \phi_1 \end{bmatrix} \quad \lambda^\dagger = i \begin{pmatrix} \phi_2^\dagger & \phi_2^\dagger \sigma_2 \end{pmatrix}$$

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8 real, independent functions

- Satisfy $(CPT)^2 = I$
- Dirac Lagrangian

$$\mathfrak{L}_{Dirac} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi$$

Elkos (λ)

- charge conjugation operator

$$\hat{C}|\lambda\rangle = c|\lambda\rangle$$

- Form of Elko and conjugate

$$\lambda = \begin{bmatrix} \sigma_2 \phi_1^* \\ \phi_1 \end{bmatrix} \quad \lambda^\dagger = i\begin{pmatrix} \phi_2^\dagger & \phi_2^\dagger \sigma_2 \end{pmatrix}$$

8 real, independent functions

- Satisfy $(CPT)^2 = -I$
- Elko Lagrangian

$$\mathfrak{L}_{elko} = \frac{1}{2}\mathfrak{D}_\mu \lambda^\dagger \mathfrak{D}^\mu \lambda - m^2 \lambda \lambda^\dagger$$

Mass dimension and interactions

Ahluwalia and Grumiller '05

- Standard matter particles satisfy $(CPT)^2 = 1$.
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Unlike Dirac fields, Elkos can ONLY interact with standard matter particles via Higgs and/or gravity.

FRW background

Boehmer '07 '08, SS '09, Gredat & SS '10

- Consider the following $(3+1)-d$ action

$$S = \int d^4x\sqrt{-g}\left(R + \underset{\overset{\uparrow}{\frac{1}{2}\left[\frac{1}{2}g^{\mu\nu}(\mathfrak{D}_\mu\lambda^\dagger\mathfrak{D}_\nu\lambda + \mathfrak{D}_\nu\lambda^\dagger\mathfrak{D}_\mu\lambda)\right] - V(\lambda^\dagger\lambda)}}{\mathcal{L}_{\mathrm{Elko}}}\right)$$

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- FRW line-element:

$$ds^2 = \underset{\overset{\uparrow}{\text{cosmic time}}}{dt^2} - a^2(t)\,\underset{\overset{\uparrow}{\text{expanding 3-space}}}{d\bar{x}^2} = a^2(\eta)\left[\underset{\overset{\uparrow}{\text{conformal time}}}{d\eta^2} - d\bar{x}^2\right]$$

▸ Form of Elko

FRW background

Boehmer '07 '08, SS '09, Gredat & SS '10

- Effective density and pressure

$\mathcal{H} = a'/a$

$$\begin{aligned}\rho &= \frac{1}{2}\frac{(\varphi'(\eta))^2}{a^2(\eta)} + V(\varphi) - \frac{3}{8}\frac{\mathcal{H}^2}{a^2(\eta)}\varphi^2(\eta)\\ p &= \frac{1}{2}\frac{(\varphi'(\eta))^2}{a^2(\eta)} - V(\varphi) + \frac{1}{8}\frac{\mathcal{H}^2}{a^2(\eta)}\varphi^2(\eta)\end{aligned}$$

extra terms

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extra terms

$$\rho + 3p = 2\left[\frac{(\varphi'(\eta))^2}{a^2(\eta)} - V(\varphi)\right] \quad \Longrightarrow$$

Identical to canonical scalar field

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extra terms

- Acceleration equation is identical to canonical scalar field driven inflation

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3M_{\rm Pl}^2}(\rho + 3p) = \frac{8\pi}{3M_{\rm Pl}^2}\left[\, V(\varphi) - \dot{\varphi}^2 \,\right] \qquad M_{\rm Pl} \equiv G^{-1/2} \sim 10^{19}\,\mathrm{GeV}$$

Impossible to distinguish the two models from the acceleration equation

FRW background

Boehmer '07 '08, SS '09, Gredat & SS '10

Scalar

$$\frac{8\pi}{3M_{\rm Pl}^2}\frac{\left[V(\varphi)+\dot{\varphi}^2/2\right]}{\left[1+\mathcal{F}\right]}=H^2$$

Modified Friedman equation

$$\ddot{\varphi}+3H\dot{\varphi}+\mathcal{G}(\varphi)+V_{,\varphi}=0$$

Equation of condensate

$$\mathcal{F}=\frac{\varphi^2}{8M_{\rm Pl}^2}$$

$$\mathcal{G}(\varphi)=-\frac{3M_{\rm Pl}^2}{\dot{\varphi}}\left[a^2H^2\mathcal{F}\right]^{\bullet}$$

➤ Salient Features:

1. Elko (and its dual) depends on a single scalar function (φ)

   Physically, this can be interpreted an Elko-pair (similar to Copper-pair) forming a scalar condensate — spinflaton.

2. Friedman and spinflaton equations receive non-trivial corrections

   Elko modification to the inflaton equations are determined by $\mathcal{F}$

Slow-roll parameters

SS '09, Gredat & SS '10

- Exact de Sitter solution [Boehmer '08]

$$V(\varphi) = 3q^2 M_{\mathrm{Pl}}^2 + \frac{q^2}{4}\varphi^2 \qquad \varphi \propto \exp\left(\pm\frac{qt}{2}\right) \qquad a \propto \exp(qt)$$

Different from the canonical scalar field

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- First order exact slow-roll parameters are: ▸ Results

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} = \varepsilon_{\mathrm{can}}[1+\mathcal{F}] - \mathcal{F} \qquad \varepsilon_{\mathrm{can}} = 3\frac{\dot{\varphi}^2/2}{\dot{\varphi}^2/2+V}$$

$$\delta \equiv -\frac{\ddot{\varphi}}{H\dot{\varphi}} = \delta_{\mathrm{can}} + \mathcal{F}(\varepsilon_{\mathrm{can}} - 1) - \frac{\ln(1+\mathcal{F})'}{2\mathcal{H}} \qquad \delta_{\mathrm{can}} = \varepsilon_{\mathrm{can}} - \frac{\dot{\varepsilon}_{\mathrm{can}}}{2H\varepsilon_{\mathrm{can}}}$$

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- Slow-roll approximation corresponds to

$$\epsilon, \delta \ll 1 \qquad \Longrightarrow \qquad \varepsilon_{\rm can}, \delta_{\rm can} \ll 1, \mathcal{F} \ll 1$$

Linear perturbation

SS '09, Gredat & SS '10

- Consider small inhomogeneties:

$$\lambda = \overline{\lambda} + \delta\lambda \qquad g_{\mu\nu} = \overline{g_{\mu\nu}} + \delta g_{\mu\nu} \qquad \delta g_{\mu\nu} = \delta g_{\mu\nu}^{(S)} + \delta g_{\mu\nu}^{(T)} \qquad \left|\frac{\delta g_{\mu\nu}}{\overline{g_{\mu\nu}}}\right| \ll 1$$

$$\delta g_{\mu\nu}{}^{(S)} = a^2(\eta)\begin{pmatrix} 2\Phi & 0 \\ 0 & -2\Psi\delta_{ij} \end{pmatrix} \qquad \delta g_{\mu\nu}{}^{(T)} = a^2(\eta)\begin{pmatrix} 0 & 0 \\ 0 & h_{ij} \end{pmatrix}$$

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- Elkos do not source the tensor perturbation equations and they are free gravitational waves:

$$\mu_T'' + \left(k^2 - \frac{a''(\eta)}{a(\eta)}\right)\mu_T = 0$$

Linear perturbation

SS '09, Gredat & SS '10

Issues

- Scalar perturbations are harder to compute even for the scalar fields which have one free real function.
-

Linear perturbation

SS '09, Gredat & SS '10

Issues

- Scalar perturbations are harder to compute even for the scalar fields which have one free real function.
- Elkos have 8 real functions and not all are independent.

  Such an analysis has not be done for any spinor in the literature!
-

Linear perturbation

SS '09, Gredat & SS '10

Approach

- Assume the anisotropic stress of the perturbed Elko is zero

$$\Phi \to \Psi \qquad \Rightarrow \qquad \delta T_{ij} = 0 \qquad \forall \quad i \neq j$$

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- Set the vector perturbations to zero

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- Perturbed Elko must satisfy [$\delta\varphi(x)$ is perturbed condensate]

$$\overline{\lambda^{\dagger}}\delta\lambda + \delta\lambda^{\dagger}\overline{\lambda} = 2\overline{\varphi}\delta\varphi$$

▸ Technical details

Linear perturbation

SS '09, Gredat & SS '10

▸ MS variable

- Scalar perturbation equation is

$$\mu_s'' - \left[-k^2 + \frac{z''}{z} - \ln[1-\mathcal{F}_\varepsilon]'' + \frac{7\mathcal{H}'\mathcal{F}_\varepsilon^{\frac{1}{2}}}{2} + \frac{\mathcal{H}\varepsilon'\mathcal{F}_\varepsilon^{\frac{1}{2}}}{\varepsilon} \right] \mu_s = 0$$

Different from the canonical scalar field

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Salient Features

- Elko modification to the canonical MS equation is determined by $\mathcal{F}$.
- This equation is exact.

Power spectra

SS '09, Gredat & SS '10

- Upon quantization, in the slow-roll limit $\epsilon, \delta \ll 1$, power-spectra are:

$$\begin{aligned}
\mathcal{P}_S(k) &\simeq \left(\frac{H^2}{8M_{\rm Pl}^2\pi^2}\right)\left(\frac{\varepsilon+\mathcal{F}}{\varepsilon^2}\right)\left[1-2(c_0+1)\varepsilon_{\rm can}\right] \\
\mathcal{P}_T(k) &= \left(\frac{2H^2}{M_{\rm Pl}^2\pi^2}\right)\left[1-2(c_0+1)\varepsilon_{\rm can}+2\varepsilon_{\rm can}x\right]
\end{aligned}$$

Results and implications

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SS '09, Gredat & SS '10

- $\mathcal{P}_S(k), \mathcal{P}_T(k)$, during slow-roll, are nearly scale-invariant

◂ Slow-roll

-

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- $\mathcal{P}_S(k), \mathcal{P}_T(k)$, during slow-roll, are nearly scale-invariant

◂ Slow-roll

- Predicts running of spectral index at the leading order of ϵ

$$\frac{dn_S}{d\ln k} = -\frac{\varepsilon_{\rm can}}{2} - 4\varepsilon_{\rm can}\mathcal{F}_\varepsilon^{1/2} + \frac{\varepsilon_{\rm can}}{2}\frac{\mathcal{F}}{1+\mathcal{F}}$$

$$\frac{dn_T}{d\ln k} = 2\,\varepsilon_{\rm can}\,\mathcal{F}_\varepsilon^{1/2}$$

Consistent with WMAP data

-

Results and implications

SS '09, Gredat & SS '10

- $\mathcal{P}_S(k), \mathcal{P}_T(k)$, during slow-roll, are nearly scale-invariant

◂ Slow-roll

- Modified consistency relations: Scalar and tensor perturbations originate from the scalar condensate and they are not independent. Consistency relations link them.

  1. tensor-to-scalar ratio is $r \simeq 16\,\varepsilon_{\rm can}\,\left[1 - 2\mathcal{F}_\varepsilon\right]$

     Tensor contribution is smaller compared to canonical inflation

  2. The other observationally useful is the relation between n_T and r:

$$n_T = \frac{r}{8}(1+\mathcal{F}_\varepsilon)\left[1 + \varepsilon_{\rm can}\left[\frac{11}{6}c + \mathcal{F}_\varepsilon - \mathcal{F}\right] - 2\delta_{\rm can}\,c\right]$$

Different from the scalar field inflation

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Different from the scalar field inflation

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Conclusions, Issues and Outlook

Conclusions

- Fermions forming a scalar condensate is a real alternative to the scalar field model of inflation
- It leads to attractor behavior [Basak, Bhat & SS '11]
- For the first time perturbation equations for a spinor field are derived
- Scalar condensate from Elkos lead to observationally consistent primordial power spectra
- Predicts running of spectral index and modified consistency relations

Conclusions, Issues and Outlook

Issues

- As in the canonical scalar field the form of the potential is unclear.
- Power spectra calculations relies on the slow-roll condition.

Outlook

- Can it lead to large non-Gaussianity? [Basak & SS, Work in progress]
- Can Elko condensate lead to growing vorticies and hence magnetic field? [Work in progress]

Mass dimension

Back

- Propagator of the Elko field is

$$G^{\mathrm{Elko}} = \int d^4p \frac{1}{(2\pi)^4} \exp^{-ip_\mu(x'^\mu - x^\mu)} \frac{\mathbb{I}}{p^\mu p_\mu - m^2 + i\epsilon}$$

Compare this with the propagator of the Dirac spinor

$$G^{\mathrm{Dirac}} = \int d^4p \frac{1}{(2\pi)^4} \exp -ip_\mu(x'^\mu - x^\mu) \frac{\gamma^\mu p_\mu + m\,\mathbb{I}}{p^\mu p_\mu - m^2 + i\epsilon}$$

Mass dimension of Elkos is different from Dirac spinors
while it is same as Klein-Gordan field

Form of Elkos in the background

SS '09, Gredat & SS '10

◂ Back

- Form of Elko which leads to $T_{ti} = T_{ij} = 0$: $\alpha_1 = \alpha_2^{-1} = \sqrt{1+\sqrt{3}}$

$$\overline{\lambda} = \frac{\overline{\varphi}(t)}{\sqrt[4]{12}} \begin{pmatrix} -\alpha_1 e^{i\pi/4} \\ \alpha_2 \frac{i}{\sqrt{2}} \\ \alpha_2 \frac{1}{\sqrt{2}} \\ \alpha_1 e^{i\pi/4} \end{pmatrix} \qquad \overline{\lambda}^\dagger = \frac{\overline{\varphi}(t)}{\sqrt[4]{12}} \begin{pmatrix} -\alpha_1 e^{-i\frac{\pi}{4}} & -\frac{i\alpha_2}{\sqrt{2}} & \frac{\alpha_2}{\sqrt{2}} & \alpha_1 e^{-i\frac{\pi}{4}} \end{pmatrix}$$

Scalar perturbation equations

◂ Back

$$\Delta\Psi - 3\mathcal{H}\Psi' - \left(\mathcal{H}' + 2\mathcal{H}^2[1+\mathcal{F}(\overline{\varphi})]\right)\Psi = \frac{1}{2M_{\mathrm{Pl}}^2}[\overline{\varphi}'\delta\varphi' + a^2 V_{,\overline{\varphi}}\varphi] + 3\mathcal{F}(\overline{\varphi})\mathcal{H}[\Psi' - \frac{\mathcal{H}}{\overline{\varphi}}\delta\varphi]$$

0-0 equation

$$\Psi' + \mathcal{H}[1+\mathcal{F}(\overline{\varphi})]\Psi = \frac{1}{2M_{\mathrm{Pl}}^2}\,\overline{\varphi}'\delta\varphi$$

0-i equation

$$\Psi'' + 3\mathcal{H}\Psi' + \left(\mathcal{H}' + 2\mathcal{H}^2[1+\mathcal{F}(\overline{\varphi})]\right)\Psi = \frac{1}{2M_{\mathrm{Pl}}^2}[\overline{\varphi}'\delta\varphi' - a^2\frac{V_{,\varphi}}{2}\delta\varphi] - \mathcal{F}(\overline{\varphi})\mathcal{H}[\Psi' - \frac{\mathcal{H}}{\overline{\varphi}}\delta\varphi]$$

i-i equation

$$\delta\varphi'' - \Delta\delta\varphi - \overline{\varphi}'\left[4 - 3\,(1-\varepsilon)\,\mathcal{F}_\varepsilon - 3\sqrt{\mathcal{F}_\varepsilon}\right]\Psi' + \mathcal{H}\left[2 + 3(1-\varepsilon)\mathcal{F}_\varepsilon + 2\mathcal{F}\right]\delta\varphi'$$
$$+a^2\left[V_{,\varphi}\Psi + \frac{1}{2}V_{,\varphi\varphi}\delta\varphi\right] - \frac{3}{4}\mathcal{H}^2\left[1 - \frac{8}{3}\mathcal{F}\left[3 + \frac{\mathcal{G}}{\mathcal{H}\overline{\varphi}'} - \delta\right] + 4(1-\varepsilon)\sqrt{\mathcal{F}_\varepsilon}\right]\delta\varphi$$
$$-2\mathcal{H}\overline{\varphi}'\left[3 + \frac{\mathcal{G}}{\mathcal{H}\overline{\varphi}'} - \delta\right]\Psi + \frac{2}{\sqrt{3}}\frac{\overline{\varphi}'}{\mathcal{H}}\mathcal{F}_\varepsilon\nabla\Psi' = 0$$

Mukhanov-Sasaki (Q) variable

SS '09, Gredat & SS '10

- Q is a gauge-invariant linear combination of $\delta\varphi$ and Ψ

  Also related to the curvature perturbation $\mathcal{R}$

- Unlike scalar field, not possible to obtain Q directly from $\delta\varphi$ and Ψ

  $\delta\varphi$ is derived from $\delta\lambda$

- Approach: Assume the relation between $\mathcal{R}$ and Q is like that of canonical scalar field.

- This leads to

$$Q = a\,\delta\varphi + z\,\Psi \qquad\qquad z = [1 - \mathcal{F}_\varepsilon]\,(a\overline{\varphi}')/\mathcal{H}$$

Back

Scalar perturbation equations

SS '09, Gredat & SS '10

Scalar perturbation equations

SS '09, Gredat & SS '10

Salient Features

- Matter perturbations have one dof — perturbed condensate

Consistent linear perturbation equations

Scalar perturbation equations

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- Sound speed of perturbations is 1 $c_s^2 = 1$

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$$\delta S \propto \nabla^2 \Psi$$

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- Sound speed of perturbations is 1 $c_s^2 = 1$
- Entropic perturbations vanish at super-Hubble scales

$$\delta S \propto \nabla^2 \Psi$$

- Curvature perturbation ζ is given by $\mathcal{F}_\varepsilon = \mathcal{F}/(\mathcal{F}+\varepsilon)$

$$\zeta = \Psi + \mathcal{H}\frac{\delta\varphi}{\overline{\varphi}'}\frac{1}{(1-\mathcal{F}_\varepsilon)}$$