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String-dust perfect fluid in singularity-free models

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Abstract

We have attempted to incorporate string-dust in perfect fluid singularity free inhomogeneous models. It turns out that the stiff fluid can only sustain inclusion of string-dust.

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Following Senovilla's discovery [1] of a non-singular inhomogeneous cylindrically symmetric model, a singularity free family [2] has been identified which includes the radiation Universe $\rho = 3p$ [1] and the stiff fluid model $\rho = p$ [3-4]. It turns out the non-singular form of the metric cannot sustain viscosity [5] while the radial heat flow can be incorporated successfully [4] without disturbing the singularity-free character of the spacetime. In this note we wish to incorporate string-dust distribution along with perfect fluid in the non-singular form of the metric. We have considered string-dust in various cases; in the Kerr-NUT symmetry [6-7] and in the Einstein and Gödel Universes [8].

We take the non-singular form of the metric [4],

$$\begin{aligned} ds^2 &= C^{2\alpha}(kt)C^{2a}(mr)(dt^2 - dr^2) - C^{2\beta}(kt)C^{2b}(mr)dz^2 \\ &\quad -m^{-2}C^{2\alpha}(kt)C^{2c}(mr)S^2(mr)d\theta^2 \end{aligned} \tag{1}$$

where $C(x) = \cosh x, S(x) = \sinh x$. The regularity of the metric further requires $\alpha + \beta = 1$.

The stress tensor T_{ik} for string-dust with perfect fluid is given by

$$T_{ik} = -8\pi \left[(\rho + p)u_i u_k - p g_{ik} + \lambda(u_i u_k - w_i w_k)\right] \tag{2}$$

where $u^i u_i = 1 = -w^i w_i$, $u^i w_i = 0$ and ρ, p and λ denote fluid density, pressure and string energy density respectively. The Einstein equation will then read as

$$R_{ik} = -8\pi \left[\lambda(u_i u_k - g_{ik} - w_i w_k) + (\rho + p)u_i u_k - \frac{1}{2}(\rho - p)g_{ik}\right]. \tag{3}$$

From equation (3) we write

$$\begin{aligned} R_{22} &= -4\pi(\rho - p), \; R_{14} = 0 \\ R_{11} &= R_{33} = -4\pi(2\lambda + \rho - p) \\ R_{44} &= -4\pi(\rho + 3p) \end{aligned} \tag{4}$$

where R_{ik} referred to the orthonormal tetrad are given in the Appendix. Here onwards all quantities are referred to the tetrad frame. We write

$$u_i = (0,0,0,1), \qquad w_i = (0,1,0,0). \tag{5}$$

We have taken the string fibre to lie along the z-axis. From the Appendix, $R_{14} = 0$ will give

$$\alpha = \frac{b-a}{2b} \tag{6}$$

while $R_{11} = R_{33}$ gives

$$a = \frac{b(b-c-1)}{b+c+1}. \tag{7}$$

Substituting for a in (6) we get

$$\alpha = \frac{c+1}{b+c+1}. \tag{8}$$

From (4), the Appendix, (7) and (8) we have

$$\begin{aligned} 16\pi\rho A^2/k^2 &= \frac{2}{b+c+1}(b-c-1-2bn^2(b+2c+2)) \\ &\quad -\frac{2bn^2}{b+c+1}(b^2+2c^2+3bc+b-2)T^2(mr) \\ &\quad +\frac{2(c+1)(2b+c+1)}{(b+c+1)^2}\,T^2(kt) \end{aligned} \tag{9}$$

$$\begin{aligned} 16\pi p A^2/k^2 &= 2\left(\frac{2b^2n^2}{b+c+1}-1\right)+\frac{2b^2n^2(b+c-1)}{b+c+1}T^2(mr) \\ &\quad +\frac{2(c+1)(2b+c+1)}{(b+c+1)^2}T^2(kt) \end{aligned} \tag{10}$$

$$\begin{aligned} 8\pi\lambda A^2/k^2 &= n^2(b-1-3c)+\frac{c+1-b}{c+1+b} \\ &\quad +n^2(b-c)(b+c-1)T^2(mr) \end{aligned} \tag{11}$$

where $A^2 = C^{2\alpha}(kt)C^{2a}(mr)$, $n^2 = m^2/k^2$ and $T(x) = \tanh x$.

By putting $\lambda = 0$ we get back the singularity free perfect fluid models [4] and $\lambda = 0$ implies

$$(b-c)(b+c-1) = 0 \tag{12}$$

and

$$\begin{aligned} n^2 &= \frac{1-b+c}{(b+c+1)(1-b+3c)} \\ &= \frac{1}{(1+2b)^2} \text{ for } b = c \\ &= 1/4 \qquad \text{for } b+c = 1. \end{aligned} \tag{13}$$

Let us now consider the two cases; $b = c$ and $b+c = 1$ separately.

Case (i): $b = c$. Equations (9-11) will read as

$$\begin{aligned} 16\pi\rho A^2/k^2 &= \frac{2b}{(1+2b)^2}(3b+2) - \frac{2bn^2}{1+2b}(6b^2+7b+2) \\ &+ \frac{2bn^2}{1+2b}(6b^2+b-2)S_e^2(mr) \\ &- \frac{2}{(1+2b)^2}(b+1)(3b+1)S_e^2(kt) \end{aligned} \tag{14}$$

$$\begin{aligned} 16\pi pA^2/k^2 &= 2b^2\left(n^2 - \frac{1}{(1+2b)^2}\right) + \frac{2b^2n^2(1-2b)}{(1+2b)}S_e^2(mr) \\ &- \frac{2(b+1)(3b+1)}{(1+2b)^2}S_e^2(kt) \end{aligned} \tag{15}$$

$$8\pi\lambda A^2/k^2 = -(1+2b)(n^2 - \frac{1}{(1+2b)^2}) \tag{16}$$

where $S_e(x) = \text{sech}\, x$. It is easy to see that both p and λ cannot be made positive for all values of r and t simultaneously. Hence this case does not give a physically acceptable model.

Case (ii): $b+c=1$. Here we shall have

$$16\pi\rho A^2/k^2 = \frac{1}{2}b(4-b)(1-4n^2)+\frac{1}{2}(b^2-4)S_e^2(kt) \tag{17}$$

$$16\pi p A^2/k^2 = \frac{b^2}{2}(4n^2-1)+\frac{1}{2}(b^2-4)S_e^2(kt) \tag{18}$$

$$8\pi\lambda A^2/k^2 = (b-1)(4n^2-1). \tag{19}$$

The positivity of the above parameters is ensured by $n^2 \geq 1/4$ and $b \geq 4$. It reduces to the stiff fluid $\rho = p$ non-singular models for $n^2 = 1/4$ [3].

We have thus seen that string-dust along with perfect fluid can be incorporated in the singularity-free family of models only in the case of stiff fluid.

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Appendix

$$
\begin{aligned}
A^2R_{11} &= (b+3c+1)m^2 - \alpha k^2 + m^2(b^2+c^2-(1+a)(b+c)-a)T^2(mr) \\
A^2R_{22} &= 2bm^2 - (1-\alpha)k^2 + bm^2(b+c-1)T^2(mr) \\
A^2R_{33} &= m^2(1+b+3c) - \alpha k^2 + m^2c(b+c-1)T^2(mr) \\
A^2R_{44} &= -2am^2 + (1+\alpha)k^2 - m^2a(b+c-1)T^2(mr) \\
&\quad -2\alpha(2-\alpha)T^2(kt) \\
A^2R_{14} &= mk(b(1-2\alpha)-a)T(mr)T(kt)
\end{aligned}
$$

where $A^2 = C^{2\alpha}(kt)C^{2a}(mr), \qquad T(x) = \tanh x.$

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