On active and passive gravitational mass of a fluid sphere

Naresh Dadhich*

Inter-University Centre for Astronomy and Astrophysics

Post Bag 4, Ganeshkhind, Pune 411 007, India

Abstract

* e-mail address : naresh@iucaa.ernet.in

Before we go ahead let us recall, for the benefit of a general reader, the definitions of the three masses : m_i is the measure of body's resistance to motion, m_a is the measure of body's ability to produce gravitational field and m_p is the measure of body's response to gravitational field. A priory there is no reason why should these three measures be equal.

Let us begin by writing the Einstein equation ($G = C = 1$)

$$R_{ab} = -8\pi(T_{ab} - \frac{1}{2}Tg_{ab}) \tag{1}$$

where for a perfect fluid we have

$$T_{ab} = (\rho + p)k_a k_b - pg_{ab} \tag{2}$$

and

$$(\rho + p)k^a; \quad k^b = \frac{\partial p}{\partial x^b}(g^{ab} - k^a k^b). \tag{3}$$

This indicates, as Bounor notes[1], $(\rho + p)$ as the inertial mass density which by the Principle of Equivalence is equal to the passive mass density. It may be noted that in density perturbation calculations in cosmological models also $(\rho + p)$ acts as inertial density.

Bounor obtains m_p by

$$m_p = \int (\rho + p)\sqrt{-^3g}d^3x \tag{4}$$

$$R_{ab}k^a k^b = -8\pi(\rho + p) \qquad (7)$$

where $k^a k_a = 1 = k^a k_a$ and $k^a k_a = 0$. This shows $(\rho + 3p)$ and $(\rho + p)$ are invariants of spacetime.

The above equation (6) approximates to the Newtonian equation $\nabla^2\phi = 4\pi\rho$ which clearly brings out $(\rho + 3p)$ as the active mass density. Thus the integral (5) actually gives m_a and m_p will be obtained by replacing $(\rho + 3p)$ by $(\rho + p)$. Let us now evaluate these masses for a static sphere of uniform density.

Consider the Schwarzschild interior solution given by

$$ds^2 = \frac{D^2}{4}dt^2 - (1 - \beta^3)^{-1}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2)$$

$$8\pi p = \frac{3}{R^2}, \quad 8\pi p = \frac{3}{DR^2}(\sqrt{1 - \beta^2} - \sqrt{1 - \beta_1^2}) \qquad (8)$$

where

$$D = 3\sqrt{1 - \beta_1^2} - \sqrt{1 - \beta^2}, \quad \beta = \frac{r}{R}, \quad \beta_1 = \frac{r_1}{R}.$$

Its matching to the Schwarzschild exterior solution implies

$$m = \frac{1}{2}\beta_1^3 R = \frac{4\pi}{3}\rho r_1^3. \qquad (9)$$

For the metric (8), we now obtain

$$m_a = m \qquad (10)$$

and very serious consideration. This as well as the consequences of $m_a \neq m_p$ are the issues for future investigation.

Finally, it is however interesting to note that $(\rho + 3p) \geq 0$ may not always be respected in unusual settings of present-day astroparticle cosmology involving exotic matter but $(\rho + p) \geq 0$ is always adhered to. That is m_a may not always be positive implying gravity may turn repulsive in some circumstances but $m_i = m_p$ does always remain positive.