

Can dark energy be decaying?

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Abstract. We explore the fate of the universe given the possibility that the density associated with ‘dark energy’ may decay slowly with time. Decaying dark energy is modeled by a homogeneous scalar field which couples minimally to gravity and whose potential has *at least one* local quadratic maximum. Dark energy decays as the scalar field rolls down its potential, consequently the current acceleration epoch is a transient. We examine two models of decaying dark energy. In the first, the dark energy potential is modeled by an analytical form which is generic close to the potential maximum. The second potential is the cosine, which can become negative as the field evolves, ensuring that a spatially flat universe collapses in the future. We examine the feasibility of both models using observations of high redshift type Ia supernovae. A maximum likelihood analysis is used to find allowed regions in the $\{m, \phi_0\}$ plane (m is the tachyon mass modulus and ϕ_0 the initial scalar field value; $m \sim H_0$ and $\phi_0 \sim M_P$ by order of magnitude). For the first model, the time for the potential to drop to half its maximum value is larger than ~ 8 Gyrs. In the case of the cosine potential, the time left until the universe collapses is always greater than ~ 18 Gyrs (both estimates are presented for $\Omega_{0m} = 0.3$, $m/H_0 \sim 1$, $H_0 \simeq 70$ km/sec/Mpc, and at the 95.4% confidence level).

1. Introduction

Observations of the luminosity distance to distant type Ia supernovae [1, 2] supported by the discovery of CMB angular temperature fluctuations on degree scales [3] and measurements of the power spectrum of galaxy clustering [4] convincingly show that our Universe is (approximately) spatially flat, with $\sim 30\%$ of its critical energy density in non-relativistic matter (cold dark matter (CDM) and baryons) and the remaining $\sim 70\%$ in a smooth component having a large negative pressure (‘dark energy’). Dark energy is clearly the most abundant form of matter in the Universe (in terms of the effective energy density), yet both its nature and its cosmological origin remain enigmatic at present. It is clear that the observational properties of dark energy (DE) are tantalizingly close to those of a cosmological constant ‘ Λ ’, yet dark energy *need not* be Λ exactly (see reviews [5, 6]).

An intriguing possibility, carrying far reaching consequences both for the current Universe and its ultimate fate, is that the dark energy may be *decaying*. The possibility that dark energy could be unstable is in fact suggested by the remarkable *qualitative*

analogy between the presence of dark energy today and the properties of a different type of ‘dark energy’ – the inflaton field – postulated in the inflationary scenario of the early Universe. This analogy works in two ways. On one hand, the fact that a form of matter having a large negative pressure dominates the universe today makes it not unnatural that a similar form of matter (having $w < 0$) could have dominated the universe in the distant past. On the other hand, since dark energy in the early Universe (the inflaton) was unstable and decayed aeons ago, one might be tempted to ask whether the nature of dark energy observed today will be any different. In this paper we address in detail the possibility that dark energy, like the inflaton which preceded it, may be decaying.

Decaying dark energy (DDE) leads to numerous interesting possibilities including the fact that the current epoch of cosmic acceleration could be a transient which ends after the dark energy density has dropped to sufficiently small values. Such a universe will clearly be very different from the standard Λ CDM cosmology. For instance, a DDE universe may not possess horizons which are characteristic of Λ CDM as well as tracker-driven quintessence models (see [7] for a brief review). (Interesting implications for the future of our universe also arise in certain braneworld models, in which the current accelerating regime is a transient between two matter dominated epochs [8, 9, 10].)

Within the context of quintessence models another interesting (though more speculative) possibility is provided by potentials which are not constrained to be positive but which can become negative for certain values of ϕ [11]. These potentials are either bounded from below, in which case $V(\phi)$ has one or more minima at which $V(\phi_i) < 0$. (A good example is furnished by the cosine potential which will be examined in detail later in this paper.) Alternatively the potential becomes unbounded, in which case $V(\phi) \rightarrow -\infty$, for some values of ϕ . Under the influence of a negative potential, DDE steadily decreases and the universe expands at an increasingly slower rate. Finally a stage is reached when the negative DDE density *exactly cancels* the positive density of dark matter (plus the curvature term, if present). At this point of time the expansion of the universe stops ($H = 0$) and thereafter the universe begins to collapse ($H < 0$). This paper examines two general classes of potentials describing decaying dark energy in the light of recent high redshift supernova observations. In the first class $V(\phi) = V_0 - \frac{1}{2}m^2\phi^2$, and we evolve the field equations into the future to determine the time taken for the potential to drop to half its maximum value. The second class of models involves the cosine potential. In both cases we use the agreement between our model and high redshift supernova data *in the past*, to extrapolate and predict the behaviour of our model universe *in the future*. As a result, we find the range of model parameters allowed by the supernovae data and determine: (i) the *minimum* time for the universe to *stop accelerating*, and, (ii) the *minimum* time for the universe to *collapse* in DDE models with negative potentials.

2. Decaying Dark Energy

The decaying dark energy (DDE) model which we consider is analogous to successful inflationary models in that it undergoes quasi-homogeneous decay. We model DDE by a scalar field ϕ (‘quintessence’) which couples minimally to gravity and has one or more maxima in its potential. Near any one such local maximum (which for convenience we assume is at $\phi = 0$) the field can be *generically* modeled by the potential

$$V(\phi) = V_0 - \frac{m^2\phi^2}{2}, \quad \text{where } m^2 \equiv \left. \frac{d^2V}{d\phi^2} \right|_{\phi=0}. \quad (1)$$

The modulus of the tachyonic rest-mass m will be assumed $\sim H_0$ where H_0 is the Hubble constant (the slow-roll condition, $m \ll H_0$, would result in the absence of any noticeable observational effects for ultra-light scalars, making such fields virtually indistinguishable from Λ CDM, at least within the accuracy of present data). It is known from observations that physical properties of dark energy are close to those of a cosmological constant at present. This means that the variation in ϕ must be on time scales of order H_0^{-1} and not much faster. (In the opposite case the equation of state w would vary much too rapidly.) The equation of motion then implies that $m \sim H_0$ which suggests that all characteristic time-scales in our problem will be of order H_0^{-1} or greater. Therefore, in this model the initial value ϕ_0 of the scalar field ϕ and its change – during the entire process of dark energy decay – are both expected to be of the order of G^{-1} . (This follows quite simply from the Einstein equation $m^2\phi^2 \sim \rho_{\text{crit}} \sim H_0^2/G$.) However, since cosmology is now a precision science, we can hope to obtain much better *quantitative* results, beyond these simple qualitative estimates, using current observational data. This is precisely what is done in this paper.

We should at this stage mention that, in principle, decay mechanisms other than those considered in this paper are possible. As an example one should mention bubble-like decay via quantum tunneling in the case when $V'' > 0$, or strongly inhomogeneous classical decay in the case of a tachyonic mass large compared to H_0 , however these possibilities must be confronted with observational data other than the high- z supernova data which we wish to consider, and will therefore not be discussed any further by us in this paper. It may also be appropriate to mention that our model of decaying dark energy does not introduce any direct non-gravitational coupling of dark energy to non-relativistic matter. Significant coupling of dark energy to dark matter may lead to large energy transfer from dark energy to dark matter in the process of DDE decay. However this model of DDE, though potentially interesting, lies somewhat beyond the scope of the present paper and we do not consider it any further here. In our case, what we call the DE decay is actually the transition of DE from the potential-dominated regime to a regime in which the potential and kinetic energies of the dark energy field ϕ become comparable. ‡

‡ We also do not consider fields which couple non-minimally to gravity. Such a coupling, if small, would result in an insignificant change in the effective mass parameter m for time intervals of the order of H_0^{-1} which we consider in this paper. Large couplings would lead to a value of the effective Brans-Dicke

The field equations governing the behaviour of the scalar field as it rolls down its potential in a spatially flat universe are

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + V'(\phi) &= 0 , \\ H^2 &= \frac{8\pi G}{3}(\rho_m + \rho_\phi) , \end{aligned} \quad (2)$$

where $V(\phi)$ is given by (1) and

$$\rho_\phi = V(\phi) + \frac{1}{2}\dot{\phi}^2 . \quad (3)$$

At early times ($H_0 t \ll 1$), the universe was matter-dominated. Assuming $\rho_m \gg \rho_\phi$ in (2) we obtain the following exact solution for $\phi(t)$ during the matter dominated regime:

$$\phi(t) = \phi_0 \frac{\sinh mt}{mt} . \quad (4)$$

Eq (4) determines the initial conditions for ϕ and $\dot{\phi}$. At very early times ($mt \ll 1$) the very large damping experienced by the scalar field ensures that it remains close to its initial value at ϕ_0 . One should note that this initial value must be sufficiently small, $\phi_0 \lesssim \sqrt{3/8\pi G}$, in order that DE have sufficiently large negative pressure during the present epoch. This is clearly demonstrated in the next section when we compare this model of dark energy with supernova observations.

Since the point $\phi = 0$, $\dot{\phi} = 0$ is a saddle point for homogeneous solutions of the scalar field equation with the potential (1), there exists a set of non-zero measure of generic solutions which remain sufficiently close to our solution up until the present epoch. (It is sufficient for this purpose that a large initial kinetic energy – if at all it existed – is redshifted by $z \sim 3$ so that the field settles on the trajectory described by (4) by that redshift. Since the kinetic energy of the scalar field decays as $\propto a^{-6}$ during the regime when $\dot{\phi}^2 \gg m^2\phi^2$ this requirement is easily satisfied in practice.) Of course, one may with good reason ask as to whether our set of initial conditions on $\{\phi, \dot{\phi}\}$ is not too small. The answer to this question depends entirely on the (unknown) behaviour of present dark energy at large temperatures and curvatures where the form of the potential (1) need not be valid. For this reason, we will not discuss it any further here. However it should be noted that, generally speaking, the probability of such initial conditions will be strongly enhanced if there are *many* maxima in the DE potential. This occurs, for instance, in the DE model with $V(\phi) = V_0 \cos(\phi/f) + V_1$ [12, 13, 15, 16] which will be discussed by us later in this paper (in the inflationary context, such a model is called ‘natural inflation’).

In the next section, we numerically integrate the system of equations (2) with the potential (1), and determine the scale factor $a(t)$ and the Hubble parameter $H(z) \equiv \dot{a}/a$ as functions of the initial field displacement ϕ_0 for specific values of m and Ω_{0m} . Our parameter ω which violates the lower bounds on this parameter set by Solar system tests of Einstein gravity.

model is confronted against high redshift type Ia supernova observations through its luminosity distance

$$D_L = c \int \frac{dz}{H(z)}, \quad (5)$$

which is used to place constraints on the free parameters of the model, namely m and ϕ_0 (V_0 is not a free parameter since it is uniquely determined by m , ϕ_0 , Ω_{om} and the current value of the Hubble parameter – H_0).

Having determined the permitted range of parameters values, we shall proceed to determine the future evolution of our model universe. As emphasized in [14], reliable future predictions are only possible for finite intervals of time. Previous predictions (of the kind ‘our Universe will keep expanding and not encounter a singularity’) had a depth of approximately 20 Gyrs [14] (see [17, 18, 19] for similar estimates and [9] for the braneworld context). It is entirely reasonable to use the form (1) up to the point when $m^2\phi^2 = V_0$. This occurs when the potential has declined to half its maximum value. In the next section we shall determine the minimal period of time for the potential to reach its half-way mark (this time could be referred to as the ‘half-decay time of dark energy’). Note that this estimate is rather robust since it depends upon the fairly general form of the potential near its maximum value given by (1) and not on any other details of dark energy.

In Sec. 3, we also consider a potential which can become negative. The form of this potential is assumed to be [12, 15, 16]:

$$V = V_0 \cos \frac{\phi}{f}, \quad f = \frac{\sqrt{V_0}}{m} \quad (6)$$

where the value of f is chosen in such a way that the potential (6) coincides with (1) at small ϕ . Of course, the hypothesis that V may acquire negative values is speculative since it does not follow from any of the current observational data. However, such potentials often arise in supergravity and M-theory models (see, e.g., [15, 17, 19]). For this potential, we calculate: (i) the minimal extent of the current acceleration epoch, and (ii) the time elapsed before the universe collapses. (We should draw the readers attention to the fact that the collapse of the universe is a generic property of flat cosmological models with negative potentials [11].) We will not however consider the subsequent period of contraction in any detail since its exact duration strongly depends both upon the behaviour of V in the region $V < 0$ and upon the properties of dark matter at high energies. We end with section 4 which contains a summary of our results and a discussion.

3. Methodology and Results

We constrain the parameter space of our cosmology by requiring that our dark energy model provides a good fit to type Ia supernova data. For this purpose we use the 54 SNe Ia from the primary ‘fit C’ of the Supernova Cosmology Project, which includes 16

low redshift Calan-Tololo SNe [1]. Fit C is a subsample of a total of 60 SNe of which six are excluded as outliers: two low redshift SNe due to suspected reddening and four high redshift SNe which are excluded due to atypical light curves. The measured quantity in this data, the bolometric magnitude m_B , is related to the luminosity distance and therefore the cosmological parameters by the following equation

$$m_B = \mathcal{M} + 5 \log_{10} D_L(z; \Omega_{0m}, m, \phi_0) , \quad (7)$$

where $D_L = H_0 d_L$ is the Hubble-parameter-free luminosity distance and $\mathcal{M} = M_B + 25 - 5 \log_{10} H_0$ is the Hubble-parameter-free absolute magnitude. We shall assume that the SNe measurements come with uncorrelated Gaussian errors in which case the likelihood function is given by the chi-squared distribution with $N-n$ degrees of freedom: $\mathcal{L} \propto \exp(-\chi^2/2)$. (In our case, $N = 54$ and $n = 3$).

The χ^2 -statistic is defined as

$$\chi^2 = \sum_{i=1}^{54} \left(\frac{m_i^{\text{eff}} - m(z_i)}{\sigma_{m_i}} \right)^2 , \quad (8)$$

where m_i^{eff} is the effective B-band magnitude of the i -th supernova obtained after correcting the observed magnitude at redshift z for the supernova width-luminosity relation, σ_{m_i} is the error in magnitude at redshift z , and $m(z_i)$ is the apparent magnitude of the i -th supernova in our cosmological model.

For a given value of Ω_{0m} , the parameters to be estimated in our cosmology are \mathcal{M} , m and ϕ_0 . For all practical purposes, the quantity \mathcal{M} is a statistical nuisance parameter, and we marginalize over it (assuming a uniform prior) when we determine m , ϕ_0 . We perform the minimization while evolving the scalar field according to the equations of motion (2). For this purpose, we shall find it convenient to measure the parameter m in units of the quantity $\sqrt{8\pi G V_0/3}$ and the parameter ϕ_0 in units of the reduced Planck mass $\tilde{M}_P = \sqrt{3/8\pi G}$. (In these units we constrain the value of m using the prior $m \geq 0.1$. This condition is set to minimize numerical uncertainties and to avoid models which are virtually indistinguishable from Λ CDM).

In Figure 1, we show the confidence levels in the $m - \phi_0$ plane for the first potential described by Eq (1). We consider two cosmological models in which the current value of Ω_{0m} is 0.3 and 0.4 respectively. (The results for the two cases differ marginally.) We see that an enormous region of parameter space which corresponds to large values of m & ϕ_0 is disallowed in both models. The reason for this is simple, for large values $m \gg 1$, $\phi_0 \gg 1$, the scalar field potential decreases much too rapidly and the matter density never gets to reach its present value of $\Omega_{0m} = 0.3$ (left panel) or $\Omega_{0m} = 0.4$ (right panel). Also, from the confidence levels we see that at higher values of m (ϕ_0), the model provides a good fit to SNe data *only if* the corresponding value of ϕ_0 (m) is very small. (It should be noted that for asymptotically small values of either m or ϕ_0 the DDE model becomes virtually indistinguishable from Λ CDM.) From the value of the χ^2 at best-fit we see that higher values of Ω_{0m} seem to be slightly favoured by models of DDE. For $\Omega_{0m} = 0.3$, $\chi_{\text{dof}}^2 = 1.053$ for the best-fit model with $m = 0.73$, $\phi_0 = 0.18$, while

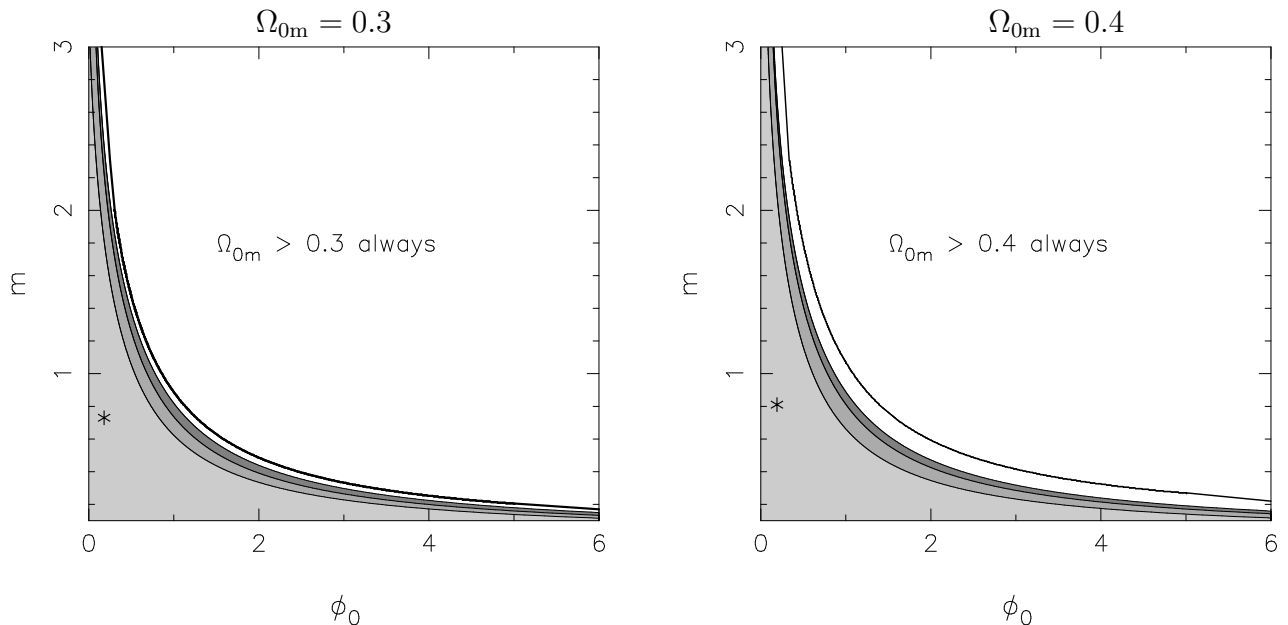


Figure 1. Confidence levels at 68.3% (light grey inner contour) 95.4% (medium grey contour) and 99.73% (dark grey outer contour) are shown in the m - ϕ_0 plane for the potential $V(\phi) = V_0 - \frac{1}{2}m^2\phi^2$. In the left panel, the present value of the matter density is $\Omega_{0m} = 0.3$, and in the right panel it is $\Omega_{0m} = 0.4$. The mass m is measured in units of $\sqrt{8\pi G V_0/3}$, and the initial field value, ϕ_0 , is in units of the reduced Planck mass $\tilde{M}_P = \sqrt{3/8\pi G}$. The best-fit point in each plot is marked by a star. The χ^2 per degree of freedom at the best-fit is 1.053 for $\Omega_{0m} = 0.3$ and 1.049 for $\Omega_{0m} = 0.4$. For both figures, in the region to the right of the thick solid line, parameter values are such that the matter density never reaches its present value, hence this region is disallowed by observations.

Λ CDM (for identical Ω_{0m}) has $\chi^2_{\text{dof}} = 1.054$. For $\Omega_{0m} = 0.4$, $\chi^2_{\text{dof}} = 1.049$ for the best-fit model $m = 0.81$, $\phi_0 = 0.19$, while $\chi^2_{\text{dof}}(\Lambda\text{CDM}) = 1.058$ (for identical Ω_{0m}). For lower values of Ω_{0m} , *e.g.* $\Omega_{0m} = 0.2$, Λ CDM is marginally favoured over the best-fit DDE.

In Figure 2, we examine this potential in greater detail by restricting ourselves to a smaller (and more probable) region in parameter space. In this region we show the lines of constant $\Delta T_{1/2}$, where $\Delta T_{1/2}$ is the time taken, as measured from the present epoch, for the potential to drop to half its maximum value: $V[\phi(t_0 + \Delta T_{1/2})] = V_0/2$. We see that, at the 95.4% confidence level, the minimum time at which this happens is $\Delta T_{1/2} \simeq 8$ Gyrs (if $H_0 \simeq 70$ km/sec/Mpc).

In Figure 3, we show the confidence levels in the $m - \phi_0$ plane for the second potential, described by Eq (6). The results in this case are qualitatively similar to those for the first potential. Here too, a large region of parameter space corresponding to higher values of m and ϕ_0 is disallowed because for these parameter values the matter density never reaches its present day value. Also, at higher values of m (ϕ_0), the model provides a good fit to SNe data only if the complementary parameter ϕ_0 (m) is very small. Again, for asymptotically small values of either m or ϕ_0 this decaying dark

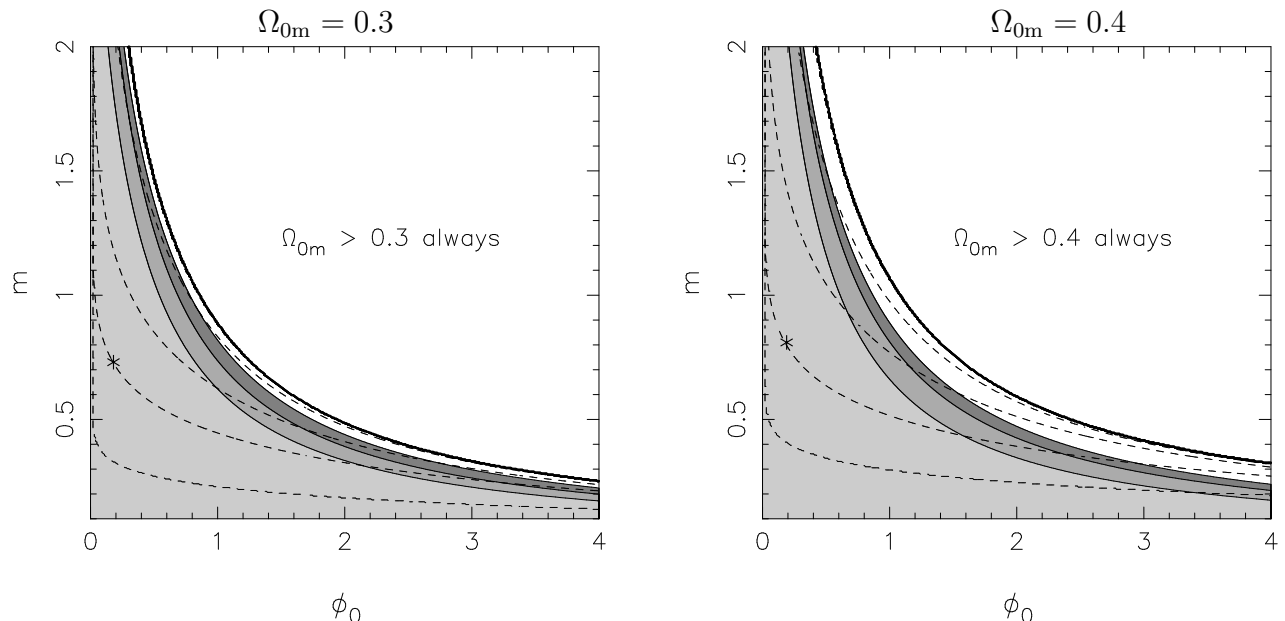


Figure 2. A magnified part of the Fig. 1 with (dashed) lines of constant $\Delta T_{1/2}$ added. $\Delta T_{1/2}$ is the time, measured from the present epoch, to when the DDE potential has dropped to half its maximum value: $V(\phi) = V_0/2$. The values of $\Delta T_{1/2}$ for the dashed curves (from top to bottom) are listed in Table 1 (from left to right). For both $\Omega_{0m} = 0.3$ and $\Omega_{0m} = 0.4$, the minimum time elapsed before the potential drops to half its maximum value is $\Delta T_{1/2} \simeq 0.6H_0^{-1} \simeq 8 \text{ Gyrs}$ ($H_0 = 70 \text{ km/s/Mpc}$) at the 95.4% confidence level. In the region to the right of the thick solid curve, parameter values are such that the matter density never reaches its present value. This region is therefore disallowed by observations.

Table 1. Time taken for the potential (1) to drop to half its maximum value.

Ω_{0m}	$\Delta T_{1/2}$ (Gyr)			
0.3	8.3	42.0	139.7	712.8
0.4	8.3	34.9	153.6	712.8

energy model becomes virtually indistinguishable from Λ CDM. For $\Omega_{0m} = 0.3$, we have $\chi_{\text{dof}}^2 = 1.050$ at the best-fit point of $m = 0.74$, $\phi_0 = 0.23$, and for $\Omega_{0m} = 0.4$, $\chi_{\text{dof}}^2 = 1.047$ for the best-fit model having $m = 0.79$, $\phi_0 = 0.24$. As in the previous analysis, Λ CDM is marginally preferred over the best-fit DDE for $\Omega_{0m} < 0.3$.

In Figure 4, we examine the results for this potential more closely by focusing on a smaller region in parameter space. In this region we show the lines of constant ΔT_{end} , which is the time left until the universe ceases to accelerate. (In terms of the deceleration parameter, $q(t_0 + \Delta T_{\text{end}}) = 0$.) We see that, at the 95.4% confidence level, the universe will continue accelerating *for at least* ~ 10 Gyrs. We also plot the lines of constant ΔT_{coll} , which is the time left until the universe collapses: $H(t_0 + \Delta T_{\text{coll}}) = 0$. We find

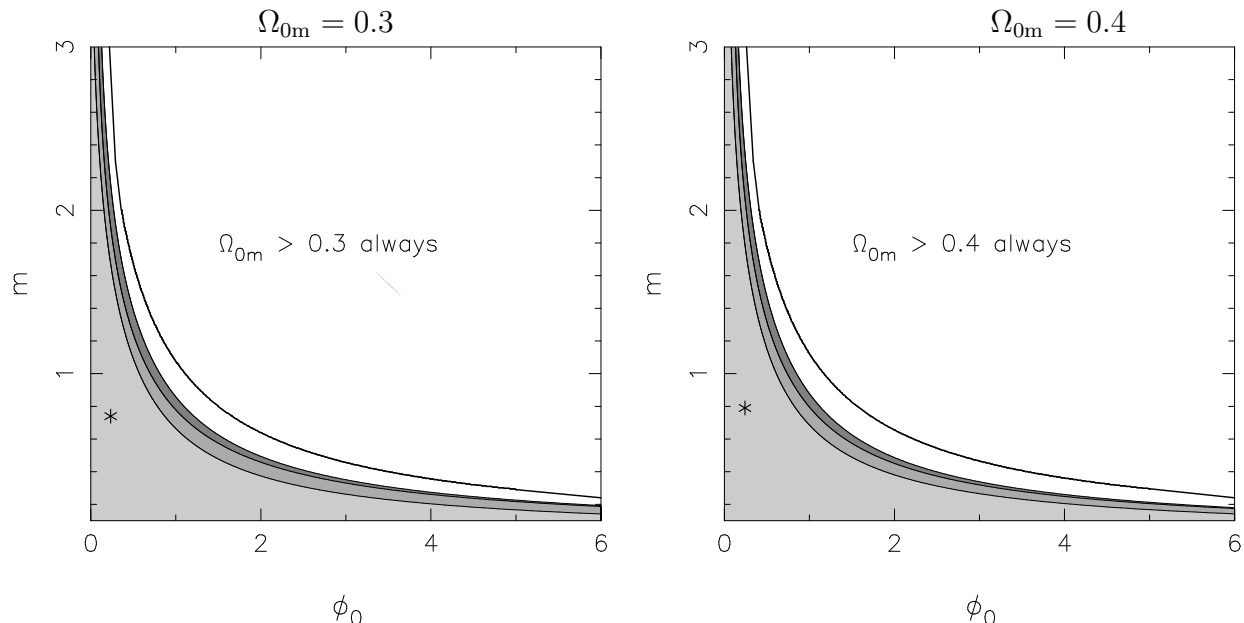


Figure 3. Confidence levels at 68.3% (light grey inner contour) 95.4% (medium grey contour) and 99.73% (dark grey outer contour) are shown in the m - ϕ_0 plane for the potential $V(\phi) = V_0 \cos(m\phi/\sqrt{V_0})$. Here m is in units of $\sqrt{8\pi G V_0/3}$, and ϕ_0 is in units of the reduced Planck mass $\tilde{M}_P = \sqrt{3/8\pi G}$. The best-fit point in each plot is marked as a star. The χ^2 per degree of freedom at the best-fit is 1.050 for $\Omega_{0m} = 0.3$ and 1.047 for $\Omega_{0m} = 0.4$. For both figures, in the region to the right of the thick solid line, parameter values are such that the matter density never reaches the present value, hence this region is disallowed by observations.

Table 2. Time until the end of acceleration, ΔT_{end} . Time until collapse, ΔT_{coll} .

Ω_{0m}	ΔT_{end} (Gyr)				ΔT_{coll} (Gyr)			
0.3	9.8	34.9	125.7	698.4	18.2	55.9	153.6	698.4
0.4	9.8	40.5	111.7	698.4	18.2	48.9	125.7	698.4

that the time to collapse is significantly larger than the time to the end of acceleration. At the 95.4% confidence level the lower bound to collapse is given by $\Delta T_{\text{coll}} \sim 18$ Gyrs for $m \lesssim 2$.

In Figure 5 we show the evolution of the deceleration parameter q and the matter density Ω_m for DDE driven by the cosine potential. We see that models allowed by the SNe data show a fairly large spread in the time span during which the universe accelerates. Although the time when the universe starts to accelerate is fairly close for different parameter values, the future epoch when the universe stops accelerating, $q(t_0 + \Delta T_{\text{end}}) = 0$, varies widely between models. The same can be said for the time when the universe collapses, which also varies significantly between models. Interestingly, for

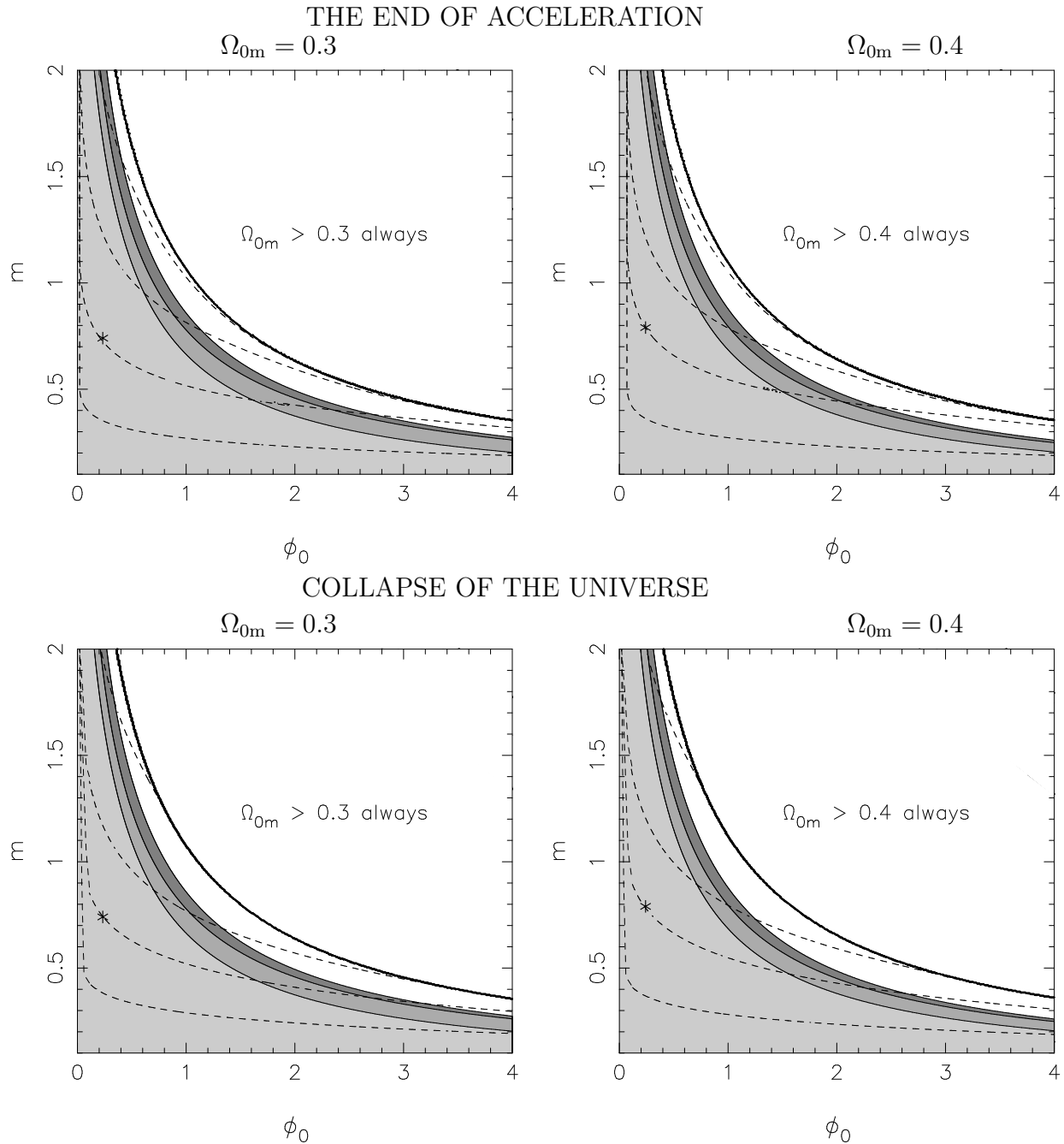


Figure 4. A magnified part of the Fig. 3 with (dashed) lines of constant ΔT_{end} (upper panel) and constant ΔT_{coll} (lower panel) added. In the upper panels, the time ΔT_{end} is measured from the present epoch to when the universe stops accelerating: $q(t_0 + \Delta T_{\text{end}}) = 0$. The values of ΔT_{end} for the dashed curves (from top to bottom) are listed in Table 2 (from left to right). For both $\Omega_{0m} = 0.3$ and $\Omega_{0m} = 0.4$, the minimum time taken for the deceleration parameter to rise to zero is $\Delta T_{\text{end}} \simeq 0.7H_0^{-1} \simeq 10$ Gyrs (at the 95.4% confidence level). For the lower panel, the values of ΔT_{coll} for the dashed curves (from top to bottom) are listed in Table 2 (from left to right). For both $\Omega_{0m} = 0.3$ and $\Omega_{0m} = 0.4$, the minimum time to collapse is $\Delta T_{\text{coll}} \simeq 1.3H_0^{-1} \simeq 18$ Gyrs at the 95.4% confidence level (we assume $H_0 = 70$ km/s/Mpc). In the region to the right of the thick solid curve the matter density never reaches its present value of $\Omega_{0m} = 0.3$ (left panel) and $\Omega_{0m} = 0.4$ (right panel), therefore this region is disallowed by observations.

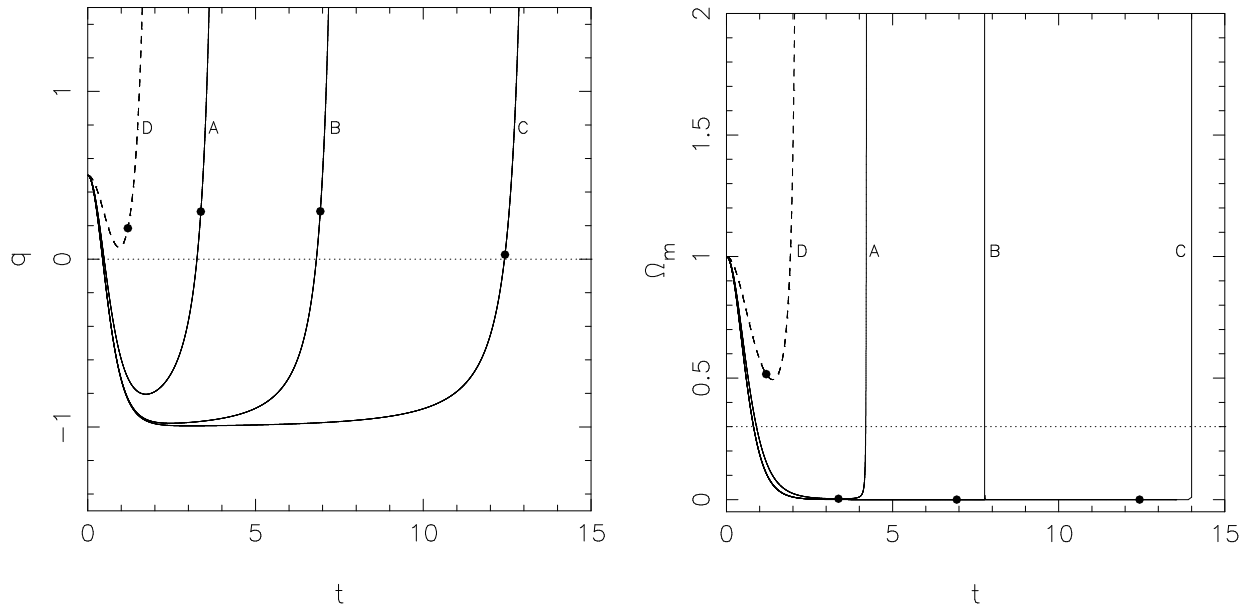


Figure 5. The evolution of the deceleration parameter q and the matter density Ω_m is shown for four different DDE models corresponding to different choices of m and ϕ_0 in the DDE potential $V(\phi) = V_0 \cos(m\phi/\sqrt{V_0})$ ($\Omega_{0m} = 0.3$). Time t is in units of $\sqrt{3/8\pi G V_0}$. The models have parameter values: $m = 1.0, \phi_0 = 0.6$ (A), $m = 1.0, \phi_0 = 0.2$ (B), $m = 0.74, \phi_0 = 0.23$ (C), $m = 1.0, \phi_0 = 1.2$ (D). Models A,B,C are allowed by supernova observations at the 95.4% confidence level. The dashed line D in both panels shows the time evolution of q and Ω_m for a DDE model with $m = 1.0, \phi_0 = 1.2$. This model is disallowed by observations since the matter density always remains larger than 0.3 (see figure 4). The horizontal dotted line in the left panel ($q = 0$) divides this panel into two regions. In the upper region $q > 0$ and the universe decelerates, whereas $q < 0$ in the lower region in which the universe accelerates. The points of intersection of $q = 0$ with A,B,C show the commencement and end of the acceleration epoch in these models. The horizontal dotted line in the right panel marks the present epoch when $\Omega_{0m} = 0.3$. The solid circles in both left and right panels show the epoch when the potential energy of the scalar field falls to zero. Note that this occurs *after* the universe stops accelerating.

most allowed models, Ω_m drops to exceedingly small values before the potential becomes sufficiently negative to initiate collapse.

4. Discussion and Conclusions

In this paper we have examined the possibility that the dark energy responsible for the acceleration of the universe decays with time. Models of decaying dark energy have recently been proposed in connection with supergravity and M-theory [17, 19]. Our analysis of decaying dark energy is carried out for two potentials. The first, defined in

Eq. (1), is a faithful local representation of a potential in the vicinity of its maximum value and therefore describes a very general situation in which the potential responsible for dark energy has one (or more) maxima. (In fact Eq. (1) generically describes a dark energy potential near its maximum value.) We follow the expansion dynamics of the universe as the scalar field rolls down this potential and, for a large region in parameter space, compare the resulting value of the luminosity distance with observations of high redshift type Ia supernovae. Our results show that the decaying dark energy model is consistent with SNe data at reasonably small values of m and ϕ_0 . We see that the potential declines to half its maximum value on a time scale not shorter than $\Delta T_{1/2} \simeq 0.6H_0^{-1}$ ($\Delta T_{1/2} \simeq 8$ Gyrs if $H_0 = 70$ km/s/Mpc) at the 95.4% confidence level.

Next, we extend this analysis to the cosine potential (6) which holds the possibility of becoming negative as the field evolves. The cosine potential has some important new features not shared by ‘tracker’ quintessence potentials. Due to its ability to grow more negative with time, the cosine potential allows for the eventual cancellation between the positive density of dark matter and the negative density of dark energy, because of which the universe collapses at late times. It is important to note that collapse occurs generically even in the case of a *spatially flat* universe, as pointed out in [11]. Comparing our model universe with supernova observations we determine the time, measured from the present epoch, to when the universe collapses. At the 95.4% confidence level, the time to collapse can never be smaller than $\Delta T_{\text{coll}} \simeq 1.3H_0^{-1}$ if $m \lesssim 2$ ($\Delta T_{\text{coll}} \simeq 18$ Gyrs for $H_0 = 70$ km/s/Mpc).

For both dark energy models (1) & (6) our current accelerating epoch is a transient which ends once the dark energy density falls below a certain critical value. For the cosine potential the time until the acceleration of the universe ends ($q(t_0 + \Delta T_{\text{end}}) = 0$) is always larger than $\Delta T_{\text{end}} \simeq 0.7H_0^{-1}$, which is about 10 Gyrs if $H_0 = 70$ km/s/Mpc (at the 95.4% CL). Thus a DDE universe will continue accelerating for at least 10 Gyrs if $H_0 = 70$ km/s/Mpc and $\Omega_{\text{om}} \simeq 0.3$. About 8 Gyrs later, this universe will collapse and head towards a ‘big crunch’ singularity.

Finally, it is worth pointing out that the decaying dark energy models discussed in this paper are devoid of horizons, which could be an attractive feature from the viewpoint of string/M-theory [7].

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