

Statistical Isotropy violation of CMB Polarization sky due to Doppler boost

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Abstract

In the frame of a moving observer, the Cosmic Microwave Background (CMB) fluctuation exhibits violation of Statistical Isotropy (SI). The SI violation effect from our local motion on CMB temperature fluctuation has been measured in the recent Planck results [1]. We calculate the effect of our local motion with velocity ($\beta \equiv |\mathbf{v}|/c = 1.23 \times 10^{-3}$) on CMB polarization field. The Lorentz transformation of the polarization field leads to aberration in the direction of incoming photons and also modulation of the Stokes parameters, which results in mixing of power between different CMB multipoles. We show that for small values of β , the effect on the angular power spectra that corresponds to the diagonal terms in the spherical harmonic space is at $O(\beta^2)$. But non-zero off-diagonal terms at the linear order in β could provide a measurable signature of SI violation in the Bipolar Spherical Harmonic (BipoSH) representation. We also calculate the measurability of β from polarization maps from experiments like Planck and PRISM. It is possible to measure β from the ideal, cosmic variance limited BipoSH spectra of EE , TE , BB , but not in EB and TB . With the instrumental noise of and angular resolution of Planck, it is not possible to measure β from BipoSH spectra of polarization. PRISM can measure β with high significance in both EE and TE BipoSH spectra, but not in BB , EB and TB BipoSH spectra.

1 Introduction

Cosmic Microwave Background (CMB) is a very important probe of our universe. It has opened an era of precision cosmology with a number of important experiments like COBE, WMAP, Planck, BOOMERanG, ACT, SPT and many others. Recent measurements from Planck of the CMB temperature power spectrum matches well with the minimal Λ CDM model [2]. Another observable is the pattern of linear polarization on CMB sky. Study of CMB polarization maps will provide more information about lensing, inflation models and gravitational waves.

The measurement of the CMB dipole implies that we observe the CMB from the reference frame moving at $v = 369.0 \pm 0.9 \text{ km s}^{-1}$ in the direction, $(l, b) = (263.99^\circ, 48.26^\circ)$ in Galactic coordinates

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[3]. This is an unavoidable systematic effect for any CMB observations. The CMB measurements have achieved the sensitivity and resolution where the subtle effect of this motion on the Statistical isotropy (SI) of correlations the fluctuations at multipoles beyond the dipole are also measurable. Recently, Planck [4] has also measured the SI violation induced by the local motion. CMB polarization map is also an important probe for measuring the presence of any Non-Statistical Isotropy (nSI) features of CMB sky. A measure of SI violation are the non zero coefficients of Bipolar Spherical Harmonics (BipoSH) representation introduced in CMB temperature measurements by Hajian and Souradeep [5] and extended to CMB polarization by Basak, Hajian and Souradeep [6].

We study the effect of our local motion on the CMB polarization field. In this work we estimate the signature of SI violation in the CMB polarization field due to the Lorentz transformation corresponding to our moving reference frame (with $\beta \equiv |\mathbf{v}|/c = 1.23 \times 10^{-3}$). Applied to the two point correlations function of polarization field, this transform leads to a mild corrections at $O(\beta^2)$ to the angular power spectra C_l^{EE} , C_l^{TE} , C_l^{BB} and C_l^{TT} values. However, it induces induces mixing of power at different CMB multipoles at linear in β and hence has a much stronger signature in SI violation. The effect of Doppler boost on covariance matrix of temperature and polarization has been studied earlier by Challinor and Leeuwen [7] where they compute the effect on diagonal and off-diagonal terms of the covariance matrix specifically white spectra of C_l . We present here the effect of Doppler boost on polarization comprehensively in the BipoSH representation of SI violation for the best fit Λ CDM model. We also study the statistics of BipoSH coefficients for polarization and estimate the measurability of β from Planck and PRISM using BipoSH coefficients by using minimum variance estimator [8, 9].

The paper is organised as follows. In Section 2, we briefly review some relevant background concepts and results used in obtaining our results. In Section 3, we calculate the effect of local motion on C_l and BipoSH coefficients. Statistics of BipoSH coefficients for polarization are derived in Section 4. In Section 5, we estimate the detectability of β from the BipoSH coefficients of polarization for Planck and PRISM using minimum variance estimator. Discussion and conclusion of our work are provided in Section 6.

2 Brief review of relevant concepts and basic results

In this section we briefly review some relevant concepts and basic results, which we need to use for this work. We discuss the basic result of Lorentz transformation on polarization vector, formalism of CMB polarization and BipoSH representation.

2.1 Lorentz transformation of Stokes parameter

Polarization of the CMB photons at a direction \hat{n} are expressed in terms of Stokes parameters (I, Q, U, V) , with respect to the set of basis vectors (\hat{e}_1, \hat{e}_2) defined on the local sky patch in the direction, \hat{n} . Hence the polarization of the Electromagnetic wave is defined by the direction of the electric field, we can define the Electric field vector of the wave as $\vec{\mathcal{E}}(\hat{n}) = \mathcal{E}_1 \hat{e}_1 + \mathcal{E}_2 \hat{e}_2$, where $(\hat{e}_1, \hat{e}_2, \hat{n})$ forms a right handed orthonormal system. The Stokes parameters can be define as,

$$\begin{aligned}
 I &= |\mathcal{E}_1|^2 + |\mathcal{E}_2|^2, \\
 Q &= |\mathcal{E}_1|^2 - |\mathcal{E}_2|^2, \\
 U &= 2\text{Re}(\mathcal{E}_1^* \mathcal{E}_2), \\
 V &= 2\text{Im}(\mathcal{E}_1^* \mathcal{E}_2).
 \end{aligned} \tag{1}$$

This Stokes parameters can be also be expressed in terms of 2×2 intensity tensor, $\mathcal{J}_{ij} = \bar{\mathcal{J}}_\nu(\hat{e}_i \otimes \hat{e}_j)$, is defined in the dimension of mean intensity ($\bar{\mathcal{J}}_\nu = \delta\mathcal{J}_\nu/(\mathcal{J}_\nu)_0$).

$$\begin{aligned} I &= \frac{1}{2}(\mathcal{J}_{11} + \mathcal{J}_{22}), \\ Q &= \frac{1}{2}(\mathcal{J}_{11} - \mathcal{J}_{22}), \\ U &= \frac{1}{2}(\mathcal{J}_{12} + \mathcal{J}_{21}), \\ V &= \frac{-i}{2}(\mathcal{J}_{12} - \mathcal{J}_{21}). \end{aligned} \tag{2}$$

where, frequency dependent intensity of the CMB field is related to CMB temperature by Planckian distribution.

$$\mathcal{J}_\nu(\hat{n}) = \frac{2h\nu^3}{c^2} \frac{1}{(\exp[h\nu/kT(\hat{n})] - 1)}. \tag{3}$$

The relation between the intensity fluctuation, $\delta\mathcal{J}_\nu$ and temperature fluctuation, δT is

$$\mathcal{J}_\nu(\hat{n}) - (\mathcal{J}_\nu)_0(\hat{n}) \equiv \delta\mathcal{J}_\nu(\hat{n}) = \left. \frac{d\mathcal{J}_\nu}{dT} \right|_{T_0} \delta T(\hat{n}) + \left. \frac{1}{2} \frac{d^2\mathcal{J}_\nu}{dT^2} \right|_{T_0} (\delta T)^2, \tag{4}$$

CMB polarization is induced by the Thompson scattering of CMB photons with the electrons, which generates only linear polarization, i.e. $V = 0$. Hence, we consider only I, Q, U in the further discussions.

Performing a rotation by an angle ϕ on this orthonormal basis, $(\hat{e}_1, \hat{e}_2, \hat{n})$, around \hat{n} , leads to change in (\hat{e}_1, \hat{e}_2) as,

$$\begin{aligned} \hat{e}_1^R &= \cos\phi \hat{e}_1 + \sin\phi \hat{e}_2, \\ \hat{e}_2^R &= -\sin\phi \hat{e}_1 + \cos\phi \hat{e}_2, \end{aligned} \tag{5}$$

where we represent the expressions in the rotated framed by a superscript R . This implies that the I, Q and U defined in eq.(2), also transform as [10, 11],

$$\begin{aligned} I^R &= I, \\ Q^R &= Q \cos 2\phi - U \sin 2\phi, \\ U^R &= Q \sin 2\phi + U \cos 2\phi. \end{aligned} \tag{6}$$

Hence $(Q \pm iU)$ transforms as a spin two variable,

$$(Q \pm iU)^R = e^{\mp i2\phi}(Q \pm iU). \tag{7}$$

Defining Stokes parameter measured by a moving observer is related to the stationary observer through Lorentz transformation. There are two different effects on the CMB polarization field due to the motion of the observer

- i. Aberration in the direction, \hat{n} , of incoming photons leading to remapping of the Stokes parameter on the sky.

ii. Modulation of Stokes parameters.

To obtain the Lorentz transformation of CMB polarization field, let \mathbf{S}' be the CMB rest frame. The observer reference frame \mathbf{S} is moving with a velocity $\vec{\beta} = \beta_0 \hat{\beta}$, with $\beta_0 = 1.23 \times 10^{-3}$ as measured by CMB dipole [3]. Lorentz transformation of CMB photons leads to aberration in the direction of incoming photon. The aberration in the direction, \hat{n} , results in a remapping of $\hat{n} \rightarrow \hat{n}'$ by the relation [4, 7],

$$\hat{n} = \frac{\hat{n}' \cdot \hat{\beta} + \beta}{1 + \hat{n}' \cdot \hat{\beta}} \hat{\beta} + \frac{\hat{n}' - \hat{n}' \cdot \hat{\beta}}{\gamma(1 + \hat{n}' \cdot \hat{\beta})}. \quad (8)$$

The Lorentz transformation of CMB temperature field is [4],

$$\delta T(\hat{n}) = \delta T' \left(\hat{n} - \nabla(\hat{n} \cdot \hat{\beta}) \right) \left(1 + \beta \cos \theta \right), \quad (9)$$

and the relation between temperature fluctuations (excluding the dipole term) and intensity fluctuations given by,

$$\delta T_I \equiv \frac{\delta \bar{\mathcal{J}}_\nu(\hat{n})}{d\bar{\mathcal{J}}_\nu/dT \Big|_{T_0}} = \frac{\delta T'}{T_0} \left(\hat{n} - \nabla(\hat{n} \cdot \hat{\beta}) \right) \left(1 + b_\nu \beta \cos \theta \right), \quad (10)$$

where, δT_I is the temperature fluctuation obtained from CMB intensity fluctuation, θ is the angle between \hat{n} and $\hat{\beta}$ and b_ν is the frequency dependent effect on Doppler boost given by,

$$b_\nu = \frac{\nu}{\nu_0} \coth \left(\frac{\nu}{2\nu_0} \right) - 1, \quad (11)$$

with $\nu_0 = 57$ GHz.

Eq.(10) relates fluctuations in the between CMB intensity and temperature. Defining eq.(2) in terms of temperature fluctuation, δT_I by $\mathcal{T}_{ij} = \delta T_I(\hat{e}_i \otimes \hat{e}_j)$, we get,

$$\begin{aligned} T_I &= \frac{1}{2}(\mathcal{T}_{11} + \mathcal{T}_{22}), \\ Q &= \frac{1}{2}(\mathcal{T}_{11} - \mathcal{T}_{22}), \\ U &= \frac{1}{2}(\mathcal{T}_{12} + \mathcal{T}_{21}). \end{aligned} \quad (12)$$

This provides the expression of Stokes parameters in terms of temperature fluctuation. So, under Lorentz transformation Q, U transforms as,

$$\begin{aligned} Q(\hat{n}) &= Q' \left(\hat{n} - \nabla(\hat{n} \cdot \hat{\beta}) \right) \left(1 + b_\nu \beta \cos \theta \right), \\ U(\hat{n}) &= U' \left(\hat{n} - \nabla(\hat{n} \cdot \hat{\beta}) \right) \left(1 + b_\nu \beta \cos \theta \right). \end{aligned} \quad (13)$$

These are the Lorentz transformation of Stokes parameters with a frequency dependence given by b_ν . In this paper we choose the frequency channel $\nu = 217$ GHz for which $b_\nu \approx 3$. This is the best Planck frequency channel for this search due to high resolution and low noise for the estimation of β . These equations relates CMB observables in the CMB rest frame (\mathbf{S}') with moving observer frame (\mathbf{S}). Statistical Isotropy(SI) in \mathbf{S}' still leads to observable Non-Statistical Isotropy (nSI) in \mathbf{S} . An alternative derivation of the Lorentz transformation of Stokes parameter from the more basic transformation of electric field is discussed in Appendix. A.

2.2 Measures of the CMB polarization field

The CMB temperature anisotropy sky map $\Delta T(\hat{n})$ can be expanded in the orthonormal space of Spherical Harmonic (SH) functions,

$$\Delta T(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n}). \quad (14)$$

Similarly, the Stokes parameters mentioned in eq.(1) for the CMB polarization field, can also be expanded in terms of Spin Weighted Spherical Harmonic (SWSH) basis ${}_{\pm 2}Y_{lm}$, due to its transformation property mentioned in eq.(7). Defining $Q \pm iU$ as [10, 11, 12],

$${}_{\pm}X(\hat{n}) = Q(\hat{n}) \pm iU(\hat{n}). \quad (15)$$

Expressing $X(\hat{n})$, in the SWSH basis [10, 11],

$${}_{\pm}X(\hat{n}) = \sum_{l=2}^{\infty} \sum_{m=-l}^{+l} {}_{\pm}X_{lm} {}_{\pm 2}Y_{lm}(\hat{n}), \quad (16)$$

and in parity eigenstates we can write,

$${}_{\pm}X(\hat{n}) = \sum_{l=2}^{\infty} \sum_{m=-l}^{+l} (E_{lm} \pm iB_{lm}) {}_{\pm 2}Y_{lm}(\hat{n}), \quad (17)$$

where,

$${}_{\pm}X_{lm} = E_{lm} \pm iB_{lm}. \quad (18)$$

Under parity transformation, E , B and ${}_sY_{lm}$ transform as,

$$\begin{aligned} {}_sY_{lm} &\rightarrow (-1)^l {}_{-s}Y_{lm}, \\ E_{lm} &\rightarrow (-1)^l E_{lm}, \\ B_{lm} &\rightarrow (-1)^{l+1} B_{lm}. \end{aligned} \quad (19)$$

For SI field, the angular power spectra of CMB polarization are related to the two point correlations of E and B , and also the cross correlation between E and T .

$$\left\langle Y_{lm}^* Y'_{l'm'} \right\rangle = C_l^{YY'} \delta_{ll'} \delta_{mm'}, \quad (20)$$

where Y and Y' can be taken to be E , B and T . Under the assumption of SI, the covariance matrix formed by $\langle Y_{lm}^* Y'_{l'm'} \rangle$, is diagonal and non zero only for EE , BB , and TE . When all processes respect parity invariance, the diagonal elements correspond to the angular power spectrum, defined as,

$$\begin{aligned} \left\langle a_{lm}^* a_{l'm'} \right\rangle &= C_l^{TT} \delta_{ll'} \delta_{mm'}, \\ \left\langle E_{lm}^* E_{l'm'} \right\rangle &= C_l^{EE} \delta_{ll'} \delta_{mm'}, \\ \left\langle B_{lm}^* B_{l'm'} \right\rangle &= C_l^{BB} \delta_{ll'} \delta_{mm'}, \\ \left\langle a_{lm}^* E_{l'm'} \right\rangle &= C_l^{TE} \delta_{ll'} \delta_{mm'}. \end{aligned} \quad (21)$$

2.3 BipoSH representation for temperature and polarization

In this section, we discuss the Non-SI (nSI) part of the covariance matrix for polarization and temperature in the Bipolar Spherical Harmonics (BipoSH) representation [5, 6]. Under the assumption that, the temperature fluctuations are Gaussian random field with zero mean, we can express the statistics of this field by the two point correlation function. In the full generality the two point correlation of SH coefficients of the CMB anisotropy $\langle a_{l_1 m_1} a_{l_2 m_2}^* \rangle$ in SH space can be expanded in the tensor product basis of two SH spaces as,

$$\langle a_{l_1 m_1} a_{l_2 m_2}^* \rangle = \sum_{LM} A_{l_1 l_2}^{LM} (-1)^{m_2} C_{l_1 m_1 l_2 - m_2}^{LM}, \quad (22)$$

Here $A_{l_1 l_2}^{LM}$ are called the Bipolar Spherical Harmonics (BipoSH) coefficients [5] and $C_{l_1 m_1 l_2 m_2}^{LM}$ are the Clebsch-Gordan coefficients. For nSI case, non-zero BipoSH coefficients are a complete representation of SI violation.

The statistics of the polarization field can be obtained by calculating the two point correlation function. Similar to temperature, BipoSH coefficients can be also generalised to include the CMB polarization [6]. The two point correlation between E , B and T can be defined as,

$$\langle Y_{l_1 m_1} Y_{l_2 m_2}^* \rangle = \sum_{LM} A_{l_1 l_2 | Y Y'}^{LM} (-1)^{m_2} C_{l_1 m_1 l_2 - m_2}^{LM}, \quad (23)$$

where Y can be any one of E , B and T . Here similar to the temperature $A_{l_1 l_2 | Y Y'}^{LM}$, are the BipoSH coefficients for polarization. Under the assumption of SI, the covariance matrix in the SH space is diagonal, which implies only $A_{l_1 l_2 | Y Y'}^{00} \neq 0$. Measurement of non-zero BipoSH values implies violation of SI.

2.4 Minimum variance estimator

We discuss here the reconstruction noise for the minimum variance estimator of the BipoSH coefficients of CMB polarization map. Minimum variance estimator is used for lensing reconstruction by Hu and Okamoto [8], Hanson et. al. [9]. It is also used by Planck for estimating several effects like lensing, SI violation and measuring β from temperature map.

BipoSH coefficients for temperature and polarization of a CMB field, can be written as,

$$\hat{A}_{l_1 l_2 | X X'}^{LM} = A_{l_1 l_2 | X X'}^{LM} + \alpha_{LM} S_{l_1 l_2 | X X'}^L, \quad (24)$$

here, $\hat{A}_{l_1 l_2 | X X'}^{LM}$ is the observed BipoSH coefficient and $A_{l_1 l_2 | X X'}^{LM}$ is the BipoSH coefficient for a SI temperature or polarization field, which on averaging over several ensembles is zero. $\alpha_{LM} S_{l_1 l_2 | X X'}^L$ is the source of SI violation with $S_{l_1 l_2 | X X'}^L$ as the shape factor and α_{LM} , strength of SI violation arising due to inevitable effects like weak lensing and Doppler boosting etc. For weak lensing, α_{LM} is equivalent to the lensing potential, ϕ_{LM} , and for Doppler boost, α_{LM} is equivalent to local velocity β_{1M} for the single CMB sky, $A_{l_1 l_2 | X X'}^{1M}$ is non-zero. To measure the β_{1M} , we define estimator, $\hat{\beta}_{1M}$ for $L = 1$ as,

$$\hat{\beta}_{1M} = \sum_{l_1 l_2} \frac{A_{l_1 l_2 | X X'}^{1M}}{S_{l_1 l_2 | X X'}^\beta} + \beta_{1M}, \quad (25)$$

To estimate the significance of signal, β from the estimator, $\hat{\beta}$, we need to evaluate the variance of the estimator for SI sky. Detection of any statistically significant signal is only possible when

variance of the first term in L.H.S. of eq.(25) is smaller than $|\beta|^2$. Taking the product of eq.(25), with its complex conjugate, we get,

$$(\hat{\sigma}_\beta)^2 \equiv \left\langle \hat{\beta}_{1M}^* \hat{\beta}_{1M'} \right\rangle = \sum_{l_1 l_2 l_3 l_4} \frac{\left\langle A_{l_3 l_4 | X X'}^{*1M} A_{l_1 l_2 | X X'}^{1M} \right\rangle}{S_{l_1 l_2 | X X'}^\beta S_{l_3 l_4 | X X'}^\beta} + |\beta_1|^2, \quad (26)$$

$$(\hat{\sigma}_\beta)^2 = N_\beta + |\beta_1|^2.$$

where,

$$N_\beta = \sum_{l_1 l_2 l_3 l_4} \frac{\left\langle A_{l_3 l_4 | X X'}^{*1M} A_{l_1 l_2 | X X'}^{1M} \right\rangle}{S_{l_1 l_2 | X X'}^\beta S_{l_3 l_4 | X X'}^\beta}. \quad (27)$$

N_β is the reconstruction noise. In this work we first computed the reconstruction noise for CMB polarization map for BipoSH coefficients. The expressions for N_β can be obtained by using the variance of BipoSH coefficients for CMB polarization sky, computed in Sec.4 . But due to weak signal strength, it is important to minimize the value of N_β further. To arrive at the minimum variance estimator, we rewrite eq.(25) as,

$$\hat{\beta}_{1M} = \sum_{l'} w_{l_1 l_2}^1 \frac{A_{l' | X X'}^{1M}}{S_{l' | X X'}^\beta} + \beta_{1M}, \quad (28)$$

where $w_{l_1 l_2}^1$ are the weights, which satisfies the constraint $\sum_{l_1 l_2} w_{l_1 l_2}^1 = 1$. This weight factors should be chosen such that, it minimises the reconstruction noise. The variance of the eq.(28) is

$$(\hat{\sigma}_\beta)^2 = \sum_{l_1 l_2} (w_{l_1 l_2}^1)^2 \frac{\left\langle A_{l_1 l_2 | X X'}^{*1M} A_{l_1 l_2 | X X'}^{1M} \right\rangle}{S_{l_1 l_2 | X X'}^\beta S_{l_1 l_2 | X X'}^\beta} + |\beta_1|^2, \quad (29)$$

Using the Lagrange multiplier method, we obtain the weight factors $w_{l_1 l_2}^1$ as,

$$w_{l_1 l_2}^1 = \left(\frac{(S_{l_1 l_2 | X X'}^\beta)^2}{\left\langle A_{l_1 l_2 | X X'}^{*1M} A_{l_1 l_2 | X X'}^{1M} \right\rangle} \right) \left[\sum_{l_1 l_2} \frac{(S_{l_1 l_2 | X X'}^\beta)^2}{\left\langle A_{l_1 l_2 | X X'}^{*1M} A_{l_1 l_2 | X X'}^{1M} \right\rangle} \right]^{-1}, \quad (30)$$

that minimizes the reconstruction noise N_β . Putting the value of weight factor in eq.(29), we get,

$$\begin{aligned} (\hat{\sigma}_\beta)^2 &= \left[\sum_{l_1 l_2} \frac{S_{l_1 l_2 | X X'}^\beta S_{l_1 l_2 | X X'}^\beta}{\left\langle A_{l_1 l_2 | X X'}^{*1M} A_{l_1 l_2 | X X'}^{1M} \right\rangle} \right]^{-1} + |\beta_1|^2, \\ (\hat{\sigma}_\beta)^2 &= N_\beta + |\beta_1|^2, \end{aligned} \quad (31)$$

where, reconstruction noise is defined as,

$$N_\beta = \left[\sum_{l_1 l_2} \frac{S_{l_1 l_2 | X X'}^\beta S_{l_1 l_2 | X X'}^\beta}{\left\langle A_{l_1 l_2 | X X'}^{*1M} A_{l_1 l_2 | X X'}^{1M} \right\rangle} \right]^{-1}, \quad (32)$$

and the minimum variance estimator is,

$$\hat{\beta}_{1M} = \sum_{l_1 l_2} \frac{S_{l_1 l_2}^\beta \hat{A}_{l_1 l_2 | XX'}^{1M}}{\left\langle A_{l_1 l_2 | XX'}^{*1M} A_{l_1 l_2 | XX'}^{1M} \right\rangle} N_\beta. \quad (33)$$

In Sec. 5 we use the above expression to infer the prospects of detecting and measuring the Doppler boost effect from polarization maps of Planck and future proposed mission PRISM and also estimate the ultimate cosmic variance limits.

3 Effect of boost on CMB covariance

In the previous section, we have discussed the power spectra and Bipolar representation of temperature and polarization field. The angular power spectra are the sole prediction of the isotropic Cosmological model. So, to determine the correct Cosmological model it is necessary to account for the Lorentz transformation of C_l from CMB rest frame to the frame of the moving observer. As the measured value of $\beta \approx 10^{-3}$, we calculate the only leading order effect of Doppler boost on C_l . From the calculated Lorentz transformation of Stokes parameters in eq.(78) for small values of β , we can relate \tilde{X} the moving frame with the CMB rest frame as,

$$\pm \tilde{X}(\hat{n}) = \pm X \left(\hat{n} - \nabla(\hat{n} \cdot \hat{\beta}) \right) \left(1 + b_\nu \beta \cos \theta \right). \quad (34)$$

We notationally represent Doppler boosted quantities with over tilde. The Taylor series expansion of eq.(34) yields,

$$\pm \tilde{X}(\hat{n}) = \left(1 + b_\nu \beta \cos \theta \right) \left(\pm X(\hat{n}) - \nabla_{i\pm} X(\hat{n}) \nabla^i (\hat{n} \cdot \hat{\beta}) + \frac{1}{2} \nabla_j \nabla_{i\pm} X(\hat{n}) \nabla^i (\hat{n} \cdot \hat{\beta}) \nabla^j (\hat{n} \cdot \hat{\beta}) \right). \quad (35)$$

Defining β in SH basis, we have,

$$\beta_{1M} = \int \beta \cdot \hat{n} Y_{1M}^*(\hat{n}) d\hat{n}. \quad (36)$$

Next we rewrite eq.(35) in SWSH retaining up to terms in $O(\beta^2)$,

$$\begin{aligned} \pm \tilde{X}_{l_1 m_1} = & \pm X_{l_1 m_1} - \sum_{l_2 m_2} \sum_{m_3} \int d\hat{n} \beta_{1m_3 \pm} X_{l_2 m_2 \pm 2} Y_{l_1 m_1}^*(\hat{n}) \nabla_{i \pm 2} Y_{l_2 m_2}(\hat{n}) \nabla^i Y_{1m_3}(\hat{n}) \\ & + \frac{1}{2} \sum_{l_2 m_2} \sum_{m_3 m_4} \int d\hat{n} \pm 2 \beta_{1m_4} \beta_{1m_3 \pm} X_{l_2 m_2} Y_{l_1 m_1}^*(\hat{n}) \nabla_j \nabla_{i \pm 2} Y_{l_2 m_2}(\hat{n}) \nabla^i Y_{1m_3}(\hat{n}) \nabla^j Y_{1m_4}(\hat{n}) \\ & + b_\nu \sum_M \sum_{l_2 m_2} \int d\hat{n} \beta_{1M \pm} X_{l_2 m_2} Y_{1M}(\hat{n}) \pm 2 Y_{l_1 m_1}^*(\hat{n}) \pm 2 Y_{l_2 m_2}(\hat{n}) \\ & - b_\nu \sum_{M m_3} \sum_{l_2 m_2} \int d\hat{n} \beta_{1M} \beta_{1m_3 \pm} X_{l_2 m_2} Y_{1M}(\hat{n}) \pm 2 Y_{l_1 m_1}^*(\hat{n}) \nabla_{i \pm 2} Y_{l_2 m_2}(\hat{n}) \nabla^i Y_{1m_3}^*(\hat{n}). \end{aligned} \quad (37)$$

Further, we define the quantities,

$$\begin{aligned}\pm 2G_{l_1 l_2 1}^{m_1 M m_2 m_3} &= \int d\hat{n} Y_{1M}(\hat{n})_{\pm 2} Y_{l_1 m_1}^*(\hat{n}) \nabla_i \pm 2 Y_{l_2 m_2}(\hat{n}) \nabla^i Y_{1m_3}(\hat{n}), \\ \pm 2H_{l_1 l_2 1}^{m_1 m_2 M} &= \int d\hat{n} Y_{1M}(\hat{n})_{\pm 2} Y_{l_1 m_1}^*(\hat{n})_{\pm 2} Y_{l_2 m_2}(\hat{n}),\end{aligned}\tag{38}$$

and as defined in [12],

$$\begin{aligned}\pm 2I_{l_1 l_2 1}^{m_1 m_2 m_3} &= \int d\hat{n} \pm 2 Y_{l_1 m_1}^*(\hat{n}) \nabla_i \pm 2 Y_{l_2 m_2}(\hat{n}) \nabla^i Y_{1m_3}(\hat{n}), \\ \pm 2J_{l_1 l_2 1}^{m_1 m_3 m_2 m_4} &= \int d\hat{n} \pm 2 Y_{l_1 m_1}^*(\hat{n}) \nabla_j \nabla_i \pm 2 Y_{l_2 m_2}(\hat{n}) \nabla^i Y_{1m_3}(\hat{n}) \nabla^j Y_{1m_4}^*(\hat{n}).\end{aligned}\tag{39}$$

With these definitions, eq.(37) becomes,

$$\begin{aligned}\pm \tilde{X}_{l_1 m_1} &= \pm X_{l_1 m_1} - \sum_{l_2 m_2} \sum_{m_3} \pm X_{l_2 m_2} \beta_{1m_3} \pm 2I_{l_1 l_2 1}^{m_1 m_2 m_3} + \frac{1}{2} \sum_{l_2 m_2} \sum_{m_3 m_4} \pm X_{l_2 m_2} \beta_{1m_3} \beta_{1m_4} \pm 2J_{l_1 l_2 1}^{m_1 m_3 m_2 m_4} \\ &+ b_\nu \sum_M \sum_{l_2 m_2} \pm X_{l_2 m_2} \beta_{1M} \pm 2H_{l_1 l_2 1}^{m_1 m_2 M} - b_\nu \sum_{M m_3} \sum_{l_2 m_2} \pm X_{l_2 m_2} \beta_{1M} \beta_{1m_3} \pm 2G_{l_1 l_2 1}^{m_1 M m_2 m_3}.\end{aligned}\tag{40}$$

The power spectrum is obtained by taking the product of eq.(40) with its complex conjugate. We calculate the effect of Lorentz transformation on both diagonal and off-diagonal terms of SH space covariance matrix. First, the effect on diagonal part (angular power spectra), followed by the effect on off-diagonal terms (BipoSH coefficients) are calculated in the next two subsections.

3.1 Effect of boost on angular power spectra $C_l^{XX'}$

In this section, we calculate the effect of boost on angular power spectra. The Lorentz transformation of these diagonal terms of SH covariance matrix can be calculated by taking the product of $\langle \pm \tilde{X}_{l_1 m_1}^* \pm \tilde{X}_{l_2 m_2} \rangle$ and using the conditions given in eq.(21). Angular power spectra is not affected by the terms at $O(\beta)$. In Appendix B, we present the details of the calculation leading to this conclusion.

The leading order effect on the diagonal elements are $O(\beta^2)$. Following the same approach as for linear effect, we take the product $-\tilde{X}_{l_1 m_2}$ with $-\tilde{X}_{l_2 m_2}^*$ and $+\tilde{X}_{l_2 m_2}^*$, and retaining only the $O(\beta^2)$ terms, gives us the leading correction $O(\beta^2)$ terms. In Appendix C, we have provided details of the intermediate steps of the calculation. The correction to the angular power spectrum are,

$$\begin{aligned}\tilde{C}_l^{EE} &= C_l^{EE} \left[1 + \beta^2 \left(\frac{\Pi_l}{4\pi} l(l+1) - 2b_\nu {}_2Q_{ll11}^{m m M M} \right) \right] + \beta^2 C_l^{BB} {}_2M_{ll1}^2 \\ &+ \beta^2 C_{l+1}^{EE} {}_2Z_{l+1l1}^2 + \beta^2 C_{l-1}^{EE} {}_2Z_{l-1l1}^2,\end{aligned}\tag{41}$$

$$\begin{aligned}\tilde{C}_l^{BB} &= C_l^{BB} \left[1 + \beta^2 \left(\frac{\Pi_l}{4\pi} l(l+1) - 2b_\nu {}_2Q_{ll11}^{m m M M} \right) \right] + \beta^2 C_l^{EE} {}_2M_{ll1}^2 \\ &+ \beta^2 C_{l+1}^{BB} {}_2Z_{l+1l1}^2 + \beta^2 C_{l-1}^{BB} {}_2Z_{l-1l1}^2,\end{aligned}\tag{42}$$

$$\begin{aligned}\tilde{C}_l^{TE} &= C_l^{TE} \left[1 - \beta^2 b_\nu \left({}_2Q_{ll11}^{mmMM} + {}_0Q_{ll11}^{mmMM} \right) \right] \\ &\quad + \beta^2 C_{l+1}^{TE} {}_2Z_{l+1l1} {}_0Z_{l+1l1} + \beta^2 C_{l-1}^{TE} {}_2Z_{l-1l1} {}_0Z_{l-1l1},\end{aligned}\tag{43}$$

we define,

$$\begin{aligned}\pm_s M_{l'l1} &= \frac{\Pi_{l'}}{\sqrt{4\pi}} C_{l'sl-s}^{10}, \\ \pm_s N_{l+1l1} &= \frac{\Pi_{l+1}}{\sqrt{4\pi}} (l+2) C_{l+1'sl-s}^{10}, \\ \pm_s N_{l-1l1} &= \frac{\Pi_{l-1}}{\sqrt{4\pi}} (l-1) C_{l-1'sl-s}^{10}.\end{aligned}\tag{44}$$

$$\begin{aligned}\sum_M \pm_s G_{ll11}^{mmMM} &\equiv \pm_s Q_{ll11}^{mmMM} = \sum_J \frac{\Pi_J^2 \left[2 + l(l+1) - J(J+1) \right]}{8\pi} C_{l'sJ-s}^{10} C_{l'sJ-s}^{10}, \\ {}_s Z_{l'l1} &= ({}_s N_{l'l1} - b_\nu {}_s M_{l'l1}).\end{aligned}$$

with the notation $\Pi_{l_1 l_2 \dots l_n} = \sqrt{(2l_1 + 1)(2l_2 + 1) \dots (2l_n + 1)}$ [13].

The eqs.(41), (42) and (43) give the correction due to the Lorentz transformation on the diagonal elements of EE , BB and TE . The β^2 multiplier in each correction term is less than unity. In the angular power spectra there is a mild mixing between EE and BB polarization. In the next section we derive the expression of BipoSH coefficients assuming SI in CMB rest frame and show that the effects appear at linear order in β .

3.2 Effect of boost on BipoSH spectra

As we have mentioned earlier, the two point correlation function of the CMB temperature anisotropy can be expressed as

$$\left\langle \pm \tilde{X}_{l_1 m_1}^* \pm \tilde{X}_{l_2 m_2} \right\rangle = \sum_{LM} \tilde{A}_{l_1 l_2 | \pm X \pm X}^{LM} (-1)^{m_1} C_{l_1 - m_1 l_2 m_2}^{LM},\tag{45}$$

where, we define $\tilde{A}_{l_1 l_2 | \pm X \pm X}^{LM}$ are the BipoSH coefficients. These completely encode the off-diagonal elements of the SH space Covariance matrix and non zero value of these coefficients are the signature of SI violation. The BipoSH coefficients can be decomposed into real (${}^R \tilde{A}_{l'l' | \pm X \pm X}^{LM}$) and imaginary (${}^I \tilde{A}_{l'l' | \pm X \pm X}^{LM}$) parts,

$$\tilde{A}_{l_1 l_2 | \pm X \pm X}^{LM} = {}^R \tilde{A}_{l_1 l_2 | \pm X \pm X}^{LM} + i {}^I \tilde{A}_{l_1 l_2 | \pm X \pm X}^{LM}.\tag{46}$$

Now using the property,

$$\pm \tilde{X}_{l_1 m_1}^* = \mp \tilde{X}_{l_1 -m_1},\tag{47}$$

the product of eq.(47) with its complex conjugate leads to,

$$\left\langle \pm \tilde{X}_{l_1 m_1}^* \pm \tilde{X}_{l_2 m_2} \right\rangle^* = \left\langle \mp \tilde{X}_{l_1 -m_1} \mp \tilde{X}_{l_2 -m_2} \right\rangle.\tag{48}$$

This implies the EE , BB and TE correlations are the real part of the BipoSH coefficients, which we can express for $L = 1, M = 0$ as,

$$\begin{aligned}
2\tilde{A}_{l_1 l_2 | EE}^{10} &= \sum_{m_1 m_2} \left\langle +\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} \right\rangle + \left\langle -\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} \right\rangle (-1)^{m_1} C_{l_1 - m_1 l_2 m_2}^{10}, \\
2\tilde{A}_{l_1 l_2 | BB}^{10} &= \sum_{m_1 m_2} \left\langle +\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} \right\rangle - \left\langle -\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} \right\rangle (-1)^{m_1} C_{l_1 - m_1 l_2 m_2}^{10}, \\
2\tilde{A}_{l_1 l_2 | TE}^{10} &= \sum_{m_1 m_2} \left\langle +\tilde{X}_{l_1 m_1}^* \tilde{a}_{l_2 m_2} \right\rangle + \left\langle -\tilde{X}_{l_1 m_1}^* \tilde{a}_{l_2 m_2} \right\rangle (-1)^{m_1} C_{l_1 - m_1 l_2 m_2}^{10},
\end{aligned} \tag{49}$$

and the imaginary parts of the covariance matrix are EB and TB , which are related to corresponding BipoSH coefficients for $L = 1, M = 0$ by,

$$\begin{aligned}
\tilde{A}_{l_1 l_2 | EB}^{10} &= \sum_{m_1 m_2} \left\langle E_{l_1 m_1}^* B_{l_2 m_2} \right\rangle (-1)^{m_1} C_{l_1 - m_1 l_2 m_2}^{10} \\
&= \frac{i}{2} \sum_{m_1 m_2} \left\langle +\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} \right\rangle + \left\langle -\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} \right\rangle (-1)^{m_1} C_{l_1 - m_1 l_2 m_2}^{10}, \\
\tilde{A}_{l_1 l_2 | TB}^{10} &= \sum_{m_1 m_2} \left\langle B_{l_1 m_1}^* a_{l_2 m_2} \right\rangle (-1)^{m_1} C_{l_1 - m_1 l_2 m_2}^{10} \\
&= \frac{i}{2} \sum_{m_1 m_2} \left\langle +\tilde{X}_{l_1 m_1}^* \tilde{a}_{l_2 m_2} \right\rangle - \left\langle -\tilde{X}_{l_1 m_1}^* \tilde{a}_{l_2 m_2} \right\rangle (-1)^{m_1} C_{l_1 - m_1 l_2 m_2}^{10}.
\end{aligned} \tag{50}$$

For simplicity we choose the direction of β along \hat{z} direction, which gives

$$\beta_{1M} = \beta \delta_{M0}. \tag{51}$$

The effect in any other sky coordinate can be recovered by rotation transformation of BipoSH coefficients. The transformation is identical to that of SH coefficients. The non-zero BipoSH coefficients arising due to Lorentz transformation of Stokes parameters from CMB rest frame to the moving observer frame are,

$$\begin{aligned}
\left\langle -\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} \right\rangle &= \beta \left[-\frac{1}{2} (C_{l_2}^{EE} - C_{l_2}^{BB}) [l_2(l_2 + 1) + 2 - l_1(l_1 + 1)] (-1)^{l_1 + l_2 + 1} \right. \\
&\quad + b_\nu (C_{l_2}^{EE} - C_{l_2}^{BB}) (-1)^{l_1 + l_2 + 1} \\
&\quad - \frac{1}{2} (C_{l_1}^{EE} - C_{l_1}^{BB}) [l_1(l_1 + 1) + 2 - l_2(l_2 + 1)] \\
&\quad \left. + b_\nu (C_{l_1}^{EE} - C_{l_1}^{BB}) \right] (-1)^{m_1} \frac{\Pi_{l_1} l_2}{\sqrt{4\pi} \Pi_1} C_{l_1 2l_2 - 2}^{10} C_{l_1 - m_1 l_2 m_2}^{10},
\end{aligned} \tag{52}$$

$$\begin{aligned}
\left\langle +\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} \right\rangle &= \beta \left[-\frac{1}{2} (C_{l_2}^{EE} + C_{l_2}^{BB}) [l_2(l_2 + 1) + 2 - l_1(l_1 + 1)] \right. \\
&\quad + b_\nu (C_{l_2}^{EE} + C_{l_2}^{BB}) \\
&\quad - \frac{1}{2} (C_{l_1}^{EE} + C_{l_1}^{BB}) [l_1(l_1 + 1) + 2 - l_2(l_2 + 1)] \\
&\quad \left. + b_\nu (C_{l_1}^{EE} + C_{l_1}^{BB}) \right] (-1)^{m_1} \frac{\Pi_{l_1} l_2}{\sqrt{4\pi} \Pi_1} C_{l_1 2l_2 - 2}^{10} C_{l_1 - m_1 l_2 m_2}^{10}.
\end{aligned} \tag{53}$$

$$\begin{aligned}
\left\langle +\tilde{X}_{l_1 m_1}^* \tilde{a}_{l_2 m_2} \right\rangle &= \beta \left[-\frac{1}{2} C_{l_2}^{TE} [l_2(l_2+1) + 2 - l_1(l_1+1)] \frac{\Pi_{l_1 l_2}}{\sqrt{4\pi\Pi_1}} C_{l_2 2l_1-2}^{10} \right. \\
&\quad + b_\nu C_{l_2}^{TE} \frac{\Pi_{l_1 l_2}}{\sqrt{4\pi\Pi_1}} C_{l_2 2l_1-2}^{10} \\
&\quad - \frac{1}{2} C_{l_1}^{TE} [l_1(l_1+1) + 2 - l_2(l_2+1)] \frac{\Pi_{l_1 l_2}}{\sqrt{4\pi\Pi_1}} C_{l_1 0 l_2 0}^{10} \\
&\quad \left. + b_\nu C_{l_1}^{TE} \frac{\Pi_{l_1 l_2}}{\sqrt{4\pi\Pi_1}} C_{l_1 0 l_2 0}^{10} \right] (-1)^{m_1} C_{l_1 - m_1 l_2 m_2}^{10}.
\end{aligned} \tag{54}$$

BipoSH coefficients can be decomposed into even ($+A_{ll'}^{LM}$) and odd ($-A_{ll'}^{LM}$) parity [14]. By decomposing the BipoSH coefficients for polarization into odd and even parity, we can write,

$$A_{ll'|\pm X_\pm X'}^{LM} = + A_{ll'|\pm X_\pm X'}^{+LM} \left[\frac{1 + (-1)^{l+l'+L}}{2} \right] + - A_{ll'|\pm X_\pm X'}^{LM} \left[\frac{1 - (-1)^{l+l'+L}}{2} \right]. \tag{55}$$

Even (odd) parity BipoSH are zero for the value of the sum $l + l' + L$ being odd (even). Also even (odd) parity BipoSH are symmetric (antisymmetric) in l and l' . Since eqs.(52) and (53) are symmetric in l_1 and l_2 for $l_2 = l_1 \pm 1$, these terms are the even parity BipoSH coefficients. But for $l_2 = l_1$, we have odd parity BipoSH coefficients. Using eq.(49) we obtain even BipoSH coefficients the EE , BB and TE from eqs.(52) and (53) as,

$$\begin{aligned}
\tilde{A}_{l+1|EE}^{10} &= \beta \left[C_l^{EE} (l + b_\nu) - (l + 2 - b_\nu) C_{l+1}^{EE} \right] \frac{\Pi_{ll+1}}{\sqrt{4\pi\Pi_1}} C_{l+1-2l2}^{10}, \\
&= 2\pi \frac{D_l^{EE}}{l(l+1)} \frac{\Pi_{ll+1}}{\Pi_1} C_{l+1-2l2}^{10},
\end{aligned} \tag{56}$$

$$\begin{aligned}
\tilde{A}_{l+1|BB}^{10} &= \beta \left[C_l^{BB} (l + b_\nu) - (l + 2 - b_\nu) C_{l+1}^{BB} \right] \frac{\Pi_{ll+1}}{\sqrt{4\pi\Pi_1}} C_{l+1-2l2}^{10}, \\
&= 2\pi \frac{D_l^{BB}}{l(l+1)} \frac{\Pi_{ll+1}}{\Pi_1} C_{l+1-2l2}^{10},
\end{aligned} \tag{57}$$

where, we define BipoSH spectra for $Y = E, B$ polarization as,

$$D_l^{YY} = \frac{l(l+1)\beta \left[(l + b_\nu) C_l^{YY} - (l + 2 - b_\nu) C_{l+1}^{YY} \right]}{2\pi}. \tag{58}$$

The cross correlation between T and E can be obtained as,

$$\begin{aligned}
\tilde{A}_{l+1|TE}^{10} &= \beta \left[(l + b_\nu) C_l^{TE} C_{l0l+10}^{10} - (l + 2 - b_\nu) C_{l+1}^{TE} C_{l-2l+12}^{10} \right] \frac{\Pi_{ll+1}}{\sqrt{4\pi\Pi_1}}, \\
&= 2\pi \frac{D_l^{TE}}{l(l+1)} \frac{\Pi_{ll+1}}{\sqrt{4\pi\Pi_1}} C_{l-2l+12}^{10},
\end{aligned} \tag{59}$$

where, we define BipoSH spectra for TE correlation as,

$$D_l^{TE} = \frac{\beta l(l+1) \left[(l+b_\nu) C_l^{TE} \frac{C_{l_0 l+1 0}^{10}}{C_{l-2l+1 2}^{10}} - (l+2-b_\nu) C_{l+1}^{TE} \right]}{2\pi}. \quad (60)$$

Similar result for $A_{l+1|TT}^{10}$ is obtained [4],

$$\tilde{A}_{l+1|TT}^{10} = 2\pi \frac{D_l^{TT}}{l(l+1)} \frac{\Pi_{l+1}}{\sqrt{4\pi}\Pi_1} C_{l-2l+1 2}^{10}, \quad (61)$$

where,

$$D_l^{TT} = \frac{\beta l(l+1) \left[(l+b_\nu) C_l^{TT} - (l+2-b_\nu) C_{l+1}^{TT} \right]}{2\pi}. \quad (62)$$

From the above expression it is clear that there is non zero BipoSH coefficient only for $L = 1$ arising from the first order terms in β . It is also interesting to point out that there is no mixing of EE and BB polarization in BipoSH coefficients because of the Doppler boost. Similar to the even parity BipoSH coefficients there are odd parity BipoSH coefficients with $l_1 = l_2$. The two mixing terms which are parity violating are cross-correlation between EB and TB . This implies,

$$\begin{aligned} \tilde{A}_{l|EB}^{10} &= i 2\pi \frac{D_l^{EB}}{l(l+1)} \frac{\Pi_l^2}{\Pi_1} C_{l 2l-2}^{10}, \\ \tilde{A}_{l|TB}^{10} &= i 2\pi \frac{D_l^{TB}}{l(l+1)} \frac{\Pi_{l+1}}{\sqrt{4\pi}\Pi_1} C_{l-2l+1 2}^{10}, \end{aligned} \quad (63)$$

where BipoSH spectra for odd parity BipoSH can be written as,

$$\begin{aligned} D_l^{EB} &= \frac{\beta l(l+1)(C_l^{EE} + C_l^{BB})(b_\nu - 1)}{2\pi}, \\ D_l^{TB} &= \frac{\beta l(l+1)(b_\nu - 1)C_l^{TE}}{2\pi}. \end{aligned} \quad (64)$$

We compute BipoSH spectra in the above eq.(56), (57), (59) using the lensed C_l^{EE} , C_l^{BB} and C_l^{TE} from CAMB [15] with the best fit Λ CDM parameters from Planck [16]. We use the value of tensor scale ratio, r , as 0.1. The estimated BipoSH spectra are computed and plotted in fig.1 with $\beta = 1.23 \times 10^{-3}$ [3] for frequency channel $\nu = 217$ GHz, for which $b_\nu = 3$. The BipoSH spectra for EB and TB mentioned in eq.(64) are plotted in fig.2. This also sets the inevitable bias in the measure of SI violation of CMB polarization while searching for Cosmological signature of SI violation.

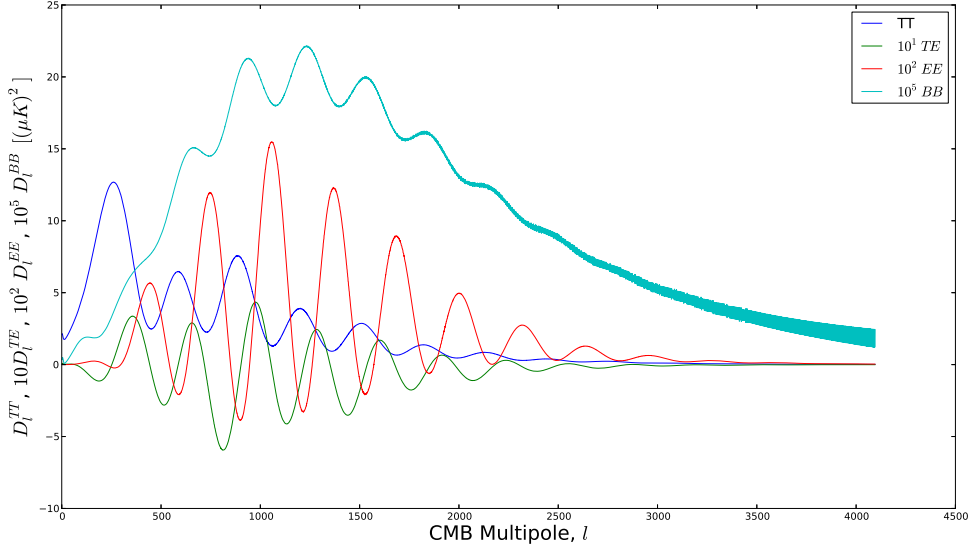


Figure 1: BipoSH spectra, D_l^{TT} , D_l^{EE} , D_l^{BB} and D_l^{TE} , with $b_\nu = 3$ for $\nu = 217$ GHz arising due to boost with $\beta = 1.23 \times 10^{-3}$. Here we have used the best-fit Λ CDM C_l^{TT} , C_l^{EE} , C_l^{BB} and C_l^{TE} generated from CAMB[15].

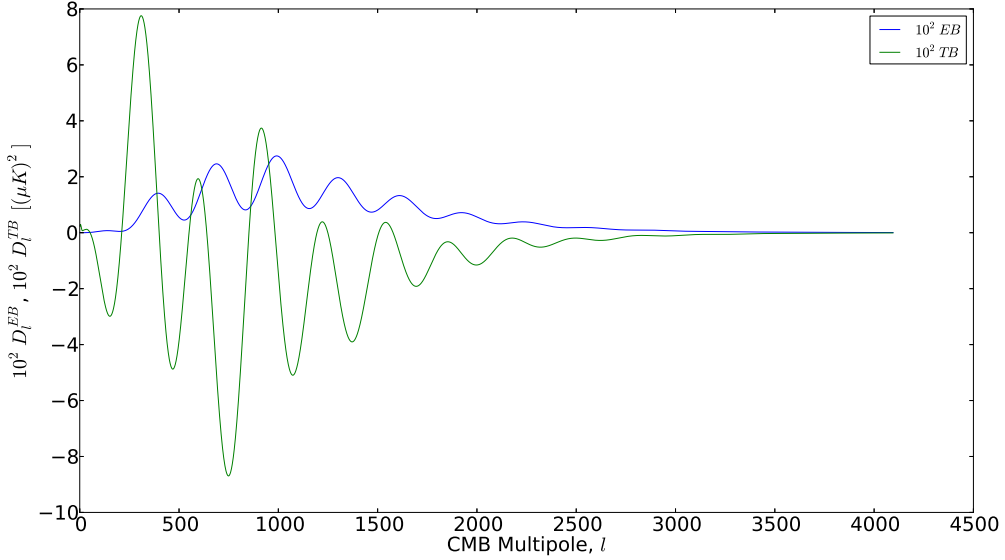


Figure 2: BipoSH spectra, D_l^{EB} and D_l^{TB} with $b_\nu = 3$ for $\nu = 217$ GHz arising due to Doppler boost with $\beta = 1.23 \times 10^{-3}$. Here we have used the best-fit Λ CDM C_l^{TT} , C_l^{EE} , C_l^{BB} and C_l^{TE} generated from CAMB[15].

4 Statistics of BipoSH coefficients for polarization

In the previous section we estimate the BipoSH coefficients arising due to our local motion. However, to estimate β from the BipoSH measurements, we need to study the statistics of the BipoSH coefficients for polarization. Statistics of diagonal elements for polarization are discussed by [10, 17, 18]. The statistics of the BipoSH coefficients, $A_{ll'YY'}^{LM}$, can be obtained with the assumption that $\langle A_{ll'YY'}^{LM} \rangle = 0$. The variance of the BipoSH coefficients can be written as,

$$\begin{aligned}
\langle A_{l_1 l_2 | EE}^{LM} A_{l_3 l_4 | EE}^{L'M'*} \rangle &= \frac{1}{16} \left[\sum_{m_1 m_2 m_3 m_4} \left\langle \left(+\tilde{X}_{l_3 m_3}^* + \tilde{X}_{l_4 m_4} + -\tilde{X}_{l_3 m_3}^* - \tilde{X}_{l_4 m_4} + -\tilde{X}_{l_3 m_3}^* + \tilde{X}_{l_4 m_4} \right. \right. \right. \\
&\quad \left. \left. + +\tilde{X}_{l_3 m_3}^* - \tilde{X}_{l_4 m_4} \right) \left(+\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} + -\tilde{X}_{l_1 m_1}^* - \tilde{X}_{l_2 m_2} + +\tilde{X}_{l_1 m_1}^* - \tilde{X}_{l_2 m_2} \right. \right. \\
&\quad \left. \left. + -\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} \right) \right\rangle C_{l_4 m_4 l_3 - m_3}^{L'M'} C_{l_2 m_2 l_1 - m_1}^{LM} \Big], \\
\langle A_{l_1 l_2 | BB}^{LM} A_{l_3 l_4 | BB}^{L'M'*} \rangle &= \frac{1}{16} \left[\sum_{m_1 m_2 m_3 m_4} \left\langle \left(+\tilde{X}_{l_3 m_3}^* + \tilde{X}_{l_4 m_4} + -\tilde{X}_{l_3 m_3}^* - \tilde{X}_{l_4 m_4} - -\tilde{X}_{l_3 m_3}^* + \tilde{X}_{l_4 m_4} \right. \right. \right. \\
&\quad \left. \left. - +\tilde{X}_{l_3 m_3}^* - \tilde{X}_{l_4 m_4} \right) \left(+\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} + -\tilde{X}_{l_1 m_1}^* - \tilde{X}_{l_2 m_2} - +\tilde{X}_{l_1 m_1}^* - \tilde{X}_{l_2 m_2} \right. \right. \\
&\quad \left. \left. - (-\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2}) \right) \right\rangle C_{l_4 m_4 l_3 - m_3}^{L'M'} C_{l_2 m_2 l_1 - m_1}^{LM} \Big], \\
\langle A_{l_1 l_2 | TE}^{LM} A_{l_3 l_4 | TE}^{L'M'*} \rangle &= \frac{1}{4} \left[\sum_{m_1 m_2 m_3 m_4} \left\langle \left(+\tilde{X}_{l_3 m_3}^* T_{l_4 m_4} + -\tilde{X}_{l_3 m_3}^* T_{l_4 m_4} \right) \left(+\tilde{X}_{l_1 m_1}^* T_{l_2 m_2} + -\tilde{X}_{l_1 m_1}^* T_{l_2 m_2} \right) \right\rangle \right. \\
&\quad \left. C_{l_4 m_4 l_3 - m_3}^{L'M'} C_{l_2 m_2 l_1 - m_1}^{LM} \right], \\
\langle A_{l_1 l_2 | EB}^{LM} A_{l_3 l_4 | EB}^{L'M'*} \rangle &= \frac{1}{16} \left[\sum_{m_1 m_2 m_3 m_4} \left\langle \left(+\tilde{X}_{l_3 m_3}^* + \tilde{X}_{l_4 m_4} - -\tilde{X}_{l_3 m_3}^* - \tilde{X}_{l_4 m_4} + -\tilde{X}_{l_3 m_3}^* + \tilde{X}_{l_4 m_4} \right. \right. \right. \\
&\quad \left. \left. - +\tilde{X}_{l_3 m_3}^* - \tilde{X}_{l_4 m_4} \right) \left(+\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} - -\tilde{X}_{l_1 m_1}^* - \tilde{X}_{l_2 m_2} - +\tilde{X}_{l_1 m_1}^* - \tilde{X}_{l_2 m_2} \right. \right. \\
&\quad \left. \left. + -\tilde{X}_{l_1 m_1}^* + \tilde{X}_{l_2 m_2} \right) \right\rangle C_{l_4 m_4 l_3 - m_3}^{L'M'} C_{l_2 m_2 l_1 - m_1}^{LM} \Big], \\
\langle A_{l_1 l_2 | TB}^{LM} A_{l_3 l_4 | TB}^{L'M'*} \rangle &= \frac{1}{4} \left[\sum_{m_1 m_2 m_3 m_4} \left\langle \left(+\tilde{X}_{l_3 m_3}^* T_{l_4 m_4} - -\tilde{X}_{l_3 m_3}^* T_{l_4 m_4} \right) \left(+\tilde{X}_{l_1 m_1}^* T_{l_2 m_2} - -\tilde{X}_{l_1 m_1}^* T_{l_2 m_2} \right) \right\rangle \right. \\
&\quad \left. C_{l_4 m_4 l_3 - m_3}^{L'M'} C_{l_2 m_2 l_1 - m_1}^{LM} \right].
\end{aligned} \tag{65}$$

With the assumption that the random temperature fluctuations are Gaussian distribution, we can reduce the four point correlation function, $\langle A_{l_1 l_2 | XX'}^{L'M'} A_{l_3 l_4 | XX'}^{LM} \rangle$ as products of two point correlation functions. Applying this to eq.(65), we get the variance of BipoSH coefficients with $L \neq 0$ for

polarization as ,

$$\begin{aligned}
\left\langle A_{l_1 l_2 | EE}^{LM} A_{l_3 l_4 | EE}^{L' M'^*} \right\rangle &= (-1)^{l_1 + l_2 + L} C_{l_1}^{EE} C_{l_2}^{EE} \delta_{LL'} \delta_{MM'} \delta_{l_1 l_4} \delta_{l_3 l_2} \\
&\quad + C_{l_1}^{EE} C_{l_2}^{EE} \delta_{LL'} \delta_{MM'} \delta_{l_2 l_4} \delta_{l_3 l_1}, \\
\left\langle A_{l_1 l_2 | BB}^{LM} A_{l_3 l_4 | BB}^{L' M'^*} \right\rangle &= (-1)^{l_1 + l_2 + L} C_{l_1}^{BB} C_{l_2}^{BB} \delta_{LL'} \delta_{MM'} \delta_{l_1 l_4} \delta_{l_3 l_2} \\
&\quad + C_{l_1}^{BB} C_{l_2}^{BB} \delta_{LL'} \delta_{MM'} \delta_{l_2 l_4} \delta_{l_3 l_1}, \\
\left\langle A_{l_1 l_2 | TE}^{LM} A_{l_3 l_4 | TE}^{L' M'^*} \right\rangle &= (-1)^{l_1 + l_2 + L} C_{l_1}^{TE} C_{l_2}^{TE} \delta_{LL'} \delta_{MM'} \delta_{l_1 l_4} \delta_{l_3 l_2} \\
&\quad + C_{l_1}^{EE} C_{l_2}^{TT} \delta_{LL'} \delta_{MM'} \delta_{l_2 l_4} \delta_{l_3 l_1}, \\
\left\langle A_{l_1 l_2 | EB}^{LM} A_{l_3 l_4 | EB}^{L' M'^*} \right\rangle &= C_{l_1}^{EE} C_{l_2}^{BB} \delta_{LL'} \delta_{MM'} \delta_{l_2 l_4} \delta_{l_3 l_1}, \\
\left\langle A_{l_1 l_2 | TB}^{LM} A_{l_3 l_4 | TB}^{L' M'^*} \right\rangle &= C_{l_1}^{BB} C_{l_2}^{TT} \delta_{LL'} \delta_{MM'} \delta_{l_2 l_4} \delta_{l_3 l_1}.
\end{aligned} \tag{66}$$

Variance of BipoSH coefficients for temperature was obtained by Hajian [19] and Nidhi et. al. [20]. For the BipoSH coefficients due to Doppler boost mentioned in eq.(56), (57), (59), the variance of BipoSH coefficients for polarization mentioned in eq.(66) becomes,

$$\begin{aligned}
\left\langle A_{l+1 | EE}^{*10} A_{l+1 | EE}^{10} \right\rangle &= 2C_l^{EE} C_{l+1}^{EE}, \\
\left\langle A_{l+1 | BB}^{*10} A_{l+1 | BB}^{10} \right\rangle &= 2C_l^{BB} C_{l+1}^{BB}, \\
\left\langle A_{l+1 | TE}^{*10} A_{l+1 | TE}^{10} \right\rangle &= C_l^{TE} C_{l+1}^{TE} + C_l^{EE} C_{l+1}^{TT}, \\
\left\langle A_{l | EB}^{10*} A_{l | EB}^{10} \right\rangle &= C_l^{EE} C_l^{BB}, \\
\left\langle A_{l | TB}^{10*} A_{l | TB}^{10} \right\rangle &= C_l^{BB} C_l^{TT}.
\end{aligned} \tag{67}$$

In presence of instrumental noise, the noise power spectrum \mathcal{N}_l^Y , depends upon beam width and sensitivity of the instrument by [10, 18, 21].

$$\mathcal{N}_l^Y = \frac{\theta_b^2 (\sigma_{pix}^2)^Y}{n_{det}} e^{l(l+1) \frac{\theta_b^2}{8\pi^2}}; \quad Y = T, E, B \tag{68}$$

where, θ_b^2 is FWHM of the Gaussian beam and $(\sigma_{pix}^2)_Y$ is the rms of the instrumental noise per detector for temperature ($Y = T$) and polarization ($Y = P$), and n_{det} are the number of detectors. For a equal integration time for two polarization states, pixel noise for temperature and polarization are related by [10],

$$(\sigma_{pix}^2)_T = \frac{(\sigma_{pix}^2)_P}{2}. \quad (69)$$

For TE we have taken the instrumental noise as zero as taken by FUTURCMB [18]. The variance in eq.(67) in the presence of instrumental noise becomes,

$$\begin{aligned} \left\langle A_{l+1|EE}^{*10} A_{l+1|EE}^{10} \right\rangle &= 2(C_l^{EE} + \mathcal{N}_l^P)(C_{l+1}^{EE} + \mathcal{N}_{l+1}^P), \\ \left\langle A_{l+1|BB}^{*10} A_{l+1|BB}^{10} \right\rangle &= 2(C_l^{BB} + \mathcal{N}_l^P)(C_{l+1}^{BB} + \mathcal{N}_{l+1}^P), \\ \left\langle A_{l+1|TE}^{*10} A_{l+1|TE}^{10} \right\rangle &= C_l^{TE} C_{l+1}^{TE} + (C_l^{EE} + \mathcal{N}_l^P)(C_{l+1}^{TT} + \mathcal{N}_{l+1}^T), \\ \left\langle A_{l|EB}^{*10} A_{l|EB}^{10} \right\rangle &= (C_l^{EE} + \mathcal{N}_l^P)(C_l^{BB} + \mathcal{N}_l^P), \\ \left\langle A_{l|TB}^{*10} A_{l|TB}^{10} \right\rangle &= (C_l^{BB} + \mathcal{N}_l^P)(C_l^{TT} + \mathcal{N}_l^T). \end{aligned} \quad (70)$$

5 Reconstruction noise for Doppler boost from Planck and PRISM

In the previous section we obtained the expression for non-zero BipoSH coefficients, arising due to local motion. In this work we derive the minimum variance reconstruction noise for polarization map for BipoSH coefficients. Instrumental noise for Planck is high, and is not suitable for making significant detection of β from polarization. However, noise level for future mission PRISM is smaller than Planck. This will make it possible to detect the magnitude and direction of β from the polarization result. By measuring the non-zero BipoSH values from experiments like Planck and PRISM, we can infer the value of our local motion. We discussed the minimum variance estimator in more details in Sec. 2.4. Using eq.(32), we obtain the expression of theoretical reconstruction

noise for the ideal cosmic variance limited of the BipoSH spectra as,

$$\begin{aligned}
N_{\beta}^{EE} &= \left[\sum_{l'} \frac{S_{l'|EE}^{\beta} S_{l'|EE}^{\beta}}{2C_l^{EE} C_{l'}^{EE}} \right]^{-1}, \\
N_{\beta}^{BB} &= \left[\sum_{l'} \frac{S_{l'|BB}^{\beta} S_{l'|BB}^{\beta}}{2C_l^{BB} C_{l'}^{BB}} \right]^{-1}, \\
N_{\beta}^{TE} &= \left[\sum_{l'} \frac{S_{l'|TE}^{\beta} S_{l'|TE}^{\beta}}{C_l^{TE} C_{l'}^{TE} + C_l^{EE} C_{l'}^{TT}} \right]^{-1}, \\
N_{\beta}^{EB} &= \left[\sum_l \frac{S_{l|EB}^{\beta} S_{l|EB}^{\beta}}{C_l^{EE} C_l^{BB}} \right]^{-1}, \\
N_{\beta}^{TB} &= \left[\sum_l \frac{S_{l|TB}^{\beta} S_{l|TB}^{\beta}}{C_l^{TT} C_l^{BB}} \right]^{-1}.
\end{aligned} \tag{71}$$

where shape factor, $S_{l'}^{\beta}$ are,

$$\begin{aligned}
S_{l+1|EE}^{\beta} &= 2\pi \frac{D_l^{EE}}{\beta l(l+1)} \frac{\Pi_{ll+1}}{\Pi_1} C_{l+1}^{10} C_{-2l2}, \\
S_{l+1|BB}^{\beta} &= 2\pi \frac{D_l^{BB}}{\beta l(l+1)} \frac{\Pi_{ll+1}}{\Pi_1} C_{l+1}^{10} C_{-2l2}, \\
S_{l+1|TE}^{\beta} &= 2\pi \frac{D_l^{TE}}{\beta l(l+1)} \frac{\Pi_{ll+1}}{\Pi_1} C_{l+1}^{10} C_{-2l2}, \\
S_{l|EB}^{\beta} &= 2\pi \frac{D_l^{EB}}{\beta l(l+1)} \frac{\Pi_l^2}{\Pi_1} C_l^{10} C_{-2l2}, \\
S_{l|TB}^{\beta} &= 2\pi \frac{D_l^{TB}}{\beta l(l+1)} \frac{\Pi_l^2}{\Pi_1} C_l^{10} C_{-2l2}.
\end{aligned} \tag{72}$$

The value of the reconstruction noise sets the maximum possible significance of the measurement of β from the corresponding observable. For the theoretical best-fit value of $C_l^{XX'}$, we obtain the reconstruction noise as mentioned in eq.(71). This is the theoretical noise curve due to the cosmic variance which is inevitable. But for any experiment, the variance is also enhanced by instrumental sensitivity and finite angular resolution. Presence of instrumental noise leads to increase in the reconstruction noise, N_{β} . As a result of which measuring the value of β becomes difficult. Similar to polarization, theoretical reconstruction noise for temperature is also mentioned by Planck [1] as,

$$\begin{aligned}
N_{\beta}^{TT} &= \left[\sum_{l'} \frac{S_{l'|TT}^{\beta} S_{l'|TT}^{\beta}}{2C_l^{TT} C_{l'}^{TT}} \right]^{-1}, \\
S_{l+1|TT}^{\beta} &= 2\pi \frac{D_l^{TT}}{\beta l(l+1)} \frac{\Pi_{ll+1}}{\Pi_1} C_{l+1}^{10} C_{l+1}^{0l0},
\end{aligned} \tag{73}$$

We estimate the value of reconstruction noise N_β for Planck and also for proposed future experiments like PRISM [22] in the presence of instrumental noise, \mathcal{N}_l^Y as,

$$\begin{aligned}
N_\beta^{EE} &= \left[\sum_{l+1} \frac{S_{l'|EE}^\beta S_{l'|EE}^\beta}{2(C_l^{EE} + \mathcal{N}_l^P)(C_{l+1}^{EE} + \mathcal{N}_{l+1}^P)} \right]^{-1}, \\
N_\beta^{BB} &= \left[\sum_{l+1} \frac{S_{l'|BB}^\beta S_{l'|BB}^\beta}{2(C_l^{BB} + \mathcal{N}_l^P)(C_{l+1}^{BB} + \mathcal{N}_{l+1}^P)} \right]^{-1}, \\
N_\beta^{TE} &= \left[\sum_{l+1} \frac{S_{l'|TE}^\beta S_{l'|TE}^\beta}{C_l^{TE} C_{l+1}^{TE} + (C_l^{EE} + \mathcal{N}_l^P)(C_{l+1}^{TT} + \mathcal{N}_{l+1}^T)} \right]^{-1}, \\
N_\beta^{EB} &= \left[\sum_l \frac{S_{l|EB}^\beta S_{l|EB}^\beta}{(C_l^{EB} + \mathcal{N}_l^P)(C_l^{BB} + \mathcal{N}_l^P)} \right]^{-1}, \\
N_\beta^{TB} &= \left[\sum_l \frac{S_{l|TB}^\beta S_{l|TB}^\beta}{(C_l^{TT} + \mathcal{N}_l^T)(C_l^{BB} + \mathcal{N}_l^P)} \right]^{-1}, \\
N_\beta^{TT} &= \left[\sum_{l'} \frac{S_{l'|TT}^\beta S_{l'|TT}^\beta}{2(C_l^{TT} + \mathcal{N}_l^T)(C_{l+1}^{TT} + \mathcal{N}_{l+1}^T)} \right]^{-1}.
\end{aligned} \tag{74}$$

For Planck, we made the estimation for the frequency channel, $\nu = 217$ GHz. The FWHM of Gaussian beam is $\theta_{FWHM} = 5$ arcminute, and the sensitivity per pixel for polarization and for temperature is $9.8\mu K/K$ and $4.8\mu K/K$ respectively [23]. This relative sensitivity of temperature and polarization nearly match the eq.(69).

Similar to Planck, we also estimate the reconstruction noise, N_β for future experiment, PRISM. From the PRISM white paper [22], we obtain the instrumental noise for $\nu = 220$ GHz with FWHM = $2.3'$ arc minute, $n_{det} = 350$ and sensitivity per detector for temperature and polarization as $50.9\mu K$ and $71.9\mu K$ respectively. Since PRISM proposes to achieve much higher angular resolution than Planck, and also much lower instrumental noise, we expect the reconstruction noise for PRISM to be smaller than that obtained for Planck. The comparison between reconstruction noise eq.(74) for Planck and PRISM with the theoretical reconstruction noise eq.(71) are given in fig.3 for TT , EE , BB and TE , and in fig.4 for EB and TB . We used the best-fit parameters from Planck [16] for the value of $C_l^{YY'}$ ($Y = T, E, B$) to evaluate eq.(65). We plotted $(N_\beta)^{1/2}$ in fig.3, 4 along with $\beta = 1.23 \times 10^{-3}$.

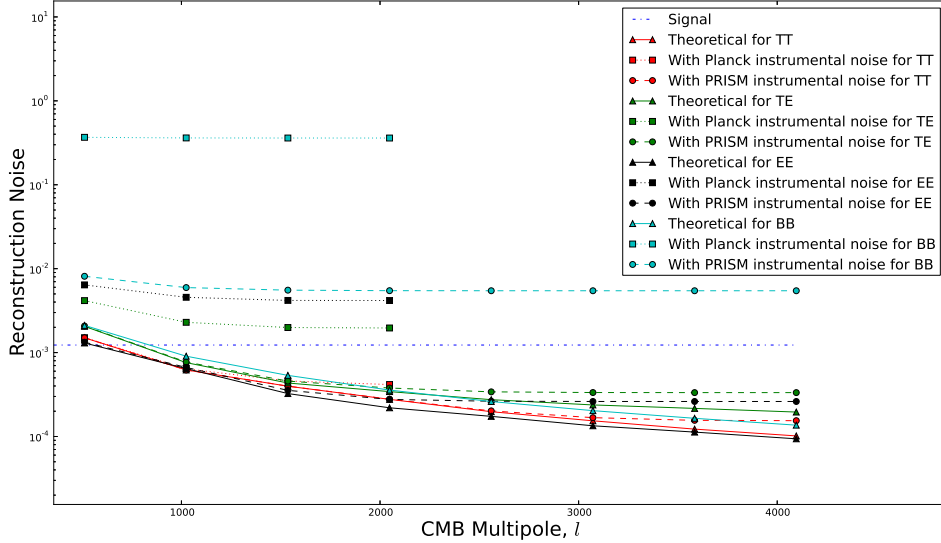


Figure 3: Cumulative binned reconstruction noise for TT , EE , TE and BB with $\Delta l = 512$. We have taken instrumental noise for Planck (dash-square), and PRISM (dash-circle). The theoretical reconstruction noise curves are plotted (dash-triangle), which indicates the minimum noise level. Doppler boost signal, $\beta = 1.23 \times 10^{-3}$ (dot-dash). Here we have used the best-fit Λ CDM C_l^{EE} , C_l^{BB} , C_l^{TE} and C_l^{TT} generated from CAMB[15] and value of $b_\nu \approx 3$ for $\nu = 217$ GHz and 220 GHz for Planck and PRISM respectively.

In principle, β is measurable in all the BipoSH spectra, except for EB and TB . We summarise the possibility of detection of β for Planck and PRISM, from the calculated reconstruction noise, plotted in fig.3 and 4.

- i. Reconstruction noise for EE , TE and BB with Planck instrumental noise and angular resolution is much above the signal ($\beta = 1.23 \times 10^{-3}$), as shown by curve (dash-square) in fig.3. This implies we cannot make statistically significant detection of β in EE , TE and BB by Planck in the frequency channel $\nu = 217$ GHz.
- ii. The detection of β is consistent with our expectation, in TT . A 3σ detection has been already reported by Planck [4] at $l_{max} = 2048$.
- iii. A measurement of β from TT BipoSH coefficients with a significance of 8σ at $l_{max} = 4096$ is possible with PRISM in the frequency channel $\nu = 220$ GHz.
- iv. Using PRISM we can make a measurement of β from both EE and TE with a significance of 4.7σ and 3.7σ respectively at $l_{max} = 4096$ in the frequency channel $\nu = 220$ GHz.
- v. In BB BipoSH coefficients measurement of β is not possible by PRISM. This is due to low value of C_l^{BB} signal relative to the instrumental noise. The theoretical reconstruction noise

curve for BB is lower than β , fig.3. This clearly indicates the possibility of detecting β from experiments with better resolution and sensitivity than PRISM.

- vi. Measurement of β from EB and TB BipoSH coefficients are not possible even in principle. The reconstruction noise for PRISM as well as the theoretical reconstruction noise are much above the value of β fig.4.

Our result shows us that we can estimate the velocity of our local motion, β with the polarization maps from PRISM, which is not possible from the experiments like Planck.

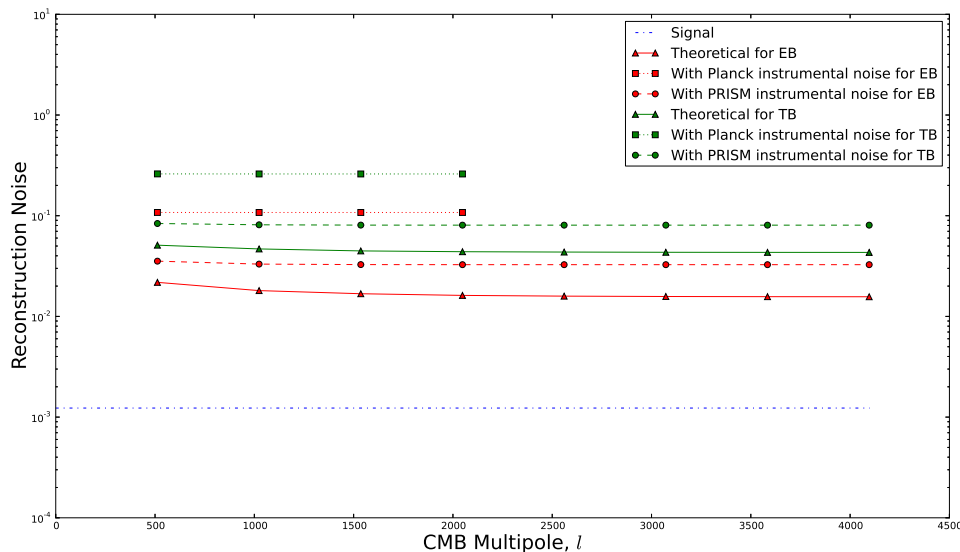


Figure 4: Cumulative binned reconstruction noise for EB and TB with $\Delta l = 512$. We have taken instrumental noise for Planck (dash-square), and PRISM(dash-circle). The theoretical reconstruction noise curve are plotted (dash-triangle), which indicates the minimum noise level. Doppler boost signal, $\beta = 1.23 \times 10^{-3}$ (dot-dash). Here we have used the best-fit Λ CDM $C_l^{EE}, C_l^{BB}, C_l^{TE}$ and C_l^{TT} generated from CAMB[15] and value of $b_\nu \approx 3$ for $\nu = 217$ GHz and 220 GHz, for Planck and PRISM respectively.

6 Conclusion

The local motion of the observer as measured from CMB dipole [3] (with $\beta \equiv v/c = 1.23 \times 10^{-3}$) causes an unavoidable violation of Statistical Isotropy effect on the CMB temperature and polarization field. Recently, Planck [4] has shown that Doppler boost of temperature fluctuation leads to mixing of power between different CMB multipoles, l , which results in non-zero value of the off-diagonal (BipoSH) terms in the SH space covariance matrix. The Doppler boost of temperature fluctuation results in violation of Statistical Isotropy (SI) in the observer's frame. This has been recently measured in the temperature map in the Planck results. Even more, recently Jeong. et al. [24] showed that the effect of aberration from boost on temperature angular power spectrum on partial sky measurements could be responsible for the differences in Planck and SPT measurements at high multipoles.

In this paper we have discussed the effect of boost on the CMB polarization field. Boost affects the CMB polarization field by modulation of the Stokes parameters and aberration in the direction of incoming photons. Because of this, both diagonal and off-diagonal terms in SH space covariance matrix of polarization field get affected. The leading order effect to angular power spectrum is $O(\beta^2)$ as mentioned in eq.(41), eq.(42) and eq.(43). This means, angular power spectra are mildly affected by boost. But the off-diagonal terms of the covariance matrix in SH space, have effect linear in β . This leads to the non-zero potentially measurable BipoSH spectra ($A_{ll+1|XX'}^{10}$) as mentioned in eq.(56), eq.(57) and eq.(59) and plotted in fig.1. This implies, an SI CMB polarization field would appear Non-SI (nSI) in our frame. The measurement of these non-zero BipoSH coefficients will lead to an estimation of β . Statistically significant measurement of β depends upon the reconstruction noise of any future experiments like Planck and PRISM. We estimated the reconstruction noise for Planck and PRISM by using minimum variance estimator method. Reconstruction noise depends upon the variance of the BipoSH coefficients and instrumental noise. The variance of BipoSH coefficients for polarization are calculated, and using the instrumental noise of Planck and PRISM, we estimated the reconstruction noise for TT , EE , TE and BB . Reconstruction noise is smaller than $\beta = 1.23 \times 10^{-3}$ in TT for Planck, and in TT , EE , TE for PRISM. But β is not measurable in BB , EB and TB BipoSH coefficients by Planck and PRISM. The theoretical reconstruction noise for BB is smaller than β . But due to higher value of instrumental noise relative to the signal, detection of β from BB spectra is not possible. Whereas, EB and TB spectra can never be use for measuring the value of β using minimum variance estimator because of high theoretical reconstruction noise. The results here can be used to derive similar implication for comparing CMB polarization angular power spectra measured by CMB experiments covering small patches of the sky as shown for temperature anisotropy in [24]. The computed BipoSH spectra from the Doppler boost need to be accounted in all future SI violation searches from the upcoming temperature and polarization maps such as Planck and PRISM.

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Appendices

A Lorentz Transformation of Stokes parameter from Electric field

In this appendix we present an alternative derivation of the Stokes parameter in terms of Electric field. The polarization of the Electromagnetic wave is defined by the direction of the electric field, we can define the Electric field vector of the wave as $\vec{\mathcal{E}}(\hat{n}) = \mathcal{E}_1 \hat{e}_1 + \mathcal{E}_2 \hat{e}_2$, where $(\hat{e}_1, \hat{e}_2, \hat{n})$ forms a right handed orthonormal system.

Lorentz transformation for Stokes parameters can also be obtained by transforming electric field from moving frame to the rest frame. Let \mathbf{S}' be the CMB rest frame. The observer reference frame \mathbf{S} is moving with a velocity $\beta = \beta_0 \hat{\beta}$, with $\beta_0 = 1.23 \times 10^{-3}$ as measured by CMB dipole [3]. On taking the direction of propagation such that $\hat{n} \cdot \hat{\beta} = \cos \theta$, the Lorentz transformation of the electric field is

$$\begin{aligned}\mathcal{E}'_1(\hat{n}) &= \gamma\mathcal{E}_1(1 - \beta \cos \theta), \\ \mathcal{E}'_2(\hat{n}) &= \gamma\mathcal{E}_2(1 - \beta \cos \theta),\end{aligned}\tag{75}$$

and the corresponding effect on Stokes parameters are [25],

$$\begin{aligned}I'(\hat{n}) &= \gamma^2 I(\hat{n})(1 - \beta \cos \theta)^2, \\ Q'(\hat{n}) &= \gamma^2 Q(\hat{n})(1 - \beta \cos \theta)^2, \\ U'(\hat{n}) &= \gamma^2 U(\hat{n})(1 - \beta \cos \theta)^2.\end{aligned}\tag{76}$$

As it is seen in eq.(76), Stokes parameters (I, Q, U) transforms similarly under Lorentz transformation and it depends only on the angle between $\hat{\beta}$ and \hat{n} . So, the effect of modulation is identical for all the Stokes parameters.

The combined effect of modulation and aberration on Stokes parameters are,

$$\begin{aligned}I'(\hat{n}') &= \gamma^2 I(\hat{n})(1 - \beta \cos \theta)^2, \\ Q'(\hat{n}') &= \gamma^2 Q(\hat{n})(1 - \beta \cos \theta)^2, \\ U'(\hat{n}') &= \gamma^2 U(\hat{n})(1 - \beta \cos \theta)^2.\end{aligned}\tag{77}$$

As the value of $\beta \approx 10^{-3}$, it is sufficient to limit to only the linear order effect of β in eq.(77), as done in the study of temperature [4]. The linear order effect of β on Stokes parameters are,

$$\begin{aligned}I(\hat{n}) &= I'(\hat{n} - \nabla(\hat{n} \cdot \hat{\beta}))(1 + 2\beta \cos \theta), \\ Q(\hat{n}) &= Q'(\hat{n} - \nabla(\hat{n} \cdot \hat{\beta}))(1 + 2\beta \cos \theta), \\ U(\hat{n}) &= U'(\hat{n} - \nabla(\hat{n} \cdot \hat{\beta}))(1 + 2\beta \cos \theta).\end{aligned}\tag{78}$$

In the above expression Stokes parameters are defined in dimension of flux.

B Absence of linear order effect of boost on angular power spectra

The angular power spectra of CMB is related to the diagonal terms in the covariance matrix. Taking the product of $+\tilde{X}_{l_1 m_1}$ defined in eq.(40) with $+\tilde{X}_{l_2 m_2}^*$ and with $-\tilde{X}_{l_2 m_2}^*$, and retaining only $O(\beta)$ terms, we get

$$\begin{aligned}\left\langle +\tilde{X}_{l_1 m_1}^* + \tilde{X}'_{l_2 m_2} \right\rangle &= (C_{l_1}^{EE} + C_{l_1}^{BB})\delta_{l_1 l_2} \delta_{m_1 m_2} - \sum_{m_3} (C_{l_2}^{EE} + C_{l_2}^{BB})\beta_{1m_3+2} I_{l_1 l_2 1}^{m_1 m_2 m_3} \\ &\quad - \sum_{m_3} (C_{l_1}^{EE} + C_{l_1}^{BB})\beta_{1m_3+2} I_{l_2 l_1 1}^{m_2 m_1 m_3} + b_\nu \left(\sum_M (C_{l_2}^{EE} + C_{l_2}^{BB})\beta_{1M+2} H_{l_1 l_2 1}^{m_1 m_2 M} \right. \\ &\quad \left. + \sum_M (C_{l_1}^{EE} + C_{l_1}^{BB})\beta_{1M+2} H_{l_2 l_1 1}^{m_2 m_1 M} \right),\end{aligned}\tag{79}$$

and

$$\begin{aligned}
\left\langle -\tilde{X}_{l_1 m_1}^* + \tilde{X}'_{l_2 m_2} \right\rangle &= (C_{l_1}^{EE} - C_{l_1}^{BB}) \delta_{l_1 l_2} \delta_{m_1 m_2} - \sum_{m_3} (C_{l_2}^{EE} - C_{l_2}^{BB}) \beta_{1 m_3 - 2} I_{l_1 l_2 1}^{m_1 m_2 m_3} \\
&\quad - \sum_{m_3} (C_{l_1}^{EE} - C_{l_1}^{BB}) \beta_{1 m_3 + 2} I_{l_2 l_1 1}^{m_2 m_1 m_3} + b_\nu \left(\sum_M (C_{l_2}^{EE} - C_{l_2}^{BB}) \beta_{1 M - 2} H_{l_1 l_2 1}^{m_1 m_2 M} \right. \\
&\quad \left. + \sum_M (C_{l_1}^{EE} - C_{l_1}^{BB}) \beta_{1 M + 2} H_{l_2 l_1 1}^{m_2 m_1 M} \right).
\end{aligned} \tag{80}$$

Using [13] and [12] respectively, ${}_{\pm 2}H_{l_2 l_1 1}^{m_2 m_1 M}$ and ${}_{\pm 2}I_{l_1 l_2 1}^{m_1 m_2 m_3}$ can be obtained as,

$$\begin{aligned}
{}_{\pm 2}H_{l_1 l_2 1}^{m_1 m_2 M} &= \int d\hat{n} Y_{1M}(\hat{n}) {}_{\pm 2}Y_{l_1 m_1}^*(\hat{n}) {}_{\pm 2}Y_{l_2 m_2}(\hat{n}), \\
&= (-1)^{m_2} \frac{\Pi_{l_1 l_2}}{\sqrt{4\pi} \Pi_1} C_{l_2 \pm 2 l_1 \mp 2}^{10} C_{l_2 m_2 l_1 - m_1}^{1-M}.
\end{aligned} \tag{81}$$

$$\begin{aligned}
{}_{\pm 2}I_{l_1 l_2 1}^{m_1 m_2 M} &= \int d\hat{n} {}_{\pm 2}Y_{l_1 m_1}^*(\hat{n}) \nabla_i {}_{\pm 2}Y_{l_2 m_2}(\hat{n}) \nabla^i Y_{1M}(\hat{n}), \\
&= (-1)^{m_2} \frac{1}{2} \left[l_2(l_2 + 1) + 2 - l_1(l_1 + 1) \right] \frac{\Pi_{l_1 l_2}}{\sqrt{4\pi} \Pi_1} C_{l_2 \pm 2 l_1 \mp 2}^{10} C_{l_2 m_2 l_1 - m_1}^{1-M}.
\end{aligned} \tag{82}$$

where, $\Pi_{l_1 l_2 \dots l_n} = \sqrt{(2l_1 + 1)(2l_2 + 1) \dots (2l_n + 1)}$ and $C_{l_2 m_2 l_1 m_1}^{LM}$ are the Clebsch-Gordon coefficients.

As mentioned in Section.2.3, the general covariance matrix elements can be written as

$$\left\langle \pm \tilde{X}_{l_1 m_1}^* \pm \tilde{X}'_{l_2 m_2} \right\rangle = \sum_{JM} \tilde{A}_{l_1 l_2 | \pm X \pm X'}^{LM} (-1)^{m_1} C_{l_1 - m_1 l_2 m_2}^{LM}. \tag{83}$$

Here $L = 0, M = 0$ gives the diagonal terms i.e., $\tilde{C}_l^{X'X} = (-1)^l (\Pi_l)^{-1} A_{l_1 l_1 | \pm X' \pm X}^{00}$, and eq.(83) can be written more explicitly as,

$$\left\langle \pm \tilde{X}_{l_1 m_1}^* \pm \tilde{X}'_{l_2 m_2} \right\rangle = \tilde{C}_{l_1}^{X'X} \delta_{l_1 l_2} \delta_{m_1 m_2} + \sum_{L \neq 0, LM} \tilde{A}_{l_1 l_2 | \pm X \pm X'}^{LM} (-1)^{m_1} C_{l_2 m_2 l_1 - m_1}^{LM}. \tag{84}$$

Hence, expressions in eq.(79) and eq.(80) can be written in two parts, the diagonal part and the off-diagonal part. The diagonal part is just the terms with value of $L = 0$ and $M = 0$. Then it can be easily shown that up to linear order in β ,

$$\tilde{C}_l^{EE} = C_l^{EE}, \tag{85}$$

and

$$\tilde{C}_l^{BB} = C_l^{BB}. \tag{86}$$

This implies that there is no $O(\beta)$ effect in the diagonal terms of Covariance matrix. The corresponding cross term TE can also be obtained. TE correlation also has no $O(\beta)$ effect.

$$\tilde{C}_l^{TE} = C_l^{TE}. \quad (87)$$

This implies that the corrections due to Doppler boost to the angular power spectrum given by the diagonal terms is negligible since there is no $O(\beta)$ effect.

C Leading second order corrections to the angular power spectra

In the appendix B we mentioned that there is no linear order effect of β on the angular power spectra of CMB. Here we provide details of the calculation of the leading order effect at $O(\beta^2)$ on the angular power spectra. By taking the product of $+\tilde{X}_{l_1 m_1}$ defined in eq.(40) with $+\tilde{X}_{l_2 m_2}^*$ and with $-\tilde{X}_{l_2 m_2}^*$, and retaining only upto $O(\beta^2)$ terms, we get

$$\begin{aligned} \tilde{C}_{l_1}^{EE} = & C_{l_1}^{EE} + \frac{1}{2}\beta^2 \left[\sum_{l_2 m_2} \sum_M +2I_{l_1 l_2 1}^{m_1 m_2 M} \left[(C_{l_2}^{EE} + C_{l_2}^{BB}) +2I_{l_1 l_2 1}^{m_1 m_2 M} + (C_{l_2}^{EE} - C_{l_2}^{BB}) -2I_{l_1 l_2 1}^{m_1 m_2 M} \right] \right. \\ & + \sum_M \left[(C_{l_1}^{EE} + C_{l_1}^{BB}) +2J_{l_1 l_1 11}^{m_1 m_1 M M} + \frac{1}{2} (C_{l_1}^{EE} - C_{l_1}^{BB}) \left[+2J_{l_1 l_1 11}^{m_1 m_1 M M} + -2J_{l_1 l_1 11}^{m_1 m_1 M M} \right] \right] \\ & + (b_\nu)^2 \sum_M \sum_{l_2 m_2} +2H_{l_1 l_2 1}^{m_1 m_2 M} \left[(C_{l_2}^{EE} + C_{l_2}^{BB}) +2H_{l_1 l_2 1}^{m_1 m_2 M} + (C_{l_2}^{EE} - C_{l_2}^{BB}) -2H_{l_1 l_2 1}^{m_1 m_2 M} \right] \\ & - 2b_\nu \sum_M \sum_{l_2 m_2} +2H_{l_1 l_2 1}^{m_1 m_2 M} \left[(C_{l_2}^{EE} + C_{l_2}^{BB}) +2I_{l_1 l_2 1}^{m_1 m_2 M} + (C_{l_2}^{EE} - C_{l_2}^{BB}) -2I_{l_1 l_2 1}^{m_1 m_2 M} \right] \\ & \left. - b_\nu \sum_M \left[2(C_{l_1}^{EE} + C_{l_1}^{BB}) +2G_{l_1 l_1 11}^{m_1 m_1 M M} + (C_{l_1}^{EE} - C_{l_1}^{BB}) \left[+2G_{l_1 l_1 11}^{m_1 m_1 M M} + -2G_{l_1 l_1 11}^{m_1 m_1 M M} \right] \right] \right], \quad (88) \end{aligned}$$

similarly we can obtain the expression for C_l^{BB} as,

$$\begin{aligned} \tilde{C}_{l_1}^{BB} = & C_{l_1}^{BB} + \frac{1}{2}\beta^2 \left[\sum_{l_2 m_2} \sum_M +2I_{l_1 l_2 1}^{m_1 m_2 M} \left[(C_{l_2}^{EE} + C_{l_2}^{BB}) +2I_{l_1 l_2 1}^{m_1 m_2 M} - (C_{l_2}^{EE} - C_{l_2}^{BB}) -2I_{l_1 l_2 1}^{m_1 m_2 M} \right] \right. \\ & + \sum_M \left[(C_{l_1}^{EE} + C_{l_1}^{BB}) +2J_{l_1 l_1 11}^{m_1 m_1 M M} - \frac{1}{2} (C_{l_1}^{EE} - C_{l_1}^{BB}) \left[+2J_{l_1 l_1 11}^{m_1 m_1 M M} + -2J_{l_1 l_1 11}^{m_1 m_1 M M} \right] \right] \\ & + (b_\nu)^2 \sum_M \sum_{l_2 m_2} +2H_{l_1 l_2 1}^{m_1 m_2 M} \left[(C_{l_2}^{EE} + C_{l_2}^{BB}) +2H_{l_1 l_2 1}^{m_1 m_2 M} - (C_{l_2}^{EE} - C_{l_2}^{BB}) -2H_{l_1 l_2 1}^{m_1 m_2 M} \right] \\ & - 2b_\nu \sum_M \sum_{l_2 m_2} +2H_{l_1 l_2 1}^{m_1 m_2 M} \left[(C_{l_2}^{EE} + C_{l_2}^{BB}) +2I_{l_1 l_2 1}^{m_1 m_2 M} - (C_{l_2}^{EE} - C_{l_2}^{BB}) -2I_{l_1 l_2 1}^{m_1 m_2 M} \right] \\ & \left. - b_\nu \sum_M \left[2(C_{l_1}^{EE} + C_{l_1}^{BB}) +2G_{l_1 l_1 11}^{m_1 m_1 M M} - (C_{l_1}^{EE} - C_{l_1}^{BB}) \left[+2G_{l_1 l_1 11}^{m_1 m_1 M M} + -2G_{l_1 l_1 11}^{m_1 m_1 M M} \right] \right] \right]. \quad (89) \end{aligned}$$

The TE correlation can be obtained as,

$$\begin{aligned}
\tilde{C}_{l_1}^{TE} &= C_{l_1}^{TE} + \beta^2 \left[\sum_{l_2 m_2} \sum_M C_{l_2}^{TE} {}_{+2}I_{l_1 l_2 1}^{m_1 m_2 M} {}_0I_{l_1 l_2 1}^{m_1 m_2 M} + \sum_M \frac{1}{2} C_{l_1}^{TE} {}_{+2}J_{l_1 l_1 11}^{m_1 m_1 M M} \right. \\
&+ \sum_M \frac{1}{2} C_{l_1}^{TE} {}_0J_{l_1 l_1 11}^{m_1 m_1 M M} + (b_\nu)^2 \sum_M \sum_{l_2 m_2} C_{l_2}^{TE} {}_{+2}H_{l_1 l_2 1}^{m_1 m_2 M} {}_0H_{l_1 l_2 1}^{m_1 m_2 M} \\
&- b_\nu \left(\sum_M \sum_{l_2 m_2} C_{l_2}^{TE} {}_{+2}H_{l_1 l_2 1}^{m_1 m_2 M} {}_0I_{l_1 l_2 1}^{m_1 m_2 M} - \sum_M \sum_{l_2 m_2} C_{l_2}^{TE} {}_{+2}I_{l_1 l_2 1}^{m_1 m_2 M} {}_0H_{l_1 l_2 1}^{m_1 m_2 M} \right. \\
&\left. \left. \sum_M C_{l_1}^{TE} {}_{+2}G_{l_1 l_1 11}^{m_1 M m_1 M} - \sum_M C_{l_1}^{TE} {}_0G_{l_1 l_1 11}^{m_1 M m_1 M} \right) \right]. \tag{90}
\end{aligned}$$

After summing over m_2, M by using the properties of Clebsch-Gordan coefficients in eq.(88), (89), (90), we can get,

$$\begin{aligned}
\sum_{m_2 M} {}_{+2}H_{l_1 l_2 1}^{m_1 m_2 M} {}_{+2}H_{l_1 l_2 1}^{m_1 m_2 M} &= \frac{\Pi_{l_2}^2}{4\pi} C_{l_2 2 l_1 - 2}^{10} C_{l_2 2 l_1 - 2}^{10}, \\
\sum_{m_2 M} -{}_{-2}H_{l_1 l_2 1}^{m_1 m_2 M} {}_{+2}H_{l_1 l_2 1}^{m_1 m_2 M} &= (-1)^{l_1 + l_2 + 1} \frac{\Pi_{l_2}^2}{4\pi} C_{l_2 2 l_1 - 2}^{10} C_{l_2 2 l_1 - 2}^{10}, \\
\sum_{m_2 M} {}_{+2}I_{l_1 l_2 1}^{m_1 m_2 M} {}_{+2}I_{l_1 l_2 1}^{m_1 m_2 M} &= \frac{\Pi_{l_2}^2}{16\pi} \left[l_2(l_2 + 1) + 2 - l_1(l_1 + 1) \right]^2 C_{l_2 2 l_1 - 2}^{10} C_{l_2 2 l_1 - 2}^{10}, \\
\sum_{m_2 M} -{}_{-2}I_{l_1 l_2 1}^{m_1 m_2 M} {}_{+2}I_{l_1 l_2 1}^{m_1 m_2 M} &= (-1)^{l_1 + l_2 + 1} \frac{\Pi_{l_2}^2}{16\pi} \left[l_2(l_2 + 1) + 2 - l_1(l_1 + 1) \right]^2 C_{l_2 2 l_1 - 2}^{10} C_{l_2 2 l_1 - 2}^{10}, \tag{91}
\end{aligned}$$

and defining,

$$\begin{aligned}
{}_{\pm 2}M_{l+1l1} &= \frac{\Pi_{l+1}}{\sqrt{4\pi}} C_{l+1 \pm 2 l_1 \mp 2}^{10}, \\
{}_{\pm 2}M_{l-1l1} &= \frac{\Pi_{l-1}}{\sqrt{4\pi}} C_{l-1 \pm 2 l_1 \mp 2}^{10}, \\
{}_{\pm 2}N_{l+1l1} &= \frac{\Pi_{l+1}}{\sqrt{4\pi}} (l_1 + 2) C_{l+1 \pm 2 l_1 \mp 2}^{10}, \\
{}_{\pm 2}N_{l-1l1} &= \frac{\Pi_{l-1}}{\sqrt{4\pi}} (l_1 - 1) C_{l-1 \pm 2 l_1 \mp 2}^{10}. \tag{92}
\end{aligned}$$

Also, the integral in ${}_{\pm 2}G_{l_1 l_1 11}^{m_1 m_1 m_3 M}$ can be obtained by using the properties of the spherical harmonics from [13],

$$\begin{aligned}
\sum_M {}_{\pm 2}G_{l_1 l_2 l_3 L}^{m_1 m_2 m_3 M} &= \sum_J \frac{\Pi_J^2 \left[l_3(l_3 + 1) + l_2(l_2 + 1) - J(J + 1) \right]}{8\pi} C_{l_2 \pm 2 J \mp 2}^{l_3 0} C_{l_1 \pm 2 J \mp 2}^{L 0} \delta_{l_1 l_2} \delta_{m_1 m_2}, \\
\sum_M {}_0G_{l_1 l_2 l_3 L}^{m_1 m_2 m_3 M} &= \sum_J \frac{\Pi_J^2 \left[l_3(l_3 + 1) + l_2(l_2 + 1) - J(J + 1) \right]}{8\pi} C_{l_2 0 J 0}^{l_3 0} C_{l_1 0 J 0}^{L 0} \delta_{l_1 l_2} \delta_{m_1 m_2}, \tag{93}
\end{aligned}$$

and the value of the $\sum_M +2 J_{l_1 l_1 11}^{m_1 m_1 M M}$ can be obtained from the [12] as,

$$\begin{aligned} \sum_M {}_0 J_{l l l l}^{m m M M} &= \int d\hat{n} Y_{lm}^* \nabla_i Y_{LM}(\hat{n}) \nabla_j Y_{LM}^*(\hat{n}) \nabla^i \nabla^j Y_{lm}(\hat{n}), \\ &= -\frac{\Pi_l}{8\pi} l L(l+1)(L+1), \end{aligned} \tag{94}$$

$$\begin{aligned} \sum_M {}_{\pm 2} J_{l l l l}^{m m M M} &= \int d\hat{n}_{\pm} Y_{lm}^* \nabla_i Y_{LM}(\hat{n}) \nabla_j Y_{LM}^*(\hat{n}) \nabla^i \nabla^j {}_{\pm 2} Y_{lm}(\hat{n}), \\ &= -\frac{\Pi_l}{8\pi} l(l+1) \left[L(L+1) - 4 \right]. \end{aligned} \tag{95}$$

The final results obtained using in these calculations are given in eqs.(41), (42), (43) in Sec. 3.1. As we mentioned in Sec. 3.1, the coefficients of β^2 in the correction terms are less than unity.

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