

## Magnetic helicity in galactic dynamos

Kandaswamy Subramanian\*

*Inter University Centre for Astronomy and Astrophysics,  
Post bag 4, Pune University campus, Ganeshkhind, Pune 411 007*

2008 February 1

**Abstract.** Magnetic fields correlated on kiloparsec scales are seen in spiral galaxies. Their origin could be due to amplification of a small seed field by a turbulent galactic dynamo. We review the current status of the galactic dynamo, especially the constraints imposed by magnetic helicity conservation. We estimate the minimal strength of the large-scale magnetic field which could arise in spite of the helicity constraint.

*Keywords :* Galaxies, Galactic magnetic fields, dynamos, magnetic helicity

### 1. The galactic dynamo

Magnetic fields in spiral galaxies have strengths of order few  $10^{-6}G$ , and are coherent on scales of several kpc (Beck et al. 1996). In several disk galaxies, like M51 and NGC 6946, they are also highly correlated (or anti-correlated) with the optical spiral arms. How do such ordered, large-scale fields arise? One possibility is the dynamo amplification of a weak but nonzero seed field. We critically review here the operation of the galactic dynamo, particularly emphasizing the constraints which arise due to the conservation of magnetic helicity in highly conducting plasma.

The evolution of the magnetic field is described by the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}). \quad (1)$$

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\*e-mail:kandu@iucaa.ernet.in

Here  $\mathbf{B}$  is the magnetic field,  $\mathbf{v}$  the velocity of the fluid and  $\eta$  the resistivity.  $\mathbf{B} = 0$  is a perfectly valid solution of the induction equation. So there would be no magnetic field generated if one were to start with a zero magnetic field. There are a number of battery mechanisms, invoking small additional source terms to Ohms law, which lead to a seed magnetic field from zero fields (cf. Rees 1994; Subramanian, Narasimha and Chitre 1994). This seed field is generically much smaller than the galactic fields. Therefore some form of dynamo action, due to motions which act to exponentiate small seed fields efficiently, is essential to explain observed galactic fields.

Galactic dynamos depend on the following two features: First, disk galaxies are differentially rotating systems. Also the magnetic flux is to a large extent frozen into the fluid. So any radial component of the magnetic field will be efficiently wound up and amplified to produce a toroidal component. But this results in only a linear amplification of the field. To obtain the observed galactic fields starting from small seed fields one needs a way to generate the radial component from the toroidal one. If this can be done, the field can grow exponentially and one has a dynamo.

A mechanism to produce the radial field from the toroidal field was invented by Parker (1955), and is known as the  $\alpha$ -effect (Steenbeck, Krause and Radler 1966). The essential feature is to invoke the effects of cyclonic turbulence in the galactic gas (cf. Ferriere 1998). The interstellar medium is assumed to be turbulent, due to for example the effect of supernovae randomly going off in different regions. In a rotating, stratified (in density and pressure) medium like a disk galaxy, such turbulence becomes helical. An upward moving fluid parcel, expands and the coriolis force makes it rotate retrograde, generating negative kinetic helicity in the northern hemisphere. Downward moving fluid contracts, and the coriolis force now makes it rotate in the prograde direction. This contributes to helicity of the same sense. Helical motions of the gas perpendicular to the disk draws out the toroidal field into a loop which looks like a *twisted*  $\Omega$ . Such a twisted loop is connected to a current which has a component parallel to the original toroidal field. If the motions have a non-zero net helicity, this parallel component of the current adds up coherently. A toroidal current then results from the toroidal field. Hence, poloidal fields can be generated from toroidal ones. (Of course microscopic diffusion is essential to make permanent changes in the field). This closes the toroidal-poloidal cycle and leads to exponential growth of the mean field.

In quantitative terms, suppose the velocity field is the sum of a mean, large-scale velocity  $\mathbf{V}_0$  and a turbulent, stochastic velocity  $\mathbf{v}_T$ . The induction equation becomes a stochastic partial differential equation. Split the magnetic field  $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$ , into a mean field  $\mathbf{B}_0 = \langle \mathbf{B} \rangle$  and a fluctuating component  $\mathbf{b}$ . Here the average  $\langle \rangle$ , is defined either as a spatial average over scales larger than the turbulent eddy scales (but smaller than the system size) or as an ensemble average. Assume the turbulence to be isotropic, homogeneous, helical and have a short (ideally delta function) correlation time  $\tau$ . Then

one can derive the mean-field dynamo equation for  $\mathbf{B}_0$ ,

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\mathbf{V}_0 \times \mathbf{B}_0 + \varepsilon - \eta_T \nabla \times \mathbf{B}_0). \quad (2)$$

$$\varepsilon = \langle \mathbf{v}_T \times \mathbf{b} \rangle = \alpha_0 \mathbf{B}_0 - \eta_T \nabla \times \mathbf{B}_0. \quad (3)$$

Here  $\varepsilon$  is the turbulent emf,  $\alpha_0 = -(\tau/3) \langle \mathbf{v}_T \cdot (\nabla \times \mathbf{v}_T) \rangle$ , is the dynamo  $\alpha$ -effect, proportional to the kinetic helicity and  $\eta_T = (\tau/3) \langle \mathbf{v}_T^2 \rangle$ , is the turbulent magnetic diffusivity proportional to the kinetic energy of the turbulence. This kinematic mean-field dynamo equation, has exponentially growing solutions, provided a dimensionless dynamo number has magnitude  $D = |\alpha_0 G h^3 \eta_T^{-2}| > D_{crit} \sim 6$  (Ruzmaikin, Shukurov and Sokoloff 1988). (Here  $h$  is the disk scale height and  $G$  the galactic shear, and we have defined  $D$  to be positive). While the  $\alpha$ -effect is crucial for regeneration of poloidal from toroidal fields, the turbulent diffusion turns out to be also essential for allowing changes in the mean field flux. The mean field grows typically on time-scales a few times the rotation time scales, of order  $10^9$  yr. Modulations of  $\alpha$ , and  $\eta_T$  due to the spiral arms, can also lead to large-scale fields, correlated (or anti-correlated) with the optical spirals (Mestel and Subramanian 1991; Moss 1998; Shukurov 1998).

The kinematic mean-field equation neglects the back-reaction on the velocity due to the Lorentz forces. This rapidly becomes a bad approximation, due to the more rapid build up of magnetic noise compared to the mean field (Kulsrud and Anderson 1992). Both direct numerical simulations of the non-linear dynamo (Brandenburg 2001, Brandenburg and Sarson 2002, Brandenburg, Dobler and Subramanian 2002) and semi-analytic modelling of the non-linear effects (Subramanian 1999; Brandenburg and Subramanian 2000) point to the crucial role played by magnetic helicity conservation in limiting mean field growth.

## 2. Magnetic helicity conservation and the galactic dynamo

The magnetic helicity associated with a field  $\mathbf{B} = \nabla \times \mathbf{A}$  is defined as  $H = \int \mathbf{A} \cdot \mathbf{B} dV$ , where  $\mathbf{A}$  is the vector potential (Moffat 1978, Berger and Field 1984). Note that this definition of helicity is only gauge invariant (and hence meaningful) if the domain of integration is periodic, infinite or has a boundary where the normal component of the field vanishes. In this case, under a gauge transformation  $\mathbf{A} \rightarrow \mathbf{A} - \nabla \psi$ , the additional term in the helicity,  $\int \nabla \psi \cdot \mathbf{B} = \int \psi \mathbf{B} \cdot d\mathbf{S} - \int \psi \nabla \cdot \mathbf{B} dV = 0$ .  $H$  measures the linkages and twists in the magnetic field. From the induction equation one can easily derive the helicity conservation equation,

$$\frac{dH}{dt} = -2\eta \int \frac{4\pi}{c} \mathbf{J} \cdot \mathbf{B} dV, \quad (4)$$

where  $\mathbf{J} = (c/4\pi) \nabla \times \mathbf{B}$  is the current density. So in ideal MHD with  $\eta = 0$ , magnetic helicity is strictly conserved. However, this does not guarantee conservation of  $H$  in the

limit  $\eta \rightarrow 0$ , because the current helicity,  $\int \mathbf{J} \cdot \mathbf{B} dV$ , may still become large. For example, the Ohmic dissipation rate of magnetic energy  $Q_{\text{Joule}} \equiv \eta(4\pi/c^2) \int \mathbf{J}^2 dV$  can be finite and balance magnetic energy input by motions, even when  $\eta \rightarrow 0$ . This is because small enough scales develop in the field (current sheets) where the current density increases with decreasing  $\eta$  as  $\propto \eta^{-1/2}$  as  $\eta \rightarrow 0$ , whilst the rms magnetic field strength,  $B_{\text{rms}}$ , remains essentially independent of  $\eta$ . Even in this case, however, the rate of magnetic helicity dissipation *decreases* with  $\eta$  like  $\propto \eta^{+1/2} \rightarrow 0$ , as  $\eta \rightarrow 0$ . Thus, under many astrophysical conditions where  $R_m$  is large ( $\eta$  small), the magnetic helicity  $H$ , is almost independent of time, even when the magnetic energy is dissipated at finite rates.

Coming back to the mean-field dynamo, we note that its operation automatically leads to the growth of linkages between the toroidal and poloidal mean fields. Such linkages measure the helicity associated with the mean field. One then wonders how this mean field (galactic) helicity arises? To understand this, we need to split the helicity conservation equation into evolution equations of the sub-helicities associated with the mean field, say  $H_0 = \int \mathbf{A}_0 \cdot \mathbf{B}_0 dV$  and the fluctuating field  $h = \int \langle \mathbf{a} \cdot \mathbf{b} \rangle dV = \langle \mathbf{a} \cdot \mathbf{b} \rangle V$ . The evolution equations for  $H_0$  and  $h$  are

$$\frac{dH_0}{dt} = \int 2\varepsilon \cdot \mathbf{B}_0 dV - 2\eta \int \frac{4\pi}{c} \mathbf{J}_0 \cdot \mathbf{B}_0 dV \quad (5)$$

$$\frac{dh}{dt} = - \int 2\varepsilon \cdot \mathbf{B}_0 dV - 2\eta \int \frac{4\pi}{c} \langle \mathbf{j} \cdot \mathbf{b} \rangle dV. \quad (6)$$

Here, and henceforth, we assume that the surface terms can either be neglected or are zero (because of boundary conditions). We see that the turbulent emf  $\varepsilon$  transfers helicity between large and small scales; it puts equal and opposite amounts of helicity into the mean field and the small-scale field, conserving the total helicity  $H = H_0 + h$ . So if one were to start with zero total helicity initially, in a system with large  $R_m$ , one will always have  $H_0 + h \approx 0$ , or  $|H_0| \approx |h|$ .

Note that for a given amount of helicity, the energy associated with the field is inversely proportional to the scale over which the field varies. If for example, the small-scale field were maximally helical, and varied on a single scale, with associated wave number,  $k_f$ , we will have  $k_f \langle \mathbf{a} \cdot \mathbf{b} \rangle = \langle \mathbf{b}^2 \rangle$ . Similarly in a periodic box, a maximally helical large scale field with wave number  $k_m$ , satisfies,  $k_m \int dV \mathbf{A}_0 \cdot \mathbf{B}_0 = \int dV \mathbf{B}_0^2$ . (Henceforth, we will denote the volume average of mean field quantities  $X_0$  over the scale of the system,  $\int (dV/V) X_0$ , by  $\overline{X_0}$ ). So, helicity conservation, with  $|H_0| \approx |h|$ , implies  $\overline{\mathbf{B}_0^2} \approx (k_m/k_f) \langle \mathbf{b}^2 \rangle$ . Now in general,  $\langle \mathbf{b}^2 \rangle^{1/2}$ , will saturate near the equipartition field strength, say  $B_{eq}^2 = 4\pi\rho v_T^2$ . (Here  $\rho$  is the fluid density). So one obtains  $\overline{\mathbf{B}_0^2} \approx (k_m/k_f) B_{eq}^2 \ll B_{eq}^2$ , for  $k_m/k_f \ll 1$ . The mean field is expected to attain at most sub-equipartition values for  $R_M \gg 1$ , if helicity is strictly conserved.

The galactic dynamo also involves shear and the generation of the toroidal field by shear, does not involve the generation of net helicity. A periodic box simulation with an

imposed periodic shear (Brandenburg, Bigazzi and Subramanian 2001), suggests that the helicity constraint applies now to the product of the mean toroidal ( $B_t$ ) and poloidal fields ( $B_p$ ). So  $B_t B_p / k_m \approx | \langle \mathbf{a} \cdot \mathbf{b} \rangle | \approx \delta \langle \mathbf{b}^2 \rangle / k_f$ . Here  $\delta < 1$  takes into account that the small-scale field will also not be fully helical. In galaxies one usually has  $B_t / B_p = Q > 1$ . This implies  $B_t^2 \sim (Q\delta)(k_m/k_f)B_{eq}^2$ . In principal one can have large  $B_t$  at the cost of  $B_p$ , with strong shear. These estimates give upper limits to the mean-field strength with and without shear. However, in both cases, the limits are smaller than  $B_{eq}$  only by the *square root* of the ratio of small to large scales (and by a further factor  $(Q\delta)^{1/2}$ , in case of shear). Whether such mean field strengths are indeed realised, depends also on detailed dynamics of mean-field dynamo saturation, to which we now turn.

### 3. Modelling dynamo saturation and minimal mean galactic fields

As a crude model of how the dynamo saturates, when the dynamo is not too supercritical (see below), one may use the quasi-linear theory applicable to weak mean fields (cf. Pouquet, Frisch and Leorat 1976; Zeldovich, Ruzmaikin and Sokoloff 1983; Gruzinov and Diamond 1994; Bhattacharjee and Yuan 1995; Subramanian 2002a). This gives a re-normalised turbulent emf, with  $\alpha = \alpha_0 + \alpha_M$ , where  $\alpha_M = (\tau/3) \langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle / (4\pi\rho)$ , is proportional to the small-scale current helicity. The turbulent diffusion  $\eta_T$  is left unchanged to the lowest order, although to the next order there arises a non-linear hyperdiffusive correction to  $\varepsilon$  (Subramanian 2002a). One can now look for a combined steady state solution to the helicity conservation equation (6), and the mean-field dynamo equation. This work is in progress (Subramanian 2000b), and preliminary results are reported here. (Similar work is also being done by Brandenburg and Blackman (private communication); see also Brandenburg 2002; Field and Blackman 2002). Assume again that the small-scale field has a scale  $k_f^{-1}$ . We then have  $\langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle = k_f^2 \langle \mathbf{a} \cdot \mathbf{b} \rangle$  (although, for fields which are not maximally helical, these helicities can not be related to the energy). Using this, we can write  $dh/dt$  in Eq. (6), in terms of  $d\alpha_M/dt$  and hence in terms of  $d\alpha/dt$ . Further, for  $R_m$  large enough that helicity is conserved, and for a large-scale field which varies on scale  $k_m^{-1}$ , we can approximately write,  $(\nabla \times \mathbf{B}_0) \cdot \mathbf{B}_0 \approx k_m^2 \overline{\mathbf{A}_0 \cdot \mathbf{B}_0} = -k_m^2 \langle \mathbf{a} \cdot \mathbf{b} \rangle = -(k_m/k_f)^2 \langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle$ . This relation is of course strictly valid only if  $H_0$  is well defined. This inturn requires that negligible mean magnetic flux leave the disk, which is likely in thin disk dynamos (cf. Ruzmaikin, Shukurov and Sokoloff 1988). (A more careful treatment will involve using the relative helicity of Berger and Field 1984). The helicity conservation equation then gives a dynamical equation for  $\alpha$ -quenching

$$\frac{1}{\eta_T k_f^2} \frac{d\alpha}{dt} = -2\alpha \frac{\overline{\mathbf{B}_0^2}}{B_{eq}^2} - 2 \frac{k_m^2}{k_f^2} (\alpha - \alpha_0) - 2 \frac{\eta}{\eta_T} (\alpha - \alpha_0) \quad (7)$$

We see that non-linear effects leads to a decrease in  $\alpha$  with time, till the RHS side Eq. (7) becomes zero. (Such quenching of  $\alpha$  has earlier been discussed by Kleorin and Ruzmaikin

1982; Zeldovich, Ruzmaikin and Sokoloff 1983). As  $\alpha$  decreases, the effective dynamo number of the galactic dynamo,  $D = |\alpha Gh^3 \eta_T^{-2}|$ , will also decrease from an initial value  $D_0 = |\alpha_0 Gh^3 \eta_T^{-2}|$  and lead to a saturation of the mean field growth when  $D = D_{crit}$ . This happens when  $\alpha = \alpha_{sat} = \alpha_0 (D_{crit}/D_0)$ . The stationary solution for *both* the dynamical  $\alpha$  quenching equation and the mean field dynamo equations is then obtained by equating the RHS of Eq. (7) to zero, and substituting a value of  $\alpha = \alpha_{sat}$  given above. For  $\eta/\eta_T \ll (k_m/k_f)^2$ , this gives an estimated mean field strength  $B_{mean} = |\mathbf{B}_0|$

$$B_{mean} \approx [(D_0/D_{crit}) - 1]^{1/2} \frac{k_m}{k_f} B_{eq}. \quad (8)$$

For galaxies  $D_0 \sim 10 - 20$ . If we adopt  $D_0/D_{crit} \sim 2$ ,  $k_m/k_f \sim l/h$ , where  $l \sim 100pc$  is the forcing scale of the turbulence,  $h \sim 400 - 1000pc$ , then the mean field strength would be 1/4 to 1/10 of equipartition at saturation. This estimate is more pessimistic than the more general limit  $B_t < (Q\delta)^{1/2} (k_m/k_f)^{1/2} B_{eq}$ , in section 2. This is basically because, for  $D_0$  exceeding but near  $D_{crit}$ , the large-scale dynamo saturates even with modest  $\alpha$  suppression, after which there is no further helicity transfer ( $\delta \ll 1$ ). On the other hand, for  $D_0 \gg D_{crit}$  Eq. (8) seems to suggest fields greater than obtained in section 2. This rather points to the limitations of the quasi-linear model for non-linear saturation, in this case, than to a violation of the more general limit.

A major caveat to the above limits is that galaxies have boundaries, and if one has a flux of helicity due to small-scale fields preferentially leaving the system then one may avoid the above constraints (cf. Blackman and Field 2000; Kleeorin et al. 2000). Artificial removal of small-scale fields in a simulation, periodically, does indeed lead to enhanced large-scale field growth; so the idea works in principle (Brandenburg, Dobler and Subramanian 2002). But so far the simulations which involve boundaries do not show a preferential out-flux of small-scale field helicity (Brandenburg and Dobler 2001). Another possibility is to think of a non-helical dynamo (Vishniac and Cho 2001), but there is no evidence yet for its working in a simulation by Arlt and Brandenburg (2001) designed to capture the effect. Clearly, thinking of ways out of the constraints implied by helicity conservation will be crucial to understand galactic magnetism.

## Acknowledgements

I thank Axel Brandenburg and S. Sridhar for insightful comments.

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