

Cosmological parameter estimation with free-form primordial power spectrum

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Abstract

Constraints on the main cosmological parameters using CMB or large scale structure data are usually based on power-law assumption of the primordial power spectrum (PPS). However, in the absence of a preferred model for the early universe, this raises a concern that current cosmological parameter estimates are strongly prejudiced by the assumed power-law form of PPS. In this paper, for the first time, we perform cosmological parameter estimation allowing the free form of the primordial spectrum. This is in fact the most general approach to estimate cosmological parameters without assuming any particular form for the primordial spectrum. We use direct reconstruction of the PPS for any point in the cosmological parameter space using recently modified Richardson-Lucy algorithm however other alternative reconstruction methods could be used for this purpose as well. We use WMAP 9 year data in our analysis considering CMB lensing effect and we report, for the first time, that the flat spatial universe with no cosmological constant is ruled out by more than 4σ confidence limit without assuming any particular form of the primordial spectrum. This would be probably the most robust indication towards dark energy using CMB data alone. Our results on the estimated cosmological parameters show that higher values of baryonic and matter density and lower value of Hubble parameter (in comparison to the estimated values by assuming power-law PPS) is preferred by the data. However, the estimated cosmological parameters by assuming free form of the PPS have overlap at 1σ confidence level with the estimated values assuming power-law form of PPS.

1 Introduction

The observables of the perturbed universe, such as CMB anisotropy, galaxy surveys and weak lensing, all depend on a set of cosmological parameters assuming a background model

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describing the current universe, as well as the parameters characterizing the presumed nature of the initial perturbations. While certain characteristics of the initial perturbations, such as the adiabatic nature and tensor contribution, can and are being tested independently, the shape of the PPS remains, at best, a well-motivated assumption. It is important to distinguish between the cosmological parameters within a model that describe the present universe from that characterizing the initial conditions, specifically the PPS, P_k . The standard model of cosmology which is the most popular and widely used cosmological model is the spatially flat Λ CDM model which incorporates a power law form of the primordial power spectrum. The model is described by 6 parameters. Four of them describes the background Λ CDM, represented by Ω_b (baryon density), Ω_{CDM} (cold dark matter density), H_0 (present rate of expansion of the universe)¹ and the reionization optical depth τ . We should mention, dark energy density Ω_Λ is directly obtained as a remainder of baryon and cold dark matter density from the total density as we have assumed spatially flat model of the universe. The other two parameters in the model describe the form of the primordial power spectrum which is assumed to be the power law defined by $P(k) = A_s \left[\frac{k}{k_*}\right]^{n_s-1}$ where A_s is the amplitude² and the tilt is given by the spectral index n_s . The imposed form of the primordial spectrum allows us to provide tight constraints on the four background parameters, however, these tight constraints are basically the result of the rigidity of the model and certain assumption of the primordial spectrum. In other words choosing different assumptions for the form of the primordial spectrum can result to different constraints on the background cosmological parameters [1]. In this paper, for the first time we study the complete Markov Chain Monte Carlo parameter estimation assuming a free form of the primordial spectrum through direct reconstruction of the PPS for each point in the background cosmological parameter space using WMAP 9 year data [2, 3]. There have been different interesting attempts to directly reconstruct the form of the primordial spectrum [1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25] and in our analysis we use the recently modified Richardson-Lucy algorithm [25]. We show that it is indeed possible to do the cosmological parameter estimation allowing free form of the primordial spectrum and for the first time we report that a spatially flat universe without cosmological constant is ruled out by more than 4σ confidence limit using CMB data alone. This is without putting any prior constraints on the Hubble parameter or using any other cosmological observation. We show that assuming the free form of the primordial spectrum, the confidence limits of the background cosmological parameters will expectedly become larger than those we get by assuming the power-law form of the PPS and we present that the data prefers larger values of baryonic and matter densities for the free form of the primordial spectrum in comparison with power-law assumption. In the next section we shall discuss the methodology of reconstruction and the parameter estimation followed by which we shall demonstrate our results in the results section. We close with a brief discussion at the end.

¹Sometimes ratio of the sound horizon to the angular diameter distance at decoupling, θ is considered to be a parameter instead of H_0

²Note that this amplitude is defined at some pivot scale k_*

2 Formalism

In this work, we have used the recently modified version of Richardson-Lucy algorithm [25] (we call it here after MRL) to reconstruct the optimal form of the primordial spectrum for each point in the background cosmological parameter space. The Richardson-Lucy algorithm [26, 27, 28, 29] has been used previously in this context to reconstruct the primordial spectrum [6, 13, 14] and in the recently modified version one can use the combination of un-binned and binned data in the analysis [25]. The modified algorithm can be formulated as,

$$\begin{aligned}
 P_k^{(i+1)} - P_k^{(i)} &= P_k^{(i)} \times \left[\sum_{\ell=2}^{\ell=900} \tilde{G}_{\ell k}^{\text{un-binned}} \left\{ \left(\frac{C_\ell^{\text{D}} - C_\ell^{\text{T}(i)}}{C_\ell^{\text{T}(i)}} \right) \tanh^2 \left[Q_\ell (C_\ell^{\text{D}} - C_\ell^{\text{T}(i)}) \right] \right\}_{\text{un-binned}} \right. \\
 &+ \left. \sum_{\ell_{\text{binned}} > 900} \tilde{G}_{\ell k}^{\text{binned}} \left\{ \left(\frac{C_\ell^{\text{D}} - C_\ell^{\text{T}(i)}}{C_\ell^{\text{T}(i)}} \right) \tanh^2 \left[\frac{C_\ell^{\text{D}} - C_\ell^{\text{T}(i)}}{\sigma_\ell^{\text{D}}} \right]^2 \right\}_{\text{binned}} \right], \quad (1)
 \end{aligned}$$

Where, $P_k^{(i+1)}$ and $P_k^{(i)}$ are the power spectrum evaluated in iterations $i+1$ and i respectively and the quantity $\tilde{G}_{\ell k}$ is normalized radiative transport kernel $G_{\ell k}$. C_ℓ^{D} and σ_ℓ^{D} are the observed C_ℓ^{TT} data points and the corresponding diagonal terms of the inverse covariance matrix. $C_\ell^{\text{T}(i)}$ is theoretically calculated angular power spectrum in i 'th iteration.

Q_ℓ is given by the following expression,

$$Q_\ell = \sum_{\ell'} (C_{\ell'}^{\text{D}} - C_{\ell'}^{\text{T}(i)}) \text{COV}^{-1}(\ell, \ell'), \quad (2)$$

where $\text{COV}^{-1}(\ell, \ell')$ is the inverse of the error covariance matrix.

The radiative transport kernel, $G_{\ell k}$ which depends on the background cosmological parameters, satisfies the following equation.

$$C_\ell = \sum_i G_{\ell k_i} P_{k_i} \quad (3)$$

We should mention that as has been indicated in eq. 1 we have used the un-binned data till $\ell = 900$ and used the binned data thereafter because of the increasing noise in higher multipoles. For more discussion on this procedure see, ref. [25].

We use publicly available CAMB [30, 31] to calculate the kernel and the C_ℓ 's and CosmoMC [32, 33] to perform the Markov Chain Monte Carlo (MCMC) analysis on the cosmological parameters. Most recent WMAP nine-year observational data [3] has been used in this analysis. We have taken into account the effect of gravitational lensing through approximating the lensing effect on different background models. Basically we assume that for each point in the background cosmological parameter space the lensing effect on the observed angular power spectrum would be the same if we assume the power-law PPS or we use the reconstructed PPS. To do so, for each point in the background parameter space one can identify the contribution of gravitational lensing towards C_ℓ^{TT} 's for the best fit power law primordial spectrum and subtract that part from the data and performed the reconstruction.

Finally during Monte Carlo analysis we include gravitational lensing corresponding to the reconstructed spectrum in our analysis. We have performed the MRL up to 40 iterations. However we have checked the consistency of our results allowing MRL to work up to different iteration numbers.

3 Results

Following the methodology described in the previous section, here we shall discuss the result of our MCMC analysis with the nine year data from WMAP. We find the best fit model provides a χ^2 ($-2 \ln \mathcal{L}$) of 7441.4 which is about 115 better than what we get from the best fit power law spectrum. We compare the bounds on the background cosmological parameters with the power law results in figure 1. As we are allowing a free form primordial power spectrum, hence having larger effective degrees of freedom, it is expected to have bigger confidence contours compared to the confidence limits derived by assuming power law PPS and fig. 1 clearly illustrates this fact. The results from Fig. 1 suggest that allowing free form PPS, data prefers higher baryon and dark matter densities compared to the canonical results. Best fit values of $\Omega_b h^2$, $\Omega_{\text{CDM}} h^2$, H_0 and τ are 0.0232, 0.132, 64 and 0.077 respectively. It should be pointed out that our analysis allows a lower value of optical depth (τ) which, in turn allows low redshift of reionization ($Z_{\text{re}} \sim 7$) compared to the power law results (see last plot of Fig. 1). Our results also indicates that independent of the form of the PPS, a flat model of universe with no cosmological constant is ruled out with a very high confidence. This is probably the first direct indication towards dark energy with high certainty from CMB data analysis alone. In figure 2 we have plotted the one dimensional marginalized likelihoods of the parameter Ω_Λ obtained using power law (in dashed red line) and allowing free form of the primordial spectrum. From the plot it can be clearly visualized that a low value of dark energy density is ruled out. Obtained result suggests that values of $\Omega_\Lambda < 0.25$ is ruled out at 4σ which implies a strong exclusion of $\Omega_\Lambda = 0$ with a very high confidence.

In Fig. 3 we have plotted a few (nearly 100) power spectra (in grey) reconstructed from the WMAP nine-year data with the kernels corresponding to the cosmological parameters within 95% (2σ) limits of the best fit. We have also demonstrated the best fit power spectrum from the punctuated inflation model [34] in green and the step models of inflation in blue [35] for comparison. The best fit power law power spectrum is also plotted (in red). We should mention that while in the reconstruction process we have used only temperature data, in the likelihood analysis polarization data also have been considered. It has been discussed in [25] that considering the WMAP polarization data cannot significantly improve the reconstruction procedure due to low quality of the polarization data.

We should mention that to check the validity of our approximation regarding lensing contribution, we have repeated our analysis without subtracting the power law lensing effects from the data and we find that the later comparison provides a χ^2 worse by 6 than the actual analysis which in turn indicates that our approximation on lensing contribution works well.

To check the robustness of our analysis and the validity of the obtained results, we performed some tests with simulated data. We have synthesized number of C_l^{TT} data from angular power spectrum obtained using power law and Λ CDM model with some fixed parameters. With the reconstructed free form spectrum we perform MCMC on the following

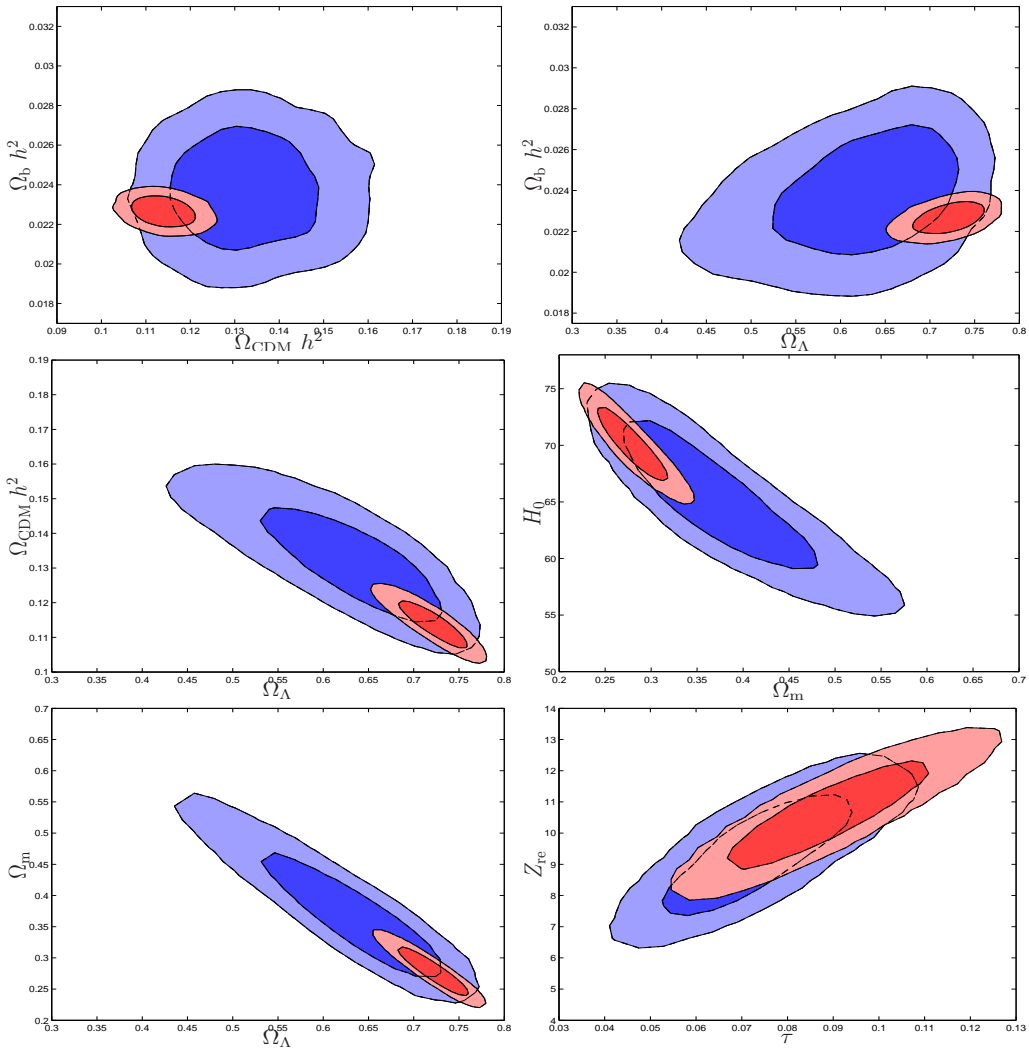


Figure 1: 2-dimensional confidence contours of the background cosmological parameters. The blue contours represents the results of the analysis allowing free form of the primordial spectrum while the red contours are by assuming the power law form of the primordial spectrum. As expected the free form of the primordial spectrum relaxes the bounds on the parameters. Throughout the analysis we have assumed a spatially flat universe and one can see that a universe with zero density for the cosmological constant is ruled out with high confidence even with no assumption for the primordial spectrum. In comparison with the results from power law assumption of the primordial spectrum, the data prefers higher values of baryonic and matter densities when we allow free form of the primordial spectrum.

datasets and calculated the likelihood assuming χ^2 distribution³. We find that in most of the cases (more than 90%) the obtained confidence contours of the cosmological parameter contains the fiducial parameter values within the 2σ region. This indicates, with the reconstruction we get our fiducial model back in almost all the cases, which, in turn proves the robustness of our analysis.

³We have shown recently [25] that this likelihood estimator is robust and can be used as an approximation to the complete WMAP likelihood

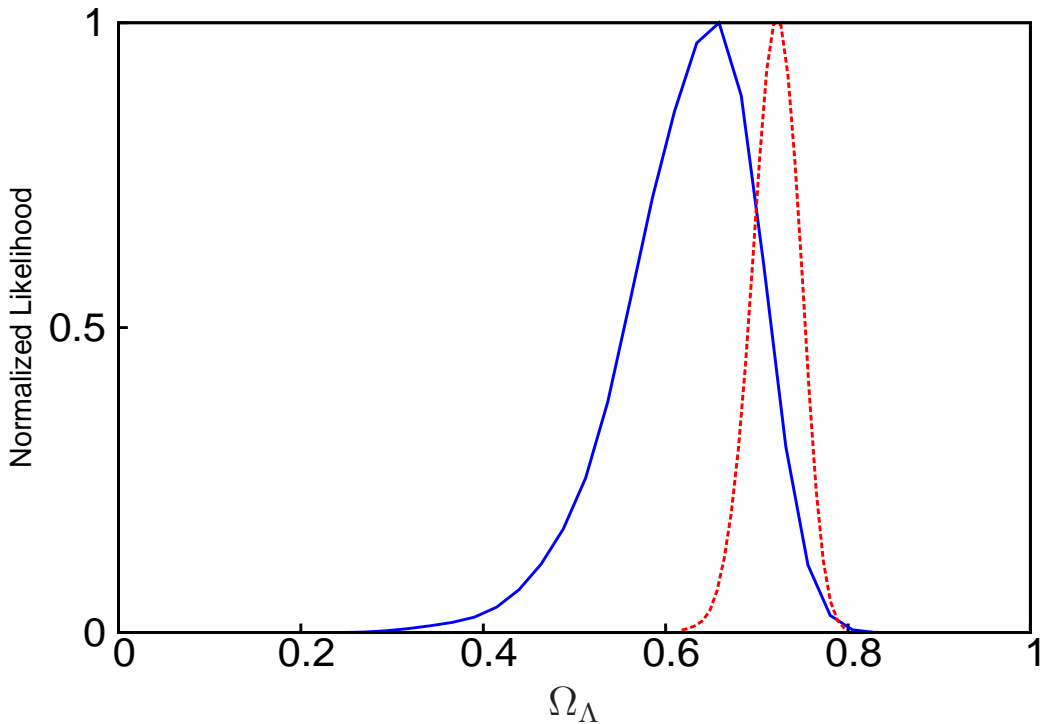


Figure 2: The one dimensional marginalized likelihood of dark energy density Ω_Λ obtained using free form of primordial spectrum (in solid blue line) and using power law (in dashed red line). $\Omega_\Lambda = 0$ is clearly not favored by the data even if we allow a power spectra free of forms. Quantitatively, in 4σ the data rules out $\Omega_\Lambda < 0.25$. This is probably the first indication towards presence of dark energy with a very high confidence using CMB data alone.

4 Discussion

In this paper we have estimated the cosmological parameters assuming free form of the primordial spectrum. The primordial spectrum for each point in the background cosmological parameter space is obtained using the MRL reconstruction procedure using the WMAP nine-year combined data of un-binned and binned angular power spectrum. We should mention that for this analysis and instead of MRL one can use trivially other alternative methods of non-parametric reconstruction of the primordial spectrum. In fact the MRL method serves just as a possible method of reconstruction to get a PPS that improves the fit of a cosmological model at different points in the cosmological parameter space to the CMB data. One is free to choose any other method to do this task. The background model is assumed to be a spatially flat Λ CDM model. Performing the MCMC analysis using CosmoMC we obtained the bounds on the background parameters and we find out that the data prefers higher baryonic and matter densities (hence lower Ω_Λ) and lower Hubble parameter when we assume the free form of the PPS in comparison with the case of power-law assumption. Our results indicates that independent of the form of the primordial spectrum and without any prior on the value of the Hubble parameter, the spatially flat universe with no cosmological constant is ruled out with a very high confidence using WMAP 9 year data alone. This is the first direct evidence of dark energy with a very high certainty from CMB data alone and

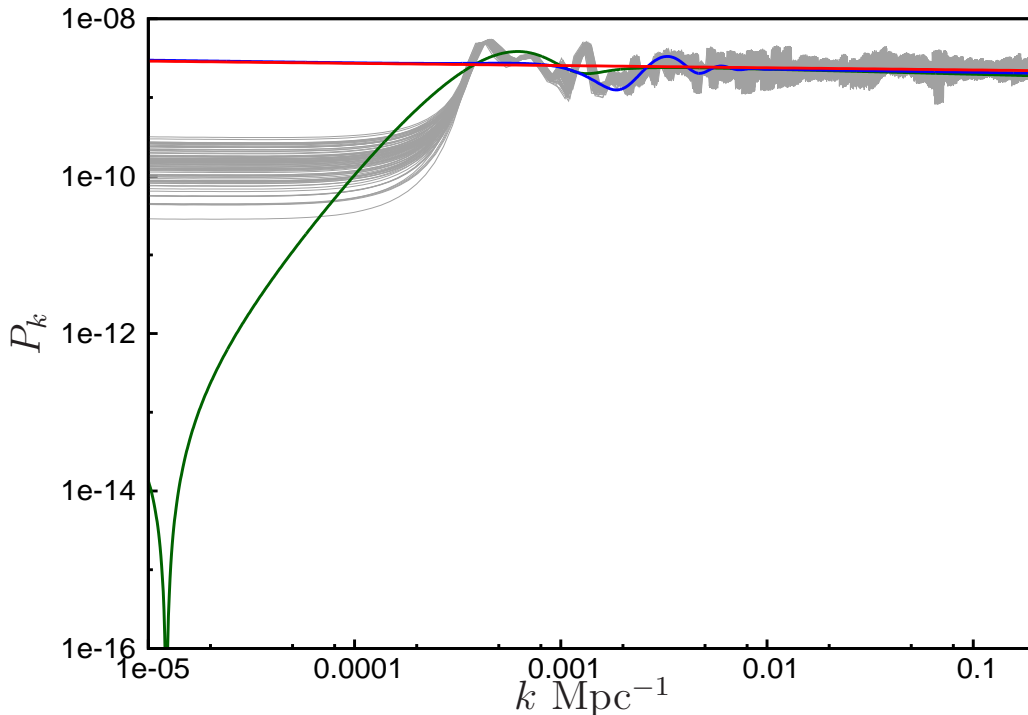


Figure 3: Reconstructed power spectrum (in grey) obtained from parameters lying within the 2σ range of the best-fit likelihood. Over the sample of the reconstructed spectra we have plotted the best fit spectra from the step model [35] (in blue) and the punctuated model [34] (in green) of inflation. The best fit power law power spectrum (in red) is plotted as well for comparison. Note that barring the low- k region (which data has low sensitivity) the sample of reconstructed spectra incorporates nearly all the models within its 2σ variation.

with no prior on the Hubble parameter or assuming the form of the primordial spectrum. We expect to get tighter constraints on the background parameters assuming the free form of the primordial spectrum using upcoming Planck data [36].

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