Indo-UK Seminar

Cosmological viability of f(R) theory of gravity

by

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Late time accelerated expansion

↙ ↘

Einstein's GR	**Modified Gravity**
+	↓
Cosmological Constant	$f(R)$ **Gravity**
or	**with**
Dark Energy	**No Dark Energy**

- **From the cosmological dynamics are $f(R)$ theories distinguishable from dark energy ?**
- **Can one design an $f(R)$ theory for each DE model ?**
- **In particular is it possible to design an $f(R)$ theory which behaves as dark energy with constant w.**

Alternative to dark energy → $f(R)$ theory

- May be Einstein's GR is not correct on large scales !

$$\mathcal{S} = \frac{1}{16\pi G}\int (\, R + f(R)\,)\,\sqrt{-g}\;\mathrm{d}^4x \; + \mathcal{S}_m$$

where $\mathcal{S}_m$ is the action for the matter content

S. Capozziello *et. al.* (2003) & S. Carroll *et. al.* (2004) suggested that

$$\mathcal{S} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R - \frac{\mu^4}{R}\right) + \mathcal{S}_m$$

can explain the accelerated expansion without dark energy!

A deeper look at the $f(R)$-theory

$$\mathcal{S} = \frac{1}{16\pi G} \int \left(R + f(R) \right) \sqrt{-g}\, \mathrm{d}^4x \; + \mathcal{S}_m$$

Modified Einstein's Equation

$$(1 + f_{R}) G^{\mu}_{\ \nu} - g^{\mu\alpha} f_{R\,,\,\alpha\,;\,\nu} + \left(\frac{2\Box f_{R} - (f - R f_{R})}{2} \right) \delta^{\mu}_{\nu} = (8\pi G)\; T^{\mu}_{(m)\ \nu}$$

where $\qquad f_{R} \equiv \dfrac{\mathrm{d}f}{\mathrm{d}R}$

Trace of the above equation

$$\Box f_{R} + \left(\frac{R f_{R} - (2\,f + R)}{3} \right) = \left(\frac{8\pi G}{3} \right) T$$

The '*hidden*' scalar field in $f(R)$ theory

$$\phi \equiv 1 + f_{R}$$

Field equations in $f(R)$ theory becomes

$$G_{\mu\,\nu} = \left(\frac{8\pi G}{\phi}\right) T_{\mu\,\nu} + \frac{1}{\phi}\left(\phi_{;\,\mu\,;\,\nu} - g_{\mu\,\nu}\Box\phi\right) + \left(\frac{V(\phi)}{2}\right) g_{\mu\,\nu}$$

$$\Box\phi + \left(\frac{1}{3}\right)\left(\phi\left(\frac{\mathrm{d}V}{\mathrm{d}\phi}\right) - 2V(\phi)\right) = \left(\frac{8\pi G}{3}\right) T$$

where $\quad V(\phi) \equiv f - R f_{R}$

These equations are *identical* to the one in Brans-Dicke theory

$$\mathcal{S} = \frac{1}{16\pi G}\int\left(\phi\, R - \omega\left(\frac{\partial_\mu\phi\partial^\mu\phi}{\phi}\right) + V(\phi)\right)\sqrt{-g}\,\mathrm{d}^4x + \mathcal{S}_m$$

but with the BD parameter $\omega = 0$.

$f(R)$ theory $\Longleftrightarrow$ Brans-Dicke theory with $\omega = 0$

- **$f(R)$ theory of gravity are equivalent to Brans-Dicke theory with the parameter $\omega = 0$**

$$\mathcal{S} = \frac{1}{16\pi G}\int \left(R + f(R) \right) \sqrt{-g}\, \mathrm{d}^4x + \mathcal{S}_m$$

$$\Updownarrow$$

$$\mathcal{S} = \frac{1}{16\pi G}\int \left(\phi\, R + V(\phi) \right) \sqrt{-g}\, \mathrm{d}^4x + \mathcal{S}_m$$

- **This equivalence is strictly valid when $f_{RR} \neq 0$**

The designer $f(R)$ theory

- **We want to design an $f(R)$ theory for which $a(t)$ satisfies the following equation**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\left[\rho_{m0}\,a^{-3} + \rho_{d0}\,a^{-3(1+w)}\right]$$

The reconstruction equations:

$$f_R' + 3\left(\frac{1+3\,w\,\Omega_d(a)}{2a}\right) f_R - \frac{f}{6\,a\,H^2} = -\frac{\Omega_d(a)}{a}$$

$$f_R'' - 3\left(\frac{1+w\,\Omega_d(a)}{2a}\right) f_R' - 3\left(\frac{1+w\,\Omega_d(a)}{a^2}\right) f_R = \frac{3(1+w)\Omega_d(a)}{a^2}$$

where $\Omega_d(a) = \dfrac{\rho_{d0}\,a^{-3(1+w)}}{\rho_{m0}\,a^{-3} + \rho_{d0}\,a^{-3(1+w)}}$

Approximate $f(R)$ at $a << 1$

In the limit $a << 1$, the reconstruction equations reduces to

$$f'_R + \frac{3}{2}\left(\frac{f_R}{a}\right) - \frac{f}{6\,a\,H^2} = -\left(\frac{\Omega_{d0}}{\Omega_{m0}}\right) a^{-(1+3w)}$$

$$f''_R - \frac{3}{2}\left(\frac{f'_R}{a}\right) - 3\left(\frac{f_R}{a^2}\right) = \left(\frac{\Omega_{d0}}{\Omega_{m0}}\right) a^{-(2+3w)}$$

On solving gives:

$$f_R(a) = C_1\, a^3$$

where C_1 is the constant of integration.

The Constant C_1

The consistency requirement implies that

$$C_1 = 3\,(1+w)\left(\frac{\Omega_{d0}}{\Omega_{m0}}\right)a^{-3(1+w)}$$

But C_1 is the constant of integration ! !

Therefore, for an $f(R)$ theory to mimic DE with constant w

- **The only allowed value of w is $w = -1$**
- **This makes $C_1 = 0$**
- **Consequently $f_R = 0$, $f'_R = 0$, $f''_R = 0$, and so and so forth**

The only possible solution is

$$f(R) = \text{ constant}$$

Conclusion

If $f(R)$ **theory with action**

$$\mathcal{S} = \frac{1}{16\pi G} \int (\, R + f(R)\,)\, \sqrt{-g}\; \mathrm{d}^4x \; + \mathcal{S}_m$$

is to drive the the accelerated expansion of the universe as if it is due to dark energy with a constant w **in Einstein's GR**

$\Longrightarrow$ **The only allowed value of** w **for which it can do so is**

$$w = -1$$

$\Longrightarrow$ **But in this case the action reduces to**

$$\mathcal{S} = \frac{1}{16\pi G} \int (\, R + 2\Lambda\,)\, \sqrt{-g}\; \mathrm{d}^4x \; + \mathcal{S}_m$$

Implications

- **The result does'nt mean that $f(R)$ theories are not a viable alternative to dark energy.**
- **In fact, it demonstrates that in $f(R)$ theories, the resulting equation of state parameter of the equivalent dark energy must necessarily be epoch dependent.**

Immediate corollary to the above result

- **$w(a)$ in $f(R)$ theories must necessarily be very close to -1 during the matter dominated epoch**

S. Unnikrishnan, S. Thakur, T. R. Seshadri, arXiv:1106.6353

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* Thank you *

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