

IMPLEMENTING POWER LAW INFLATION WITH TACHYON ROLLING ON THE BRANE

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We study a minimally coupled tachyon field rolling down to its ground state on the FRW brane. We construct tachyonic potential which can implement power law inflation in the brane world cosmology. The potential turns out to be $V_0\phi^{-1}$ on the brane and reduces to inverse square potential at late times when brane corrections to the Friedmann equation become negligible. We also do similar exercise with a normal scalar field and discover that the inverse square potential on the brane leads to power law inflation.

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I. INTRODUCTION

Cosmological inflation has become an integral part of the standard model of the universe. Apart from being capable of removing the shortcomings of the standard cosmology, the paradigm seems to have gained a fairly good amount of support from the recent observations on microwave background radiation. On the other hand there have been difficulties in obtaining accelerated expansion from fundamental theories such as M/String theory. Recently, Sen [1, 2] has shown that the decay of an unstable D-brane produces pressure-less gas with finite energy density that resembles classical dust. Gibbons has emphasized the cosmological implications of tachyonic condensate rolling towards its ground state. [3]. Rolling tachyon matter associated with unstable D-branes has an interesting equation of state which smoothly interpolates between -1 and 0. The tachyonic matter, therefore, might provide an explanation for inflation at the early epochs and could contribute to some new form of dark matter at late times [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. An effective potential for tachyon condensate is computed in reference [18]; the expression is exact in α' but tree level in g_s . Sen [19] has shown that the choice of an exponential potential for the tachyonic field leads to the absence of plane-wave solutions around the tachyon vacuum and exponential decay of the pressure at late times. However, due to the limitations of non-perturbative calculation, it is perfectly legitimate, as pointed out by Padmanabhan [8], to con-

struct a potential leading to desired cosmological evolution. The evolution of FRW cosmological models and linear perturbations of tachyon matter rolling towards a minimum of its potential is discussed by Frolov, Kofman and Starobinsky [20].

Another interesting development in cosmology inspired by String theory is related to Brane World cosmology. In this scenario, our four dimensional space time is realized as an embedding or a boundary (brane) of the higher dimensional space time (bulk). In this picture all the matter fields are confined to the brane whereas gravity can propagate in the bulk. The scenario has interesting cosmological implications, in particular, the prospects of inflation are enhanced on the brane due to an additional quadratic term in density in the Friedmann equation. In this note, following reference [8], we construct potentials which can implement power law inflation on FRW brane in presence of normal scalar field as well the tachyonic field. The fact that one can construct $V(\phi)$ for a given $a(t)$ has been known earlier [21] and the method was practically used in reference [22]. Effects of tachyon in context of brane world cosmology are discussed in reference [23]. Dynamics of gauge fields with rolling tachyon on unstable D-branes is studied in reference [24, 25, 26]

A. SCALAR FIELD POTENTIAL FOR NON-TACHYONIC MATTER ON FRW BRANE

In the 4+1 dimensional brane world scenario inspired by Randall-Sundrum model [27], the Fried-

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mann equation is modified to[28]

$$H^2 = \frac{1}{3M_p^2} \rho \left(1 + \frac{\rho}{2\lambda_b} \right) + \frac{\Lambda_4}{3} + \frac{\mathcal{E}}{a^4} \quad (1)$$

where \mathcal{E} is an integration constant which transmits bulk graviton influence onto the brane and λ_b is the brane tension. For simplicity we set Λ_4 equal to zero and also drop the last term as otherwise the inflation would render it so, leading to the expression

$$H^2 = \frac{1}{3M_p^2} \rho \left(1 + \frac{\rho}{2\lambda_b} \right) \quad (2)$$

where $\rho \equiv \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ if one is dealing with universe dominated by a single scalar field minimally coupled to gravity. The pressure of the scalar field is given by $p \equiv p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ and the equation of motion of the field propagating on the brane is

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \quad (3)$$

The conservation equation equivalent to (3) is

$$\frac{\dot{\rho}_\phi}{\rho_\phi} + 3H(1 + \omega) = 0 \quad (4)$$

where $\omega = \frac{p_\phi}{\rho_\phi}$ is the equation of state for the scalar field. The brane effects in context of inflation are most pronounced in the high energy limit $V \gg \lambda_b$; the Friedmann equation in this limit becomes

$$H = \frac{\rho}{(6\lambda_b M_p^2)^{1/2}} \quad (5)$$

Using the Friedmann equation (5) and the conservation equation (4), one obtains the expression for $(1 + \omega)$

$$1 + \omega = -\frac{1}{3} \frac{\dot{H}}{H^2} \quad (6)$$

From the evolution equation (3) and the expression $\frac{\dot{\phi}^2}{2} = V(1 + \omega)(1 - \omega)^{-1}$, one obtains the differential equation for the potential V

$$\frac{\dot{V}}{V} = -\frac{\dot{f} + 6Hf}{1 + f} \quad (7)$$

where $f = (1 + w)(1 - w)^{-1}$ Integrating (7) leads to the expression for V as a function of time

$$V(t) = \frac{C}{6H} \left(\dot{H} + 6H^2 \right) \quad (8)$$

where $C = (6\lambda_b M_p^2)^{1/2}$. Expressing $f(t)$ in terms of H and its derivative and using equation (8) leads to the expression of $\phi(t)$

$$\phi(t) = \int \left(-\frac{C}{3} \frac{\dot{H}}{H} \right)^{1/2} dt \quad (9)$$

Equations (8) and (9) can be used to determine $\phi(t)$ and $V(t)$ for a given $a(t)$ on the brane. For $a(t) \propto t^n$

$$V(t) = C \left(1 - \frac{1}{6n} \right) \frac{n}{t} \quad (10)$$

$$\phi(t) - \phi_0 = \sqrt{\frac{4C}{3}} t^{1/2} \quad (11)$$

. Combining (10) and (11) we obtain the expression for the potential as a function of ϕ

$$V(\phi) = \frac{\lambda_b}{2} \left(1 - \frac{1}{6n} \right) \frac{n}{\left(\frac{\phi(t) - \phi_0}{M_p} \right)^2} \quad (12)$$

where $n > 1/6$. In usual 4-dimensional FRW cosmology, the power law inflation is implemented by an exponential potential whereas its counter part on the brane turns out to be very different. Indeed, we earlier investigated the prospects of inflation with exponential potential on the brane, in a different context, and found the exact solution of the problem. In particular, $a(t)$ was shown to have complicated time dependence through double exponential[29] .

B. POTENTIAL FOR TACHYON ROLLING ON THE BRANE

As recently demonstrated by Sen [1, 2], a rolling tachyon condensate in a specially flat FRW cosmological model is described by an effective fluid with energy momentum tensor $T_\nu^\mu = \text{diag}(-\rho, p, p, p)$, where the energy density ρ and pressure p are given by

$$\rho = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \quad (13)$$

$$p = -V(\phi)\sqrt{1 - \dot{\phi}^2} \quad (14)$$

The Friedmann equation on the FRW brane ($V(\phi) \gg 2\lambda_b$) for the tachyonic matter is same as equation (5) with energy density

given by (13). For the tachyonic field the expression for $(1 + \omega)$ is also given by equation (6); however, ω in the present case has a form

$$\omega = \dot{\phi}^2(t) - 1 \quad (15)$$

Making use of equations (6), (13), (14) and (15) we get the expressions for $V(t)$ and $\phi(t)$

$$\phi(t) = \int \left(-\frac{\dot{H}}{3H^2} \right)^{1/2} dt \quad (16)$$

$$V(t) = CH \left(1 + \frac{\dot{H}}{3H^2} \right)^{1/2} \quad (17)$$

Equations (16) and (17) analogous to (9) and (10) determine the tachyonic field and the tachyonic potential as a function of time for a given scale factor $a(t)$. In case of power law expansion of the universe $a(t) \propto t^n$ we obtain

$$\phi(t) - \phi_0 = \left(\frac{1}{3n} \right)^{1/2} t \quad (18)$$

$$V(t) = C \left(1 - \frac{1}{3n} \right)^{1/2} \frac{n}{t} \quad (19)$$

Using Eqs. (19) and (18) we obtain the expression for $V(\phi)$

$$V(\phi) = 2\lambda_b \sqrt{n} \left(1 - \frac{1}{3n} \right)^{1/2} \frac{1}{\left(\frac{\phi(t) - \phi_0}{M_p} \right)} \quad (20)$$

where $n > 1/3$. Taking the large value of n one may have the desired accelerated expansion with $V(\phi)$ given by (20). However, a comment regarding the range of n is in order. There are string theory arguments that the tachyonic matter

asymptotically evolves to a pressure-less gas , i.e. $\dot{\phi}(\infty) = 1$ irrespective of the form of tachyonic potential. And this in turn implies that at late times $a(t) \propto t^{2/3}$. In fact, the expressions obtained here are valid in the limit $V \gg \lambda_b$ which is a legitimate limit at times inflation was operative. But at late times the tachyon field rolls down its potential to the extent that $V \ll \lambda_b$ and in this limit the brane corrections disappear and equation (2) reduces to usual Friedmann equation allowing the scale factor to evolve asymptotically as $a(t) \propto t^n$ with $n = 2/3$ [7].

To sum up, we have constructed potentials on the FRW brane with and without tachyonic matter which successfully implement power law inflation. It would be interesting to carry out phase space analysis with tachyonic potential to investigate the late time behavior of the power law inflation in usual FRW cosmology as well as in the brane world scenario.

Inspite of the very attractive features of the rolling tachyon condensate, the tachyonic inflation faces difficulties associated with reheating [30, 31]. A homogeneous tachyon field evolves towards its ground state without oscillating about it and , therefore, the conventional reheating mechanism in tachyonic model does not work. Quantum mechanical particle production during inflation provides an alternative mechanism by means of which the universe could reheat. Unfortunately, this mechanism also does not seem to work: the small energy density of radiation created in this process redshifts faster than the energy density of the tachyon field. However, the tachyon field could play the dual role of quintessence and dark matter [7].

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