

Cosmological parameter estimation using Particle Swarm Optimization (PSO)

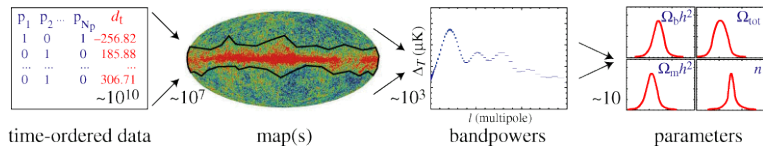
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CMBR data analysis pipeline



“In theory at least, individual members of the school can profit from the discoveries and previous experience of all other members of the school during the search for food. This advantage can become decisive, outweighing the disadvantages of competition for food items, whenever the resource is unpredictably distributed in patches”
[in reference to fish schooling]

- E. O. Wilson, 1975, *Sociobiology: The new synthesis*



Searching for the global maximum



Parameter estimation

- Search for the maximum likelihood point (ML).
 - Grid based methods - computational cost - exponentially
 - Stochastic methods - computational cost - linearly (MCMC)
- Artificial Intelligence (AI) based optimization methods - Artificial Neural Network [Phillips & Kogut (2001)], Genetic Algorithm [Yao & Sethares (1994)], Particle Swarm Optimization [Kennedy & Eberhart (1995)]
- We demonstrate the application of PSO for parameter estimation from WMAP 7 years data.



Particle Swarm Optimization (PSO)

- Equation of Motion :

$$X_{t+1}^i = X_t^i + V_{t+1}^i \quad (1)$$

- Velocity :

$$V_{t+1}^i = wV_t^i + c_1\xi_1(X_t^i - X_{Pbest}^i) + c_2\xi_2(X_t^i - X_{Gbest}) \quad (2)$$

- w : inertia weight
- c_1, c_2 : acceleration coefficients
- ξ_1, ξ_2 : random numbers
- X_{Pbest} : location of Pbest
- X_{Gbest} : location of Gbest

[Kennedy & Eberhart (1995); Wang & Mohanty (2010)]



Implementation

- Initial conditions:

$$\begin{aligned} X^i(t=1) &= X_{min}^i + \xi(X_{max}^i - X_{min}^i) \\ V^i(t=1) &= \xi V_{max}^i \end{aligned} \quad (3)$$

where

$$V_{max}^i = c_v(X_{max}^i - X_{min}^i) \quad (4)$$

- Boundary conditions (reflecting boundary):

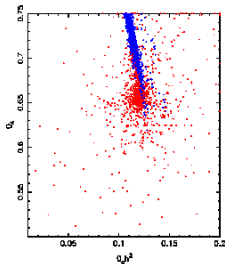
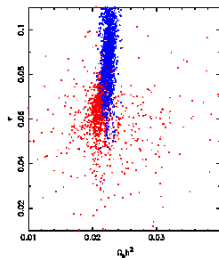
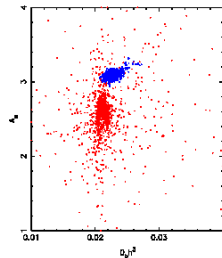
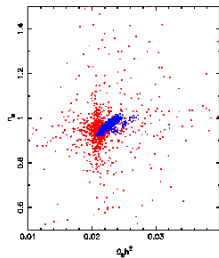
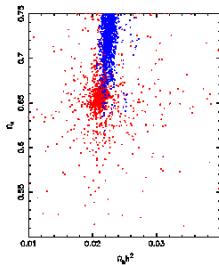
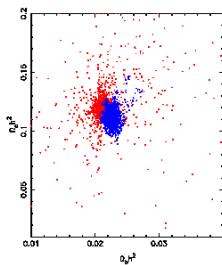
$$V_t^i = -V_t^i \quad \text{and} \quad X_t^i = \begin{cases} X_{max}^i, & \text{if } X_t^i > X_{max}^i \\ X_{min}^i, & \text{if } X_t^i < X_{min}^i \end{cases} \quad (5)$$

- Stopping criteria:

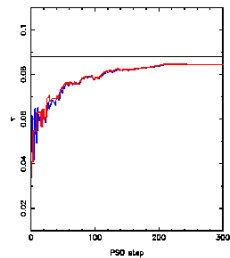
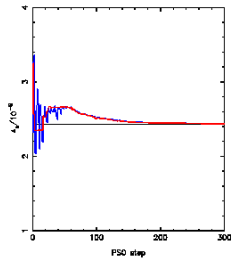
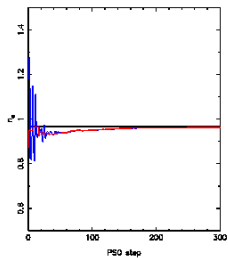
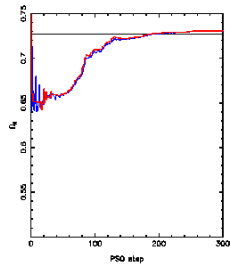
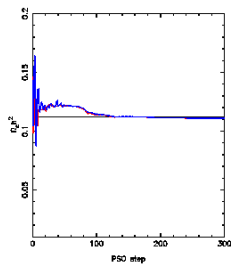
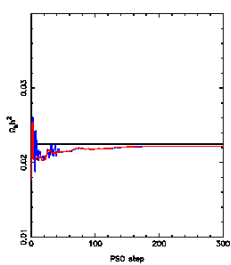
$$|\mathcal{F}(X_{Gbest,t+1} - \mathcal{F}(X_{Gbest,t}))| < \epsilon \quad \text{for } t = 1, n_{stop} \quad (6)$$



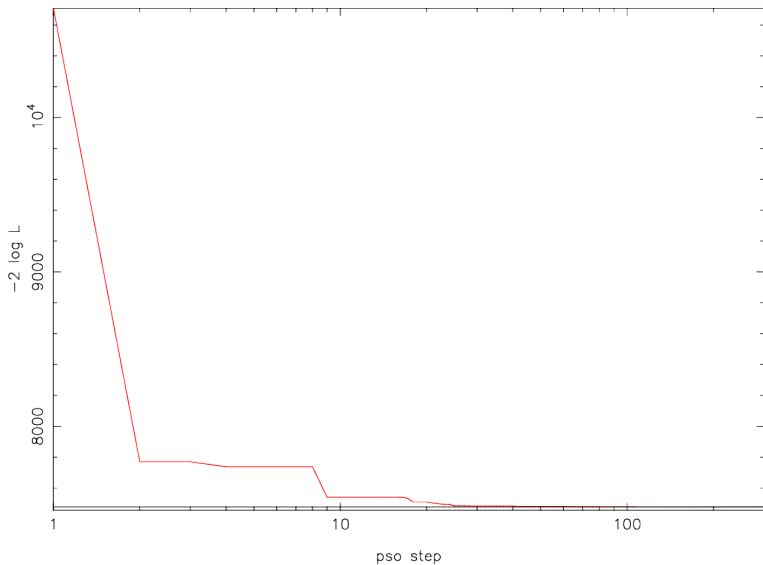
Sampling



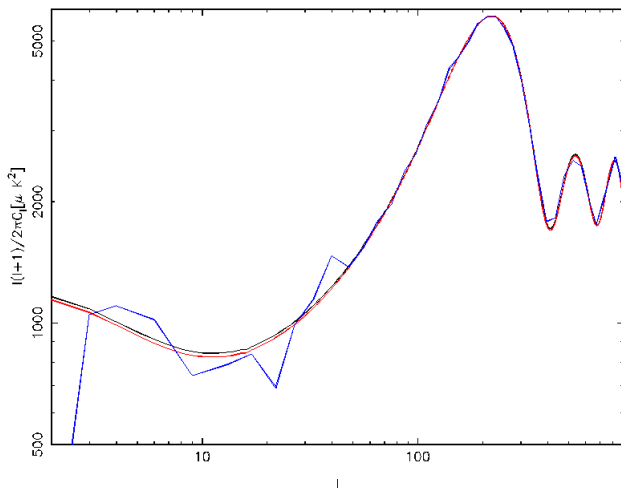
Gbest



Gbest



Power spectrum



Parameters

Cosmological parameters from PSO							
Variable	PSO best fit			WMAP best fit [30]		Difference	
	Range	Gbest	Mean	std dev.	ML		Mean
$\Omega_b h^2$	(0.01,0.04)	0.02219	0.02219	0.00049	0.02227	$0.02249^{+0.00056}_{-0.00057}$	-0.0001 (0.36 %)
$\Omega_c h^2$	(0.01,0.20)	0.1109	0.1109	0.0058	0.1116	0.1120 ± 0.0056	- 0.0051(4.40%)
Ω_Λ	(0.50,0.75)	0.731	0.731	0.034	0.729	$0.727^{+0.030}_{-0.029}$	+0.0020 (0.27%)
n_s	(0.50,1.50)	0.962	0.962	0.012	0.966	0.967 ± 0.014	-0.0040(0.41%)
$A_s/10^{-9}$	(1.0,4.0)	2.43	2.43	0.13	2.42	2.43 ± 0.11	0.01(0.41%)
τ	(0.01,0.11)	0.084	0.084	0.012	0.0865	0.088 ± 0.015	-0.0025(2.89%)

TABLE II. The first column in the above table shows the PSO fitting parameters and the second, third, fourth and fifth column show the search range, the location of the **Gbest**, average position of PSO particles and the error (which we have computed by fitting the sampled function) respectively. In the sixth and seventh columns we give the best fit (ML) and average values of parameters derived from WMAP seven years likelihood estimation respectively. In the last column we give difference between our best fit parameters (PSO parameters) and WMAP team's best fit parameters. From this table it is clear that roughly there is good agreement between the PSO best fit parameters and WMAP team's best fit parameters from the seven years data.



Summary & conclusions

- Parameter estimation is an important exercise in current cosmological research.
- We have demonstrated that PSO also can be used for parameter estimation from CMBR data.
- Parameters which we have estimated using PSO are consistent with that obtained from other commonly used methods like MCMC.
- Due to simple algorithm and few design parameters, PSO can be easily implemented and parallelized on a cluster system for accelerated search of parameters.
- PSO can be very useful in situations when the dimensionality of the search space is very high and/or there are a large number of local maxima present.



Thank You !



References

Kennedy, J., & Eberhart, R. 1995, IEEE, 1942

Phillips, N. G., & Kogut, A. 2001, ArXiv Astrophysics e-prints

Wang, Y., & Mohanty, S. D. 2010, Phys. Rev. D , 81, 063002

Yao, L., & Sethares, W. A. 1994, IEEE Transactions on Signal Processing, 927

