On the Variable-charged Black Holes Embedded into de Sitter Space: Hawking's Radiation

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Abstract

In this paper we discuss the Hawking's evaporation of the masses of variable-charged Reissner-Nordstrom and Kerr-Newman, black holes embedded into the de Sitter cosmological universe by considering the charge to be function of radial coordinate. It has been shown that every electrical radiation of variable-charged rotating or non-rotating cosmological black holes will produce a change in the mass of the body without effecting the Maxwell scalar and the cosmological constant. It is also shown that during the Hawking's radiation process, after the complete evaporation of masses of both variable-charged Reissner-Nordstrom-de Sitter and Kerr-Newman-de Sitter black holes, the electrical radiation will continue creating negative mass naked singularities embedded into de Sitter cosmological spaces

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1 Introduction

The Hawking's radiation [1] suggests that black holes which are formed by collapse, are not completely black, but emit radiation with a thermal spectrum. Hawking [2] stated that 'Because the radiation carries away energy, the black holes must presumably lose mass and eventually disappear'. In an introductory survey Hawking and Israel [3] have discussed the black hole radiation in three possible ways with creative remarks –'So far there is no good theoretical frame work with which to treat the final stages of a black hole but there seem to be three possibilities: (i) The black hole might disappear completely, leaving just the thermal radiation that it emitted during its evaporation. (ii) It might leave behind a non-radiating black hole of about the Planck mass. (iii) The emission of energy might continue indefinitely creating a negative mass naked singularity'. In [4] it has shown that Hawking's radiation could be expressed in classical spacetime metrics, by considering the charge e of the electromagnetic field to be function of the radial coordinate r of Reissner-Nordstrom as well as Kerr-Newman black holes. Such a variable charge e with respect to the coordinate r in Einstein's equations is referred to as *an electrical radiation* of the black hole. Every electrical radiation e(r) of the non-rotating as well as rotating black holes leads to a reduction in its mass by some quantity. If one considers such electrical radiation taking place continuously for a long time, then a continuous reduction of the mass will take place in the black hole body whether rotating or non-rotating, and the original mass of the black hole will evaporate completely. At that stage the complete evaporation will lead the gravity of the object depending only on the electromagnetic field, and not on the mass. We refer to such an object with zero mass as an 'instantaneous' naked singularity - a naked singularity that exists for an instant and then continues its electrical radiation to create negative mass. So this naked singularity is different from the one mentioned in Steinmular *et al.* [5], Tipler *et al.* [6] in the sense that an 'instantaneous' naked singularity, discussed in [5,6] exists only for an instant and then disappears.

We note that the time taken between two consecutive radiations is supposed to be so short that one may not physically realize how quickly radiations take place. Thus, it seems natural to expect the existence of an 'instantaneous' naked singularity with zero mass only for an instant before continuing its next radiation to create a negative mass naked singularity. This suggests that it may also be possible in the common theory of black holes that, as a black hole is invisible in nature, one may not know whether, in the universe, a particular black hole has mass or not, but electrical radiation may be detected on the black hole surface. Immediately after the complete evaporation of the mass, if one continues to radiate the remaining remnant, there will be a formation of a new mass. If one repeats the electrical radiation further, the new mass will increase gradually and then the spacetime geometry will represent the 'negative mass naked singularity'. The classical spacetime metrics, for both stationary rotating and non-rotating, which represent the negative mass naked singularities, have been presented in [4]. Here the variable charge $e(r)$ with respect to the coordinate r is followed from Boulware's suggestion [7] that the Hawking's radiation may be expressed in terms of the stress-energy tensor associated with field whose quanta are being radiated. In order to study Hawking's radiation in classical spacetime metrics, the Boulware's suggestion leads us to consider the stress-energy tensors of electromagnetic field of different forms or functions from those of Reissner-Nordstrom, as well as Kerr-Newman, black holes as these two black holes do not seem to have any direct Hawking's radiation effects. Thus, we find that (i) the changes in the mass of black holes, (ii) the formation of 'instantaneous' naked singularities with zero mass and (iii) the creation of 'negative mass naked singularities' in Reissner-Nordstrom as well as Kerr-Newman black holes [4] may presumably be the correct formulations in classical spacetime metrics of the three possibilities of black hole evaporation suggested by Hawking and Israel [3].

The aim of this paper is to study the relativistic aspect of Hawking's radiation in Reissner-Nordstrom-de Sitter as well as Kerr-Newman-de Sitter black holes by considering the variable charge with respect to the coordinate r. The results are summarized in the form of theorems as follows:

Theorem 1 *Every electrical radiation of variable-charged Reissner-Nordstrom-de Sitter and Kerr-Newman-de Sitter black holes will produce a change in the mass of the bodies without affecting the Maxwell scalar and the cosmological constant.*

Theorem 2 *The non-rotating and rotating charged de Sitter metrics describe instantaneous de Sitter naked singularity during the Hawking's evaporation process of electrical radiation of Reissner-Nordstrom-de Sitter and Kerr-Newman-de Sitter black holes.*

Theorem 3 *During the radiation process, after the complete evaporation of masses of both variable-charged Reissner-Nordstrom-de Sitter and Kerr-Newman-de Sitter black holes, the electrical radiation will continue indefinitely creating negative mass naked singularities in de Sitter spaces.*

Theorem 4 *If an electrically radiating black hole, rotating or non-rotating, is embedded into de Sitter spaces, it will continue to embed into the same space forever.*

It is found that the theorems 1, 2 and 3 are in favour of the first, second and third possibilities of the suggestions made by Hawking and Israel [3]. But theorem 3 provides a violation of Penrose's cosmic-censorship hypothesis that 'no naked singularity can ever be created' [8]. Classical spacetime metrics describing theorem 3 have been derived below. Theorem 4 suggests that, once an electrically radiating black hole is embedded into the de Sitter cosmological universe, in principle it will continue to embed forever during its radiation process. It happens because it does not seem to have any possible mathematical method to remove the cosmological constant from the Einstein's field equations of radiating universe, unless some external forces apply to it.

Here, we shall use the phrase 'change in the mass' rather then 'loss of mass' as there is a possibility of creating new mass after the exhaustion of the original mass, if one repeats the same process of electrical radiation. This can be seen latter in this paper. Hawking's radiation is being incorporated, in the classical general relativity describing the change in mass appearing in the classical space-time metrics, without quantum mechanical aspect as done by Hawking [1] or the path integral method used by Hurtle and Hawking [9] or thermodynamic viewpoint [10]. In section 2 we present classical spacetime metrics affected by the change in the mass of the 'variable-charged' black holes embedded into de Sitter spaces after electrical radiation. In Section 3 the properties of the metrics formed after the electrical radiation are being discussed with regards to the Kerr-Schild form and Chandrasekhar's relation [8]. The NP (Newman and Penrose [11]) version of original rotating de Sitter, Reissner-Nordstrom-de Sitter and Kerr-Newman-de Sitter solutions [12] are presented in the appendices. The NP quantities are calculated by using the differential form structure in NP formalism developed by McIntosh and Hickman [13] in (–2) signature.

2 Changing masses of variable-charged black holes embedded in de Sitter space

In this section we shall consider the variable-charged black holes embedded into de Sitter space with the charge $e(r)$. By solving Einstein-Maxwell field equations with the variable-charge $e(r)$ of Reissner-Nordstrom as well as Kerr-Newman, black holes embedded in de Sitter spaces, we develop the relativistic aspect of Hawking radiation in classical spacetime metrics. It is to mention that in the formulation of the relativistic aspect of Hawking's radiation, we do not impose any condition on the field equations except considering the charge e to be function of polar coordinate r and the decomposition of the Ricci scalar $\Lambda \equiv (1/24)\, R_{ab}\, g^{ab}$ into two parts, without loss of generality, as follows

$$\Lambda = \Lambda^{(\mathrm{C})} + \Lambda^{(\mathrm{E})}, \tag{2.1}$$

where $\Lambda^{(\mathrm{C})}$ is the *non-zero* cosmological Ricci scalar, and $\Lambda^{(\mathrm{E})}$ is the *zero* Ricci scalar of electromagnetic field for the rotating as well as non-rotating black holes, which can be seen in the equation (C9) of appendix cited below. This decomposition of Ricci scalar Λ is possible because the cosmological object and the electromagnetic field are two different matter fields of different physical nature, though they are supposed to exist on the same spacetime coordinates here. For our purpose of the paper, this type of decomposition of Ricci scalars Λ will serve well in the study of Hawking's radiation of black holes embedded into the de Sitter cosmological space.

2.1 *Variable-charged Reissner-Nordstrom-de Sitter solution*

The line element of the variable-charged Reissner-Nordstrom-de Sitter solution with the assumption that the charge e of the body is a function of coordinate r, is given by

$$ds^2 = \Big\{1 - \frac{2M}{r} + \frac{e^2(r)}{r^2} - \frac{\Lambda^* r^2}{3}\Big\}\, du^2 + 2du\, dr - r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \quad (2.2)$$

When we set e = constant initially, the metric will recover the charged Reissner-Nordstrom-de Sitter solution given in the appendix B. Here Λ^* is the cosmological constant of the de Sitter universe. The complex null tetrad vectors for this metric are chosen as

$$\begin{aligned} \ell_a &= \delta^1_a, \\ n_a &= \frac{1}{2}\Big\{1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{\Lambda^* r^2}{3}\Big\}\delta^1_a + \delta^2_a, \\ m_a &= -\frac{r}{\sqrt{2}}\,\{\delta^3_a + i\sin\theta\,\delta^4_a\}, \\ \overline{m}_a &= -\frac{r}{\sqrt{2}}\,\{\delta^3_a - i\sin\theta\,\delta^4_a\}. \end{aligned} \quad (2.3)$$

where ℓ_a, n_a are real null vectors and m_a is complex null vector. Using these null tetrad vectors we calculate the spin coefficients, Ricci scalars and Weyl scalars as follows:

$$\kappa = \epsilon = \sigma = \nu = \lambda = \pi = \tau = 0, \quad \rho = -\frac{1}{r}, \;\; \beta = -\alpha = \frac{1}{2\sqrt{2}r}\cot\theta,$$

$$\mu = -\frac{1}{2\,r}\Big\{1 - \frac{2M}{r} - \frac{\Lambda^* r^2}{3} + \frac{e^2(r)}{r^2}\Big\},$$

$$\gamma = \frac{1}{2\,r^2}\Big\{M + e(r)\,e'(r) - \frac{e^2(r)}{r} - \frac{\Lambda^* r^2}{3}\Big\},$$

$$\phi_{11} = \frac{1}{4\,r^2}\Big\{e'^2(r) + e(r)\,e''(r)\Big\} + \frac{1}{2\,r^4}\Big\{e^2(r) - 2\,r\,e(r)e'(r)\Big\}, \quad (2.4)$$

$$\Lambda = \frac{\Lambda^*}{6} - \frac{1}{12\,r^2}\,\{e'^2(r) + e(r)\,e''(r)\}, \quad (2.5)$$

$$\psi_2 = \frac{1}{6\,r^2}\Big\{e'^2(r) + e(r)\,e''(r)\Big\} - \frac{1}{r^3}\Big\{M + e(r)e'(r) - \frac{e^2(r)}{r}\Big\}, \quad (2.6)$$

where a prime denotes the derivative with respect to r. The Weyl curvature scalar ψ_2, Ricci scalars ϕ_{11} and Λ for the metric (2.2) are defined by

$$\psi_2 \equiv -C_{abcd}\,\ell^a\,m^b\,\overline{m}^c\,n^d,$$
$$\phi_{11} \equiv -\frac{1}{4}\,R_{ab}(\ell^a\,n^b + m^a\,\overline{m}^b), \quad \Lambda \equiv \frac{1}{24}\,R_{ab}\,g^{ab}. \tag{2.7}$$

We have seen from above that there is no Λ^* term in ϕ_{11} and ψ_2. So the Ricci scalar ϕ_{11} is purely for electromagnetic field. Hence, using the decomposition (2.1) of Λ in equation (2.5) we obtain

$$\Lambda^{(\mathrm{C})} = \frac{\Lambda^*}{6}, \tag{2.8}$$

and

$$\Lambda^{(\mathrm{E})} = -\frac{1}{12\,r^2}\,\{e'^2(r) + e(r)\,e''(r)\}, \tag{2.9}$$

For an electromagnetic field we must have the Ricci scalar $\Lambda^{(\mathrm{E})} = 0$ leading to the solution

$$e^2(r) = 2\,rm_1 + C \tag{2.10}$$

where m_1 and C are real constants. Then the Ricci scalar becomes

$$\phi_{11} = \frac{C}{2\,r^4}. \tag{2.11}$$

Thus, the Maxwell scalar $\phi_1 = \frac{1}{2}F_{ab}(\ell^a\,n^b + \overline{m}^a\,m^b)$ takes the form, by identifying the real constant $C \equiv e^2$,

$$\phi_1 = \frac{1}{\sqrt{2}}\,e\,r^{-2}. \tag{2.12}$$

This shows that the Maxwell scalar ϕ_1 does not change its form by considering the charge e to be a function of r in Einstein-Maxwell field equations. Here, by using equation (2.10) in (2.4) and (2.6), we have the changed NP quantities

$$\mu = -\frac{1}{2\,r}\Big\{1 - \frac{2}{r}(M - m_1) + \frac{e^2}{r^2} - \frac{\Lambda^* r^2}{3}\Big\}, \tag{2.13}$$

$$\gamma = \frac{1}{2\,r^2}\,\Big\{(M - m_1) - \frac{e^2}{r} - \frac{\Lambda^* r^2}{3}\Big\},$$
$$\psi_2 = -\frac{1}{r^3}\,\Big\{(M - m_1) - \frac{e^2}{r}\Big\}, \quad \phi_1 = \frac{1}{\sqrt{2}}\,e\,r^{-2}, \tag{2.14}$$

and the metric (2.2) becomes

$$ds^2 = \Big\{1 - \frac{2}{r}(M - m_1) + \frac{e^2}{r^2} - \frac{\Lambda^* r^2}{3}\Big\}\,du^2 + 2du\,dr - r^2(d\theta^2 + \sin^2\theta\,d\phi^2). \tag{2.15}$$

We observe from the above that the mass M of non-rotating black hole (2.2) is lost a quantity m_1 at the end of the first electrical radiation. This loss of mass is agreeing with Hawking's discovery that the radiating objects must lose its mass [2]. On this losing mass, Wald [14] has pointed that a black hole will lose its mass at the rate as the energy is radiated. If one considers the same process for second time taking e in (2.15) to be function of r with the mass $M - m_1$ in Einstein-Maxwell field equations, then the mass will again be decreased by another constant m_2 (say); that is, after the second radiation the total mass might become $M - (m_1 + m_2)$. This is due to the fact, that the Maxwell scalar ϕ_1 does not change its form after considering the charge e to be function of r for the second time as $\Lambda^{(\mathrm{E})}$ calculated from the Einstein-Maxwell field equations has to vanish for electromagnetic fields with $e(r)$. Hence, if one repeats the same process n times, everytime considering the decomposition of Λ and the charge e to be function of r, then one can expect the solution to change gradually and the total mass becomes $M - (m_1 + m_2 + m_3 + ... + m_n)$ and therefore the metric (2.15) takes the form:

$$ds^2 = \Big\{1 - \frac{2}{r}\,\mathcal{M} + \frac{e^2}{r^2} - \frac{\Lambda^* r^2}{3}\Big\}\,du^2 + 2du\,dr - r^2(d\theta^2 + \sin^2\theta\,d\phi^2), \quad (2.16)$$

where the mass of the black hole after the radiation for n times will be

$$\mathcal{M} = M - (m_1 + m_2 + m_3 + ... + m_n). \quad (2.17)$$

So, the changed NP quantities are

$$\mu = -\frac{1}{2\,r}\Big\{1 - \frac{2}{r}\mathcal{M} + \frac{e^2}{r^2} - \frac{\Lambda^* r^2}{3}\Big\}, \quad (2.18)$$

$$\gamma = \frac{1}{2\,r^2}\,\Big\{\mathcal{M} - \frac{e^2}{r} - \frac{\Lambda^* r^2}{3}\Big\}, \quad (2.19)$$

$$\psi_2 = -\frac{1}{r^3}\,\Big\{\mathcal{M} - \frac{e^2}{r}\Big\}. \quad (2.20)$$

This means that for every electrical radiation, the original mass M of the non-rotating black hole (2.2) will lose some quantity. Thus, it seems reasonable to expect that, taking Hawking's radiation of black holes into account, such continuous lose of mass will lead to evaporate the original mass M. In the case the black hole has evaporated down to the Planck mass, the mass M may not exactly equal to the continuously lost quantities $m_1 + m_2 + m_3 + ... + m_n$. That is, according to the second possibility of Hawking and Israel [3], a small quantity of mass may be left, say, Planck mass of about 10^{-5} g with continuous electrical radiation. Otherwise, when $M = m_1 + m_2 + m_3 + ... + m_n$ for a complete evaporation of the mass, $\mathcal{M}$ will be zero, rather than leaving behind a Planck-size mass black hole remnant. At this stage the non-rotating black hole will have the electric charge e and the cosmological constant Λ^*, but no mass; so that the line element will take the following form:

$$ds^2 = \Big(1 + \frac{e^2}{r^2} - \frac{\Lambda^* r^2}{3}\Big)\, du^2 + 2du\, dr - r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \qquad (2.21)$$

That is, the black hole embedded into de Sitter cosmological space might radiate away all its mass completely, just leaving the electrical radiation and the unchanged cosmological constant Λ^*. At this stage the gravity of the surface will depend only on electric charge, *i.e.* $\psi_2 = e^2/r^4$, and not on the mass of black holes. The metric (2.21) describes a non-rotating charged de Sitter cosmological solution. Thus, we consider this non-rotating charged de Sitter solution to be the left out remnant of the Hawking's evaporation due to the electrical radiation of Reissner-Nordstrom black hole embedded into the non-rotating de Sitter space. The metric (2.21) shows that the presence of the cosmological constant Λ^* during Hawking's radiation process could not prevent the formation of an 'instantaneous' naked singularity. The formation of 'instantaneous' naked singularity - *a naked singularity that exists for an instant and then continues its electrical radiation to create negative mass*, in standard Reissner-Nordstrom and Kerr-Newman, black holes is unavoidable during the Hawking's evaporation process, as shown in [4]. That is, if we set the cosmological constant $\Lambda^* = 0$, the metric (2.21) will certainly represent an 'instantaneous' naked singularity with zero mass, However, the Maxwell scalar ϕ is still unaffected. Thus, from (2.21) with $\Lambda^* \neq 0$ it seems natural to refer to the non-rotating charged cosmological metric as an 'instantaneous' naked singularity in de Sitter space - *a singularity that exists for an instant and then continues its electrical radiation to create its negative mass*, during the Hawking's evaporation process of electrical radiation of Reissner-Nordstrom-de Sitter black hole. This completes the proof of the first part of theorem 2 cited in the introduction.

It is suggested that the time taken between two consecutive radiations is supposed to be so short that we may not physically realize how quickly radiations take place. Thus it seems natural to expect the existence of 'instantaneous' naked singularity with zero mass in de Sitter cosmological space only for an instant before continuing its next radiation to create new mass. Immediately, after the exhaustion of the Reissner-Nordstrom mass, if the remaining solution (2.21) continues to radiate electrically with $e(r)$, there will be a formation of new mass m_1^* (say). If this electrical radiation process continues forever, the new mass will increase gradually as

$$\mathcal{M}^* = m_1^* + m_2^* + m_3^* + m_4^* + ... \qquad (2.22)$$

and then the metric with the new mass becomes

$$ds^2 = \Big(1 + \frac{2}{r}\,\mathcal{M}^* + \frac{e^2}{r^2} - \frac{\Lambda^* r^2}{3}\Big)\,du^2 + 2du\,dr - r^2(d\theta^2 + \sin^2\theta\,d\phi^2). \quad (2.23)$$

However, it appears that this new mass $\mathcal{M}^*$ will never decrease, rather it might increase gradually as the radiation continues forever. Then the Weyl scalar for the metric (2.23) becomes

$$\psi_2 = \frac{1}{r^3}\Big\{\mathcal{M}^* + \frac{e^2}{r}\Big\}, \quad (2.24)$$

which is different from the one given in (2.20) by one minus sign. But the Maxwell scalar ϕ_1 is still remained the same as in (2.12). Thus, we have shown the changes in the mass of the Reissner-Nordstrom-de Sitter black hole after every electrical radiation. Hence, it follows the theorem 1 cited above in the case of non-rotating variable-charged black hole. This also indicates the incorporation of Boulware's suggestion [7] that 'the stress-energy tensor may be used to calculate the change in the metric due to the radiation'. The classical spacetime metrics (2.16), (2.21) and (2.23) represent the relativistic aspects of Hawking's radiation in Reissner-Nordstrom-de Sitter black hole

Comparing the metrics (2.16) and (2.23), one observes that the classical spacetime (2.23) describes a non-rotating spherical symmetric star with a negative mass $\mathcal{M}^*$. Such objects with negative masses are referred to as naked singularities [1,2,3]. The metric (2.23) can be regarded as a mathematical representation of the third possibility of Hawking and Israel [3] in the case of non-rotating singularity. Here it is noted that the creation of negative mass naked singularity is mainly based on the continuous electrical radiation of the variable charge $e(r)$ in the energy-momentum tensor of Einstein-Maxwell equations under the decomposition (2.1) of Ricci scalar Λ.

2.2 *Variable-charged Kerr-Newman-de Sitter solution*

Here, we shall incorporate the relativistic aspect of Hawking's radiation in variable-charged Kerr-Newman-de Sitter black hole when the electric charge e is taken as a function of r in the Einstein-Maxwell field equations. The line element with $e(r)$ is

$$\begin{aligned} ds^2 &= \Big[1 - R^{-2}\Big\{2Mr - e^2(r) + \frac{\Lambda^* r^4}{3}\Big\}\Big]\,du^2 + 2du\,dr \\ &\quad + 2aR^{-2}\Big\{2Mr - e^2(r) + \frac{\Lambda^* r^4}{3}\Big\}\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi \\ &\quad - R^2 d\theta^2 - \Big\{(r^2+a^2)^2 - \Delta^* a^2\sin^2\theta\Big\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \quad (2.25)$$

where

$$R^2 = r^2 + a^2\cos^2\theta, \quad \Delta^* = r^2 - 2Mr + a^2 + e^2(r) - \Lambda^* r^4/3. \tag{2.26}$$

This metric will recover the rotating Kerr-Newman-de Sitter solution given in appendix (C1), when e becomes constant. The null tetrad vectors are chosen as

$$\ell_a = \delta^1_a - a\sin^2\theta\, \delta^4_a,$$

$$n_a = \frac{\Delta^*}{2\,R^2}\,\delta^1_a + \delta^2_a - \frac{\Delta^*}{2\,R^2}\,a\sin^2\theta\,\delta^4_a,$$

$$m_a = -\frac{1}{\sqrt{2}R}\Big\{ - ia\sin\theta\,\delta^1_a + R^2\,\delta^3_a + i(r^2+a^2)\sin\theta\,\delta^4_a\Big\}, \tag{2.27}$$

$$\overline{m}_a = -\frac{1}{\sqrt{2}\overline{R}}\Big\{ ia\sin\theta\,\delta^1_a + R^2\,\delta^3_a - i(r^2+a^2)\sin\theta\,\delta^4_a\Big\}.$$

where $R = r + i\,a\cos\theta$. Then we solve the Einstein-Maxwell field equations for the metric (2.27) and write only the changed NP quantities

$$\mu = -\frac{1}{2\overline{R}\,R^2}\Big\{r^2 - 2Mr + a^2 + e^2(r) - \frac{\Lambda^* r^4}{3}\Big\}, \tag{2.28}$$

$$\begin{aligned}\gamma \;\; &= \;\; \frac{1}{2\overline{R}\,R^2}\Big[\Big\{r - M + e(r)\,e'(r) - \frac{\Lambda^* r^4}{3}\Big\}\overline{R} \\ &\quad -\Big\{r^2 - 2Mr + a^2 + e^2(r) - \frac{2\Lambda^* r^4}{3}\Big\}\Big],\end{aligned} \tag{2.29}$$

$$\begin{aligned}\psi_2 \;\; &= \;\; \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big\{ - MR + e^2(r) - e(r)e'(r)\,\overline{R} + \frac{\Lambda^* r^2}{3}a^2\cos^2\theta\Big\} \\ &\quad + \frac{1}{6\,R^2}\Big\{e'^2(r) + e(r)\,e''(r)\Big\},\end{aligned} \tag{2.30}$$

$$\begin{aligned}\phi_{11} \;\; &= \;\; \frac{1}{2\,R^2\,R^2}\Big\{e^2(r) - 2r\,e(r)e'(r) - \Lambda^* r^2\,a^2\cos^2\theta\Big\} \\ &\quad + \frac{1}{4R^2}\Big\{e'^2(r) + e(r)\,e''(r)\Big\},\end{aligned} \tag{2.31}$$

$$\Lambda = \frac{\Lambda^* r^2}{6\,R^2} - \frac{1}{12\,R^2}\Big\{e'^2(r) + e(r)\,e''(r)\Big\}, \tag{2.32}$$

where a prime denotes the derivative with respect to r. We have seen that in each expression of ϕ_{11} and ψ_2 there is a cosmological Λ^* term coupling with the rotation parameter a. In the case of non-rotating Reissner-Nordstrom-de Sitter black hole, such Λ^* term does not involve in the expression of ϕ_{11} and ψ_2 as seen in (2.4) and (2.6). Hence, without loss of generality, it will be convenient here to have a decomposition of ϕ_{11} into two parts - one for the cosmological Ricci scalar $\phi_{11}^{(\mathrm{C})}$ and the other for the electromagnetic field $\phi_{11}^{(\mathrm{E})}$ as in the case of Λ in (2.1), such that

$$\phi_{11}^{(\mathrm{C})} = -\frac{1}{2\,R^2\,\overline{R}^2}\,\Lambda^* r^2\,a^2\cos^2\theta \tag{2.33}$$

$$\phi_{11}^{(\mathrm{E})} = \frac{1}{2\,R^2\,\overline{R}^2}\Big\{e^2(r) - 2r\,e(r)e'(r)\Big\} + \frac{1}{4R\overline{R}}\Big\{e'^2(r) + e(r)\,e''(r)\Big\}. \tag{2.34}$$

Similarly, we also have the decomposition of Λ as in (2.1)

$$\Lambda^{(\mathrm{C})} = \frac{\Lambda^* r^2}{6\,R\overline{R}} \tag{2.35}$$

$$\Lambda^{(\mathrm{E})} = -\frac{1}{12\,R\overline{R}}\Big\{e'^2(r) + e(r)\,e''(r)\Big\}. \tag{2.36}$$

From (2.8) and (2.35) we have seen the difference between the two cosmological Ricci scalars Λ^{C} of non-rotating and rotating black holes.

Now, the scalar $\Lambda^{(\mathrm{E})}$ for electromagnetic field must vanish for this rotating metric. Thus, the vanishing $\Lambda^{(\mathrm{E})}$ of the equation (2.36) yields that

$$e^2(r) = 2\,r m_1 + C \tag{2.37}$$

where m_1 and C are real constants of integration. Then, substituting this result in equation (2.34) we obtain the Ricci scalar for electromagnetic field

$$\phi_{11}^{(\mathrm{E})} = \frac{C}{2\,R^2\,\overline{R}^2}. \tag{2.38}$$

however, the cosmological Ricci scalar $\phi_{11}^{(\mathrm{C})}$ remains the same form as in (2.33). Accordingly, the Maxwell scalar takes, after identifying the constant $C \equiv e^2$,

$$\phi_1 = \frac{e}{\sqrt{2}\,\overline{R}\,\overline{R}}. \tag{2.39}$$

Hence, the changed NP quantities

$$\mu = -\frac{1}{2\overline{R}\,R^2}\Big\{r^2 - 2r(M - m_1) + a^2 + e^2 - \frac{\Lambda^* r^4}{3}\Big\}, \tag{2.40}$$

$$\gamma = \frac{1}{2\overline{R}\,R^2}\Big[\Big\{r - (M - m_1)\Big\}\overline{R} - \Big\{r^2 - 2r(M - m_1) + a^2 + e^2 - \frac{\Lambda^* r^4}{3}\Big\}\Big],$$

$$\psi_2 = \frac{1}{\overline{R}\,\overline{R}\,R^2}, \Big\{ -(M-m_1)R + e^2 + \frac{\Lambda^* r^2}{3}\, a^2 \cos^2\theta \Big\}, \tag{2.41}$$

$$\phi_{11} = \frac{1}{2\,R^2\,R^2} \Big\{ e^2(r) - \Lambda^* r^2\, a^2 \cos^2\theta \Big\}, \tag{2.42}$$

$$\Lambda = \Lambda^{(\mathrm{C})} = \frac{\Lambda^* r^2}{6\,R^2}. \tag{2.43}$$

We have seen the changes in μ, γ and ψ_2, but no changes in ϕ_{11} and Λ. Thus, the rotating solution (2.25) with a new constant m_1 after the first radiation becomes

$$\begin{aligned} ds^2 \;\; = \;\; & \Big[1 - R^{-2}\Big\{2r(M-m_1) - e^2 + \frac{\Lambda^* r^4}{3}\Big\}\Big]\, du^2 + 2du\, dr \\ & +2aR^{-2}\Big\{2r(M-m_1) - e^2 + \frac{\Lambda^* r^4}{3}\Big\} \sin^2\theta\, du\, d\phi - 2a \sin^2\theta\, dr\, d\phi \\ & -R^2 d\theta^2 - \Big\{(r^2+a^2)^2 - \Delta^* a^2 \sin^2\theta\Big\}\, R^{-2} \sin^2\theta\, d\phi^2, \end{aligned} \tag{2.44}$$

where

$$\Delta^* = r^2 - 2r(M-m_1) + a^2 + e^2 - \Lambda^* r^4/3. \tag{2.45}$$

This suggests that the first electrical radiation of rotating black hole leads to a reduction of the gravitational mass M by a quantity m_1 with the same Maxwell scalar ϕ_1 and the constant $\Lambda^{(\mathrm{C})}$. If we consider another radiation by taking e in (2.44) to be a function of r with the mass $M - m_1$ and the decomposition (2.1), then the Einstein-Maxwell field equations yield to reduce this mass by another constant quantity m_2 (say); i.e., after the second radiation, the mass will become $M - (m_1 + m_2)$. Here again, the Maxwell scalar ϕ_1 and the constant $\Lambda^{(\mathrm{C})}$ remain the same form after the second radiation also. Thus, if we consider n radiations everytime taking the charge e to be function of r with the decomposition of Λ, the Maxwell scalar ϕ_1 will be the same, but the metric takes the form:

$$\begin{aligned} ds^2 \;\; = \;\; & \Big[1 - R^{-2}\Big\{2r\mathcal{M} - e^2 + \frac{\Lambda^* r^4}{3}\Big\}\Big]\, du^2 + 2du\, dr \\ & +2aR^{-2}\Big\{2r\mathcal{M} - e^2 + \frac{\Lambda^* r^4}{3}\Big\} \sin^2\theta\, du\, d\phi - 2a \sin^2\theta\, dr\, d\phi \\ & -R^2 d\theta^2 - \Big\{(r^2+a^2)^2 - \Delta^* a^2 \sin^2\theta\Big\}\, R^{-2} \sin^2\theta\, d\phi^2, \end{aligned} \tag{2.46}$$

where the total mass of the black hole, after the n radiations will be of the form

$$\mathcal{M} = M - (m_1 + m_2 + m_3 + m_4 + ... + m_n). \tag{2.47}$$

Taking Hawking's radiation of black holes, we can expect that the total mass of black hole will be radiated away just leaving $\mathcal{M}$ equivalent to Planck mass of about 10^{-5} g, that is, M may not be exactly equal to $m_1 + m_2 + m_3 +$

$m_4 + ... + m_n$, but has a difference of about Planck-size mass, as in the case of non-rotating black hole. Otherwise, the total mass of black hole will be evaporated completely after continuous radiation when $\mathcal{M} = 0$, that is, $M = m_1 + m_2 + m_3 + m_4 + ... + m_n$. Here the rotating variable-charged black hole might completely radiate away its mass just leaving the electrical charge e and the cosmological constant Λ^*. We find this situation in the form of classical space-time metric as

$$\begin{aligned} ds^2 &= \Big\{1 + \Big(e^2 - \frac{\Lambda^* r^4}{3}\Big) R^{-2}\Big\} du^2 + 2du\,dr \\ &\quad -2a\,R^{-2}\Big(e^2 - \frac{\Lambda^* r^4}{3}\Big)\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi \\ &\quad -R^2 d\theta^2 - \Big\{(r^2+a^2)^2 - \Delta^* a^2\sin^2\theta\Big\} R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{2.48}$$

with the charge e and the cosmological constant Λ^*, but no mass, where $\Delta^* = r^2 + a^2 + e^2 - \Lambda^* r^4/3$. The metric (2.48) will describe a rotating 'instantaneous' naked singularity with zero mass in de Sitter space. At this stage, the Weyl scalar ψ_2 takes the form

$$\psi_2 = \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big\{e^2 + \frac{\Lambda^* r^2}{3}\,a^2\cos^2\theta\Big\} \tag{2.49}$$

showing the gravity on the surface of the remaining solution depending on the electric charge e and the cosmological constant Λ^* coupling with the rotational parameter a; however, the Maxwell scalar ϕ_1 and the Ricci scalar $\Lambda^{(C)}$ remain the same as in (2.39) and (2.35) respectively. This completes the other part of the theorem 2 cited above.

It means that there may be rotating black holes in the universe whose masses are completely radiated; their gravity depends on the electric charge of the body and the cosmological constant Λ^*, and their metrics appear similar to that in (2.48). Here the idea of this complete evaporation of masses of radiating black holes embedded in the de Sitter space is in agreement with that of Hawking's evaporation of black holes. It is worth studying the nature of such rotating black holes (2.48) or in the case of non-rotating (2.21). This might give a different physical nature, which one has not been studied in common theory of black holes embedded into the de Sitter space.

Here, we again consider the charge e to be function of radial coordinate r for next radiation in (2.48), so that we get, from the Einstein's field equations, the scalar $\Lambda^{(E)}$ as given in equation (2.36) and the same scalar $\Lambda^{(C)}$ as in (2.35). Then the vanishing of this $\Lambda^{(E)}$ for electromagnetic field will lead to create a new mass (say m_1^*) in the remaining space-time geometry (2.48). For the second radiation, we again consider the charge e to be function of r in the field equations with the mass m_1^*. Then the vanishing of $\Lambda^{(E)}$ will lead to increase the new mass by another quantity m_2^* (say) i.e., after the second radiation of (2.48) the new mass will be $m^*_1 + m^*_2$. If this radiation process continues further for a long time, the new mass will increase gradually as

$$\mathcal{M}^* = m^*_1 + m^*_2 + m^*_3 + m^*_4 + ... \tag{2.50}$$

Then the spacetime metric will take the form

$$\begin{aligned} ds^2 \;\; = \;\; & \Big[1 + R^{-2}\Big\{2r\mathcal{M}^* + e^2 - \frac{\Lambda^* r^4}{3}\Big\}\Big]\, du^2 + 2du\, dr \\ & -2aR^{-2}\Big\{2r\mathcal{M}^* + e^2 - \frac{\Lambda^* r^4}{3}\Big\}\sin^2\theta\, du\, d\phi - 2a\sin^2\theta\, dr\, d\phi \\ & -R^2 d\theta^2 - \Big\{(r^2+a^2)^2 - \Delta^* a^2\sin^2\theta\Big\}\, R^{-2}\sin^2\theta\, d\phi^2, \end{aligned} \tag{2.51}$$

where $\Delta^* = r^2 + 2r\mathcal{M}^* + a^2 + e^2 - \Lambda^* r^4/3$. The changed NP quantities ψ_2 and μ are as follows

$$\psi_2 = \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big\{\mathcal{M}^* R + e^2 + \frac{\Lambda^* r^2}{3}\, a^2\cos^2\theta\Big\}. \tag{2.52}$$

$$\mu = -\frac{1}{2\overline{R}\,R^2}\Big\{r^2 + 2r\mathcal{M}^* + a^2 + e^2 - \frac{\Lambda^* r^4}{3}\Big\}, \tag{2.53}$$

with ϕ_1 given in (2.39) and $\Lambda^{(\mathrm{C})}$ in (2.35). Comparing the metrics (2.46) and (2.51), we find that the classical spacetime (2.51) describes a rotating negative mass naked singularity embedded into the de Sitter cosmological space. Thus, from the above it follows the proof of the rotating part of theorem 3. We have also shown the possible changes in the mass of the rotating charged de Sitter black hole without affecting the Maxwell scalar ϕ_1 as well as the cosmological constant Λ^*, and accordingly, metrics are cited for future use. Thus, this completes the proof of other part of the theorem 1 for the rotating charged de Sitter black hole. Also since there is no effect on the cosmological constant Λ^* during Hawking's evaporation process, it will always remain unaffected. That is, unless some external forces apply to remove the cosmological constant Λ^* from the spacetime geometries, it will continue to exist along with the electrically radiating objects, rotating or non-rotating forever. This leads to the proof of the theorem 4 cited above for both the rotating as well as non-rotating black holes.

3 Conclusion

In the above section, we have shown that the Hawking's radiation can be expressed in classical spacetime metrics, by considering the charge e to be the function of the radial coordinate r of Reissner-Nordstrom-de Sitter as well as Kerr-Newman-de Sitter black holes. That is, every electrical radiation produces a change in the mass of the charged objects. These changes in the mass of black holes embedded in de Sitter space, after every electrical radiation, describe the relativistic aspect of Hawking's evaporation of masses of black holes in the classical spacetime metrics. This follows from the statement of theorem 1. It appears that the black hole evaporation in de Sitter space is mainly based on the decomposition of the Ricci scalar Λ into two parts - one for the cosmological object and other for the electromagnetic field. That is, the Ricci scalar for the cosmological de Sitter space is generally non-vanishing whereas that for the electromagnetic field is always vanished. Thus the decomposition of Ricci scalar Λ for the black holes embedded in de Sitter space is, without loss of generality be possible, because the cosmological objects and the electromagnetic fields are two different matter fields possessing different physical properties. So the vanishing Ricci scalar for electromagnetic field with the charge $e(r)$ makes the change in the mass of black holes embedded in the de Sitter space. Thus, we find that the black hole radiation process in de Sitter space is based on the electrical radiation of the variable-charge $e(r)$ in the energy-momentum tensor describing the change in mass in classical spacetime metrics which is in agreement with Boulware's suggestion [7] mentioned above. The Hawking's evaporation of masses and the creation of negative mass naked singularities are also due to the continuous electrical radiation. The formation of naked singularity of negative mass is also Hawking's suggestion [2] mentioned in the introduction above. This suggests that, if one accepts the continuous electrical radiation to lead the complete evaporation of the original mass of black holes, then the same radiation will also lead to the creation of new mass to form negative mass naked singularities embedded into the de Sitter space. This clearly suggests that an electrically radiating black hole, rotating or non-rotating will not disappear completely, which is against the suggestion made in [1,2,3,5]. The disappearance of such a black hole during the radiation process will be for an instant, thereby the formation of 'instantaneous' de Sitter naked singularities. This follows the statement of the theorem 2 above for both rotating and non-rotating black holes.

Now the spacetime metric (2.51) representing the rotating negative mass naked singularity in de Sitter space can be written in the Boyer-Lindquist coordinates (t, r, θ, ϕ) for future use as

$$\begin{aligned} ds^2 &= \Big[1 + R^{-2}\Big\{2r\,\mathcal{M}^* + e^2 - \frac{\Lambda^* r^4}{3}\Big\}\Big]\,dt^2 - \frac{R^2}{\Delta^*}dr^2 - R^2 d\theta^2 \\ &\quad -\{(r^2+a^2)^2 - \Delta^* a^2 \sin^2\theta\}\,R^{-2}\sin^2\theta\,d\phi^2 \\ &\quad -2a\Big(2r\,\mathcal{M}^* + e^2 - \frac{\Lambda^* r^4}{3}\Big)\,R^{-2}\sin^2\theta\,dt\,d\phi. \end{aligned} \tag{3.1}$$

where $\Delta^* = r^2 + 2r\,\mathcal{M}^* + a^2 + e^2 - \Lambda^* r^4/3$.

The metric (2.51) describing rotating negative mass naked singularity can be written in two Kerr-Schild forms on different backgrounds as follows:

$$g_{ab}^{\mathrm{NMdS}} = g_{ab}^{\mathrm{dS}} + 2Q(r,\theta)\ell_a\ell_b \tag{3.2}$$

where $Q(r,\theta) = (r\mathcal{M}^* + e^2/2)R^{-2}$, and

$$g_{ab}^{\mathrm{NMdS}} = g_{ab}^{\mathrm{NM}} + 2Q(r,\theta)\,\ell_a\,\ell_b, \tag{3.3}$$

with $Q(r,\theta) = -(\Lambda^* r^4/6)R^{-2}$. g_{ab}^{NM} is the metric tensor for rotating negative mass naked singularity whose the line element is given below

$$\begin{aligned} ds^2 \;=\; & \Big[1 + R^{-2}\Big(2r\mathcal{M}^* + e^2\Big)\Big]\,du^2 + 2du\,dr \\ & -2aR^{-2}\Big(2r\mathcal{M}^* + e^2\Big)\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi \\ & -R^2 d\theta^2 - \Big\{(r^2+a^2)^2 - \Delta^* a^2\sin^2\theta\Big\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{3.4}$$

where $\Delta^* = r^2 + 2r\mathcal{M}^* + a^2 + e^2$. The Kerr-Schild (3.2) provides the classical spacetime metric representing negative mass naked singularity embedded into the rotating de Sitter universe and the Kerr-Schild form (3.3) shows that the rotating de Sitter space is embedded into the negative mass naked singularity background. The null vector ℓ_a, appearing in (3.2) and (3.3) is one of the repeated principal null directions of g_{ab}^{NMdS}, g_{ab}^{NM} and g_{ab}^{dS} which are all Petrov type D spacetime metrics. Also ℓ_a is geodesic, shear free, expanding as well as non-zero twist null vector.

As the negative mass naked singularity metric g_{ab}^{NMdS} (2.51) describes Petrov D spacetime, it is worth mentioning that Chandrasekhar [15] has established a relation of spin coefficients ρ, μ, τ, π in the case of an affinely parameterized geodesic vector, generating an integral which is constant along the geodesic in a vacuum Petrov type D space-time

$$\frac{\rho}{\overline{\rho}} = \frac{\mu}{\overline{\mu}} = \frac{\tau}{\overline{\pi}} = \frac{\pi}{\overline{\tau}}. \tag{3.5}$$

This relation has been derived on the basis of the vacuum Petrov type D space-time with $\psi_2 \neq 0$, $\psi_0 = \psi_1 = \psi_3 = \psi_4 = 0$ and $\phi_{01} = \phi_{02} = \phi_{10} = \phi_{20} = \phi_{12} = \phi_{21} = \phi_{00} = \phi_{22} = \phi_{11} = \Lambda = 0$. In [16] for non-vacuum Petrov type D spacetimes, the Killing-Yano (KY) scalar $\chi_1 = \frac{1}{2} f_{ab}(\ell^a\, n^b + \overline{m}^a\, m^b)$ has been introduced in (3.5) as

$$\frac{\rho}{\overline{\rho}} = \frac{\mu}{\overline{\mu}} = \frac{\tau}{\overline{\pi}} = \frac{\pi}{\overline{\tau}} = -\frac{\overline{\chi}_1}{\chi_1}, \tag{3.6}$$

where f_{ab} is KY tensor satisfying the KY equations

$$f_{ab;c} + f_{ac;b} = 0. \tag{3.7}$$

The relation (3.6) can also be found in equation (31.63) of Kramer, et al. [17], if one puts $\chi_1 = A + iB$ with a little calculation. The importance of KY tensor in General Relativity seems to lie on Carter's remarkable result [18] that the separation constant of Hamilton-Jacobi equation (for charged orbits) in the Kerr space-time gives a fourth constant. In fact, this constant arises from the scalar field $K_{ab}v^a v^b$ which has vanishing divergent along a unit vector v^a tangent to an orbit of charged particle. Here $K_{ab} = f_{ma} f_b^m$.

It is emphasized that the KY equations in NP formalism given in [16] for non-vacuum Petrov type D can be solved for the negative mass naked singularity embedded into the de Sitter spacetime metric (2.51) to obtain the KY scalar χ_1 as

$$\chi_1 = iC(r - i\,a\cos\theta), \tag{3.8}$$

where C is a real constant. Ultimately, the relation (3.6) takes the form

$$\frac{\rho}{\bar\rho} = \frac{\mu}{\bar\mu} = \frac{\tau}{\bar\pi} = \frac{\pi}{\bar\tau} = -\frac{\bar\chi_1}{\chi_1} = \frac{R}{\bar R}, \tag{3.9}$$

where $R = r + i\,a\cos\theta$ and the spin coefficients ρ, μ, τ, π are unchanged quantities for the negative mass naked singularity except μ. So they are presented in the appendix C and μ is given in (2.53) above. Thus, it concludes that the negative mass naked singularity embedded into the de Sitter cosmological space satisfies the extended version of Chandrasekhar's relation (3.6).

We have seen from above that the changes in the masses of Reissner-Nordstrom-de Sitter as well as Kerr-Newman-de Sitter black holes take place due to the vanishing of Ricci scalar $\Lambda^{(\mathrm{E})}$ and both the Maxwell scalar ϕ_1 and cosmological Ricci scalar $\Lambda^{(\mathrm{C})}$ remained unchanged in the field equations for a variable charge. So, if the Maxwell scalar ϕ_1 is absent from the spacetime geometry, there will be no electrical radiation and, consequently, no change in the mass of the black hole will take place in the black hole. Therefore, one cannot expect, in principle to observe such relativistic change in the masses in the case of uncharged Schwarzschild-de Sitter as well as Kerr-de Sitter black holes. Taking Hawking radiation into account, it emphasizes that the time taken of one radiation to another will be very short that one may not, practically realize after losing m_1 from the mass how quickly m_2 is being reduced and so on, as seen above in Section 2. It is found that the metric (2.21) or (2.48) without mass will occur *only* for a very short period, as the electrical radiation continues further. Thus, we have incorporated Hawking radiation in relativistic viewpoint in classical curved spacetime geometries which will describe the possible life style of electrically radiating black holes embedded into the de Sitter universe during their radiation process. One might expect that the metrics with mass $\mathcal{M}^*$ in equations (2.23) and (2.51) might have a different physical feature to the Reissner-Nordstrom-de Sitter as well as Kerr-Newman-de Sitter solutions.

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Appendix

Here we shall present the rotating de Sitter, Reissner-Nordstrom-de Sitter as well as Kerr-Newman-de Sitter metrics in NP formalism for a quick reference. From the rotating Kerr-Newman-de Sitter solution ($M \neq 0, a \neq 0, e \neq 0$), the rotating de Sitter ($M = 0, a \neq 0, e = 0$) and Reissner-Nordstrom-de Sitter ($M \neq 0, a = o, e \neq 0$) metrics can be recovered. It is also worth mentioning that from the rotating Kerr-Newman-de Sitter metric, we can recover a rotating charged de Sitter cosmological universe ($M = 0, a \neq 0, e \neq 0$) which has been referred in section 2.

A. *Rotating de Sitter solution*:

The rotating de Sitter solution is derived by using Wang-Wu function [20] in general rotating solutions in [12]. The line element is given as

$$\begin{aligned} ds^2 &= \Big\{1 - \frac{\Lambda^* r^4}{3\,R^2}\Big\}\,du^2 + 2du\,dr \\ &+2a\frac{\Lambda^* r^4}{3}\,R^{-2}\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi \\ &-R^2 d\theta^2 - \Big\{(r^2+a^2)^2 - \Delta^* a^2\sin^2\theta\Big\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \qquad \text{(A1)}$$

where $R^2 = r^2 + a^2\cos^2\theta$, $\Delta^* = r^2 - \Lambda^* r^4/3 + a^2$. Here Λ^* denotes the cosmological constant of the de Sitter space. The null tetrad vectors are

$$\ell_a = \delta^1_a - a\sin^2\theta\,\delta^4_a,$$

$$n_a = \frac{\Delta^*}{2\,R^2}\,\delta^1_a + \delta^2_a - \frac{\Delta^*}{2\,R^2}\,a\sin^2\theta\,\delta^4_a,$$

$$m_a = -\frac{1}{\sqrt{2}R}\Big\{-ia\sin\theta\,\delta^1_a + R^2\,\delta^3_a + i(r^2+a^2)\sin\theta\,\delta^4_a\Big\}, \qquad \text{(A2)}$$

$$\overline{m}_a = -\frac{1}{\sqrt{2}\overline{R}}\Big\{ia\sin\theta\,\delta^1_a + R^2\,\delta^3_a - i(r^2+a^2)\sin\theta\,\delta^4_a\Big\}.$$

where $R = r + i\,a\cos\theta$. Then the NP quantities are

$$\kappa = \sigma = \nu = \lambda = \epsilon = 0,$$

$$\rho = -\frac{1}{\overline{R}}, \quad \mu = -\frac{\Delta^*}{2\overline{R}\,R^2}, \quad \alpha = \frac{(2ai - R\cos\theta)}{2\sqrt{2}\overline{R}\,\overline{R}\sin\theta},$$
$$\beta = \frac{\cot\theta}{2\sqrt{2}R}, \quad \pi = \frac{i\,a\sin\theta}{\sqrt{2}\overline{R}\,\overline{R}}, \quad \tau = -\frac{i\,a\sin\theta}{\sqrt{2}R^2}, \qquad \text{(A3)}$$

$$\gamma = -\frac{1}{2\overline{R}\,R^2}\left\{(1 - \frac{1}{3}\Lambda^* r^2) r\,\overline{R} + \Delta^*\right\},$$

$$\phi_{11} = -\frac{1}{2\,R^2\,R^2}\Lambda^* r^2 a^2 \cos^2\theta, \quad \psi_2 = \frac{1}{3\overline{R}\,\overline{R}\,R^2}\Lambda^* r^2 a^2 \cos^2\theta, \qquad \text{(A4)}$$

$$\Lambda = \frac{\Lambda^* r^2}{6R^2}. \qquad \text{(A5)}$$

From these NP quantities we certainly observe that the rotating de Sitter metric is a Petrov type D gravitational field, whose one of the repeated principal null vectors, ℓ_a is geodesic, shear free, expanding as well as rotating. The rotating cosmological space possesses an energy-momentum tensor

$$T_{ab} = 2\,\rho^*\,\ell_{(a}\,n_{b)} + 2\,p\,m_{(a}\overline{m}_{b)}, \qquad \text{(A6)}$$

where $K\,\rho^* = 2\,\phi_{11} + 6\,\Lambda$ and $K\,p = 2\,\phi_{11} - 6\,\Lambda.$ are related to the density and the pressure of the cosmological matter which is, however not a perfect fluid.

B. *Reissner-Nordstrom-de Sitter solution*:

This is a spherically symmetric non-rotating solution

$$ds^2 = (1 - \frac{2M}{r} + \frac{e^2}{r^2} - \frac{\Lambda^* r^2}{3})\,du^2 + 2du\,dr - r^2 d\theta^2 + r^2\sin^2\theta\,d\phi^2, \qquad \text{(B1)}$$

where M and e are the mass and charge respectively. For this metric one chooses the null tetrad vectors the covariant complex null tetrad vectors take the forms

$$\ell_a = \delta_a^1, \quad n_a = \frac{1}{2}\left\{1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{\Lambda^* r^2}{3}\right\}\delta_a^1 + \delta_a^2, \qquad \text{(B2)}$$

$$m_a = -\frac{r}{\sqrt{2}}\,\{\delta_a^3 + i\sin\theta\,\delta_a^4\}.$$

The NP quantities are

$$\kappa = \sigma = \nu = \lambda = \pi = \tau = \epsilon = 0, \quad \rho = -\frac{1}{r}, \quad \beta = -\alpha = \frac{1}{2\sqrt{2}r}\cot\theta,$$

$$\mu = -\frac{1}{2\,r}\Big(1 - \frac{2M}{r} + \frac{e^2}{r^2} - \frac{\Lambda^* r^2}{3}\Big),\;\; \gamma = \frac{1}{2\,r^3}\,(M\,r - e^2), \tag{B3}$$

$$\psi_2 = -(M\,r - e^2)\,r^{-4},\;\; \phi_{11} = \frac{1}{2}\,e^2\,r^{-4},\quad \Lambda = \frac{\Lambda^*}{6} \tag{B4}$$

We have seen that there is no Λ^* terms in ϕ_{11} and ψ_2 for this non-rotating metric. The energy momentum tensor for the Reissner-Nordstrom-de Sitter metric takes the form

$$T_{ab} = T^{(\mathrm{E})}_{ab} + \Lambda^* g_{ab}, \tag{B5}$$

where the energy-momentum tensor for the non-null electromagnetic field is given as

$$T^{(\mathrm{E})}_{ab} = \frac{4\,e^2}{K\,r^4}\{\ell_{(a}\,n_{b)} + m_{(a}\overline{m}_{b)}\} \tag{B6}$$

The EMT (B5) can be treated as Guth's modification of energy momentum tensor [19] for electromagnetic field in early inflationary universe.

C. *Kerr-Newman-de Sitter solution*:

The line element of the Kerr-Newman-de Sitter solution is given in [12] as

$$\begin{aligned} ds^2 \;\; = \;\; & \Big\{1 - R^{-2}\Big(2Mr - e^2 + \frac{\Lambda^*\,r^4}{3}\Big)\Big\}\,du^2 + 2du\,dr \\ & + 2aR^{-2}\Big(2Mr - e^2 + \frac{\Lambda^*\,r^4}{3}\Big)\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi \\ & - R^2 d\theta^2 - \Big\{(r^2 + a^2)^2 - \Delta^* a^2\sin^2\theta\Big\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{C1}$$

where $R^2 = r^2 + a^2\cos^2\theta$, $\Delta^* = r^2 - 2Mr - \Lambda^*\,r^4/3 + a^2 + e^2$. The null tetrad vectors are

$$\begin{aligned} \ell_a &= \delta^1_a - a\sin^2\theta\,\delta^4_a, \\ n_a &= \frac{\Delta^*}{2\,R^2}\,\delta^1_a + \delta^2_a - \frac{\Delta^*}{2\,R^2}\,a\sin^2\theta\,\delta^4_a, \end{aligned} \tag{C2}$$

$$m_a = -\frac{1}{\sqrt{2}R}\Big\{ - ia\sin\theta\,\delta^1_a + R^2\,\delta^3_a + i(r^2 + a^2)\sin\theta\,\delta^4_a\Big\},$$

where $R = r + i\,a\cos\theta$. Then the NP quantities are

$$\kappa = \sigma = \nu = \lambda = \epsilon = 0,$$

$$\rho \;\; = \;\; -\frac{1}{\overline{R}},\quad \mu = -\frac{\Delta^*}{2\overline{R}\,R^2},\quad \alpha = \frac{(2ai - R\cos\theta)}{2\sqrt{2}\overline{R}\,\overline{R}\sin\theta},$$

$$\beta = \frac{\cot\theta}{2\sqrt{2}R}, \quad \pi = \frac{i\,a\sin\theta}{\sqrt{2}\overline{R}\,\overline{R}}, \quad \tau = -\frac{i\,a\sin\theta}{\sqrt{2}R^2}, \tag{C3}$$

$$\gamma = \frac{1}{2\overline{R}\,R^2}\left[(r - M - \frac{2}{3}\Lambda^* r^3)\overline{R} - \Delta^*\right],$$

$$\psi_2 = \frac{1}{\overline{R}\overline{R}\,R^2}\Big\{e^2 - M\,R + \frac{\Lambda^* r^2}{3}a^2\cos^2\theta\Big\}, \tag{C4}$$

$$\phi_{11} = \frac{1}{2\,R^2\,R^2}\Big(e^2 - \Lambda^* r^2 a^2\cos^2\theta\Big), \tag{C5}$$

$$\Lambda = \frac{\Lambda^* r^2}{6R^2}. \tag{C6}$$

The energy-momentum tensor for the Kerr-Newman-de Sitter solution can be written in the form

$$T_{ab} = T^{(\mathrm{E})}_{ab} + T^{(\mathrm{C})}_{ab}, \tag{C7}$$

$$= 2\,\rho^{*(\mathrm{E})}\{\ell_{(a}\,n_{b)} + m_{(a}\overline{m}_{b)}\} + 2\{\rho^{*(\mathrm{C})}\,\ell_{(a}\,n_{b)} + p^{(\mathrm{C})}\,m_{(a}\overline{m}_{b)}\},$$

where the energy densities and pressures for the electromagnetic field as well as the cosmological matter field are given below:

$$\rho^{*(\mathrm{E})} = p^{(\mathrm{E})} = \frac{e^2}{K\,R^2\,R^2},$$

$$\rho^{*(\mathrm{C})} = \frac{\Lambda^* r^4}{K\,R^2\,R^2}, \quad p^{(\mathrm{C})} = \frac{-\Lambda^* r^2}{K\,R^2\,R^2}\Big(r^2 + 2a^2\cos^2\theta\Big).$$

From this it is observed that the energy-momentum tensor (C7) will represent Guth's modification form of T_{ab} [19] for the rotating metric (C1) with $a \neq 0$, as it will take the form (B5) for non-rotating metric with $a = 0$.

One can, without loss of generality, have a decomposition of the Ricci scalar $\Lambda = g_{ab}R^{ab}$ (C6) into two parts – one for the non-zero cosmological scalar $\Lambda^{(\mathrm{C})}$ and other for the zero Ricci scalar $\Lambda^{(\mathrm{E})}$ of electromagnetic field as

$$\Lambda = \Lambda^{(\mathrm{C})} + \Lambda^{(\mathrm{E})}, \tag{C8}$$

such that

$$\Lambda^{(\mathrm{C})} = \frac{\Lambda^* r^2}{6R^2} \quad \text{and} \quad \Lambda^{(\mathrm{E})} = 0. \tag{C9}$$

This decomposition of Λ has been used in the section 2 above. The derivation of these rotating de Sitter solution and Kerr-Newman-de Sitter black hole can be seen in [12]. The metric (C1) can be written in Kerr-Schild form on the de Sitter cosmological background space as

$$g_{ab}^{\rm KNdS} = g_{ab}^{\rm dS} + 2Q(r,\theta)\ell_a\ell_b \tag{C10}$$

where $Q(r,\theta) = -(rm - e^2/2)R^{-2}$, and the vector ℓ_a is a geodesic, shear free, expanding as well as rotating null vector of both $g_{ab}^{\rm KNdS}$ as well as $g_{ab}^{\rm dS}$ and $g_{ab}^{\rm KN}$ is the Kerr-Newman metric with $m = e =$ constant. This vector ℓ_a is one of the double repeated principal null vectors of the Weyl tensor of $g_{ab}^{\rm KNdS}$ and $g_{ab}^{\rm dS}$. The Kerr-Schild form (C10) can also be expressed on the Kerr-Newman background space as

$$g_{ab}^{\rm KNdS} = g_{ab}^{\rm KN} + 2Q(r,\theta)\,\ell_a\,\ell_b, \tag{C11}$$

where $Q(r,\theta) = -(\Lambda^* r^4/6)\,R^{-2}$, and Λ^* is the cosmological constant and $g_{ab}^{\rm KN}$ is the Kerr-Newman metric.

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