On the Validity of the Geodesic Motion Near a Black Hole: A Clarification.

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(ricevuto il 20 Ottobre 1981)

Summary. – Ot has recently been claimed by Tangherlini that the concept of a test particle following a geodesic breaks down near the Schwarzschild event horizon. We argue that this claim is not valid.

TANGHERLINI (1) has recently claimed that the test particle approximation does not remain valid as the particle approaches the Schwarzschild event horizon. If true, this would be a very surprising result. One would rather expect the basic dynamics of a static vacuum field to remain observer independent. For E is a constant of motion and corresponds to the energy of the particle at infinity, and the effective mass of the black hole at any radius also turns out to be constant as given by Whittaker's generalization of the Gauss theorem. We can write (2)

$$M_{\text{eff}} = (1/4\pi)\oint(\sqrt{g_{00}})_{,\alpha}n^\alpha\,\mathrm{d}v_2\,, \tag{1}$$

where n^α is the unit outward normal to the 2-surface whose invariant area element is denoted by $\mathrm{d}v_2$. This gives $M_{\text{eff}} = M$ for the Schwarzschild metric with $g_{00} = 1 - 2M/r$, $G = c = 1$. Since both E and M_{eff} do not depend on r, the geometry of space-time will only inflect a factor determining the local inertial frame. That is both E and M will increase by the same factor, hence maintaining their ratio constant for all observers outside the event horizon.

The said claim stems from the erroneous comparison of E as observed by a local observer with M which is *not* measured by the same observer, but by an observer at infinity. Let a test particle of energy E at infinity fall radially onto the black hole. A local

(1) F. R. TANGHERLINI: *Nuovo Cimento B*, **63**, 588 (1981).
(2) J. L. SYNGE: *Relativity: The General Theory* (Amsterdam, 1960), p. 340.

observer near the event horizon ($r > 2M$) will measure its energy as

(2) $$E_{obs} = E/g_{00} .$$

Admittedly, E_{obs} can become very large as it diverges as $r \to 2M$. But E_{obs} should only be compared with M_{obs} which would similarly be given by

(3) $$M_{obs} = M/g_{00} .$$

In ref. (1) E_{obs} is compared with M to claim that if $E \ll M$ (the test particle definition) at infinity, it will not remain so far an observer near the event horizon. This is how the surprising result follows. One should always compare the quantities measured in the same frame (not in two different frames). E_{obs} must only be compared with M_{obs} which will, however, retain the same ratio of E/M. All other results in the paper follow from this misunderstanding.

We thus conclude that so far as the Schwarzschild field is concerned, a test particle will remain a test particle for all observers at $r > 2M$. This is natural for the geometry will inflect the same factor to the physical quantities in a local inertial frame. However, the situation is quite different for the charged and rotating black holes. In these cases, M_{eff} does no longer remain constant, but becomes position dependent. For example, M_{eff} for a charded black hole (3,4) will be given by (from eq. (1))

(4) $$M_{eff} = M - Q^2/r .$$

Now E and M_{eff} could become quite comparable as $r \to Q^2/M$. This question is under study and the results would soon be published.

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One of us (N.D.) thanks TIFR for a visiting grant.

(3) J. M. Cohen and R. Gautreau: *Phys. Rev. D*, **19**, 2273 (1979).
(4) W. B. Bonnor: *GR9* (*International Relativity Conference held at Jena in July* (1980)), p. 242.