Mon. Not. R. Astron. Soc. **335,** L57–L61 (2002)

Small-scale microwave background anisotropies arising from tangled primordial magnetic fields

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Accepted 2002 July 9. Received 2002 July 3; in original form 2002 May 20

ABSTRACT

An inhomogeneous cosmological magnetic field creates vortical perturbations that survive Silk damping on much smaller scales than compressional modes. This ensures that there is no sharp cut-off in anisotropy on arcminute scales. As we had pointed out earlier, tangled magnetic fields, if they exist, will then be a potentially important contributor to small-angular-scale cosmic microwave background radiation anisotropies. Several ongoing and new experiments are expected to probe the very small angular scales, corresponding to multipoles with $l > 1000$. In view of this observational focus, we revisit the predicted signals arising from primordial tangled magnetic fields, for different spectra and different cosmological parameters. We also identify a new regime, where the photon mean-free path exceeds the scale of the perturbation, which dominates the predicted signal at very high l. A scale-invariant spectrum of tangled fields which redshifts to a present value $B_0 = 3 \times 10^{-9}$ G produces temperature anisotropies at the 10-μK level between $l \sim 1000$ and 3000. Larger signals result if the universe is lambda-dominated, if the baryon density is larger, or if the spectral index of magnetic tangles is steeper, $n > -3$. The signal will also have non-Gaussian statistics. We predict the distinctive form of the increased power expected in the microwave background at high l in the presence of significant tangled magnetic fields. We may be on the verge of detecting or ruling out the presence of tangled magnetic fields that are strong enough to influence the formation of large-scale structure in the Universe.

Key words: magnetic fields – cosmic microwave background – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

The origin of large-scale cosmic magnetic fields remains a challenging problem. It is widely assumed that magnetic fields in astronomical objects, like galaxies, grew by turbulent dynamo action on small seed magnetic fields (cf. Ruzmaikin, Shukurov & Sokoloff 1988; Beck et al. 1996). However, the efficiency of turbulent galactic dynamos is still unclear, especially in view of the constraints implied by helicity conservation (Cattaneo & Vainshtein 1991; Kulsrud & Anderson 1992; Gruzinov & Diamond 1994; Cattaneo & Hughes 1996; Subramanian 1998, 1999; Blackman & Field 2000; Kleeorin et al. 2000; Brandenburg 2001; Brandenburg & Subramanian 2000; Brandenburg, Dobler & Subramanian 2002). Magnetic fields with larger coherence scales may also be present in clusters of galaxies (Clarke, Kronberg & Bohringer 2001) and at high redshifts (Oren & Wolfe 1995). It is not yet clear that all of these large-scale coherent fields could result from dynamo action. Alternatively, galactic or cluster fields could be remnants of a primordial cosmological magnetic field (cf. Kulsrud 1990, 1999), although, as yet, there is no entirely compelling mechanism for producing the required field. It could be present in the initial conditions, be produced quantum gravitationally or at a phase transition, or be generated in some way at the end of a period of inflation, perhaps with an almost scale-invariant spectrum (Turner & Widrow 1988; Ratra 1992; cf. Grasso & Rubinstein 2001 for a review).

A primordial field that expanded to contribute a present field strength of the order of 10^{-9} G, tangled on galactic scales, could also affect the process of galaxy formation (Rees & Reinhardt 1972; Wasserman 1978; Kim, Olinto & Rosner 1996; Subramanian & Barrow 1998a, SB98 hereafter). It is of considerable interest, therefore, to find different ways of limiting or detecting such primordial fields (see Kronberg 1994 and Grasso & Rubinstein 2001 for reviews). Here, we will show that the imminent extension of cosmic microwave background radiation (CMBR) observations to very small angular scales allows us to test for the presence of dynamically significant cosmological magnetic fields in new ways. This is because tangled magnetic fields create small-scale power in CMBR

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temperature anisotropy that would have been damped out if magnetic fields were absent. Conversely, if enhanced power is found in the CMBR power spectrum on very small scales then it might be explained by the effects of tangled magnetic fields.

In an earlier paper (Subramanian & Barrow 1998b, hereafter Paper I), we argued that observations of anisotropies in the CMBR provide a potentially powerful constraint on such tangled magnetic fields. Indeed, the isotropy of the CMBR already places a limit of $6.8 \times 10^{-9}(\Omega_{\rm m}h^2)^{1/2}$ G on the present strength of any *uniform* (spatially homogeneous) component of the magnetic field (Barrow, Ferreira & Silk 1997), where $\Omega_{\rm m}$ is the present matter density parameter, and h the Hubble constant in units of 100 km s^{-1} Mpc^{-1}. In Paper I we obtained comparable constraints on tangled (inhomogeneous) magnetic fields and highlighted the distinctive fluctuation signature that they are expected to leave in the small-scale structure of the CMBR. In particular, tangled magnetic fields produce vortical perturbations, which are overdamped in the radiation era. These can then survive Silk damping (Silk 1968) on scales much smaller than the compressional modes (SB98; Jedamzik, Katalinic & Olinto 1998), so an obvious place to expect signals induced by tangled magnetic fields is below the Silk damping scale, or at multipoles of greater than $l \sim 1000$. Several new and ongoing experiments (*MAP*, VSA, ACBAR, CBI, ATCA and *Planck Surveyor*) are indeed expected to provide information in this large-l regime. This motivates us to revisit the computation of the expected signals at larger values of l, for a wider variety of cosmological parameters and spectral indices, than were made in Paper I. In particular, we calculate results for a new high-l regime, where the photon mean-free path exceeds the scale of the perturbation. These predictions will allow comparison with future observations.

2 ESTIMATES OF THE INDUCED ANISOTROPY

The evolution of temperature anisotropy for vector perturbations has been derived in detail by Hu & White (1997a) in the total angular momentum representation, and was also given in Paper I. The temperature anisotropy is expanded in terms of tensor spherical harmonics, and the angular power spectrum induced by vector perturbations is given by (see Paper I and Mack, Kashniashvili & Kosowsky 2002)

$$C_l = 4\pi \int_0^\infty \frac{k^2 {\rm d}k}{2\pi^2} \quad \frac{l(l+1)}{2} \times \left\langle \left| \int_0^{\tau_0} {\rm d}\tau\, g(\tau_0, \tau) v(k, \tau) \frac{j_l[k(\tau_0 - \tau)]}{k(\tau_0 - \tau)} \right|^2 \right\rangle . \quad (1)$$

Here, $v(k, \tau)$ is the magnitude of the vorticity, $\Omega_i = v_i^B - V_i$, in Fourier space, where v_i^B is the rotational component of the fluid velocity, and V_i is the vector metric perturbation. Note that, since Ω_i also appears in the Euler equation for baryons, we need never compute the vector metric perturbation explicitly (see also Mack et al. 2002). Also, k is the co-moving wavenumber, τ is conformal time, τ_0 its present value, and $j_l(z)$ is the spherical Bessel function of order l. We have ignored a small polarization correction to the source term and also a metric perturbation term which is subdominant at large l (cf. Seshadri & Subramanian 2001). The 'visibility function', $g(\tau_0, \tau)$, determines the probability that a photon reaches us at epoch τ_0 if it was last scattered at the epoch τ. We adopt a flat universe throughout, with a total matter density $\Omega_{\rm m}$ and a non-zero cosmological constant density $\Omega_\Lambda = 1 - \Omega_{\rm m}$ today.

In Paper I we obtained analytic estimates for C_l in various asymptotic regimes. We briefly recapitulate the arguments and results. First, we approximated the visibility function as a Gaussian: $g(\tau_0, \tau) = (2\pi\sigma^2)^{-1/2} \exp[-(\tau - \tau_*)^2/(2\sigma^2)]$, where τ_* is the conformal epoch of 'last scattering' and σ measures the width of the last scattering surface (LSS). To estimate these, we use the expressions given in Hu & Sugiyama (1995), adopting a baryon density parameter $\Omega_{\rm b} = 0.02\, h^{-2}$, and $h = 0.7$. We estimate the epoch of last scattering to be $(1 + z_*) \sim 1150$ and $\sigma/\tau_* \sim 0.064$. To convert redshift into conformal time we use $\tau = 6000h^{-1}[(a + a_{\rm eq})^{1/2} - a_{\rm eq}^{1/2}]/\Omega_{\rm m}^{1/2}$, valid for a flat universe (cf. Hu & White 1997b). Here, the expansion factor $a = (1+z)^{-1}$ and $a_{\rm eq} = 4.17 \times 10^{-5}(\Omega_{\rm m}\, h^2)^{-1}$. For an $\Omega_{\rm m} = 1$ model, we get $\tau_* = 130.0\, h^{-1}$ Mpc, and $\sigma = 8.4\, h^{-1}$ Mpc, while for a Λ-dominated model with $\Omega_{\rm m} = 0.3$, we get $\tau_* = 187.5\, h^{-1}$ Mpc and $\sigma = 12.1\, h^{-1}$ Mpc. We will use these numbers in the numerical estimates below.

The dominant contributions to the integral over τ in equation (1) then come from a range σ around the epoch $\tau = \tau_*$. Furthermore, $j_l[k(\tau_0 - \tau)]$ picks out (k, τ) values in the integrand that have $k(\tau_0 - \tau) \sim l$. Thus, following the arguments detailed in Paper I, for $k\sigma \ll 1$ we get the analytical estimate $l(l + 1)C_l/(2\pi) \approx (\pi/4)\Delta_v^2(k, \tau_*)|_{k=l/R_*}$. Here, $\Delta_v^2 = k^3\langle|v(k, \tau_*)|^2\rangle/(2\pi^2)$ is the power per unit logarithmic interval of k, residing in the *net* vorticity perturbation, and $R_* = \tau_0 - \tau_*$. In the opposite limit, $k\sigma \gg 1$, we get $l(l + 1)C_l/(2\pi) \approx (\sqrt{\pi}/4)[\Delta_v^2(k, \tau_*)/(k\sigma)]|_{k=l/R_*}$. At small wavelengths, C_l is suppressed by a $1/k\sigma$ factor owing to the finite thickness of the LSS.

To evaluate C_l, one also needs to estimate v, the vorticity induced by magnetic inhomogeneities. We assume the magnetic field to be initially a Gaussian random field. On galactic scales and above, the induced velocity is generally so small that it does not lead to any appreciable distortion of the initial field (SB98; Jedamzik et al. 1998). So, to a very good approximation, the magnetic field simply redshifts away as $\boldsymbol{B}(\boldsymbol{x}, t) = \boldsymbol{b}_0(\boldsymbol{x})/a^2$. The Lorentz force associated with the tangled field is then $\boldsymbol{F}_{\rm L} = (\nabla \times \boldsymbol{b}_0) \times \boldsymbol{b}_0/(4\pi a^5)$, which pushes the fluid and creates rotational velocity perturbations. These can be estimated as in Paper I, by using the Euler equation for the baryons. On scales larger than the photon mean-free path at decoupling, where the viscous effect arising from photons can be treated in the diffusion approximation, this reads

$$\left(\frac{4}{3}\rho_\gamma + \rho_{\rm b}\right) \frac{\partial \Omega_i}{\partial t} + \left[\frac{\rho_{\rm b}}{a}\frac{{\rm d}a}{{\rm d}t} + \frac{k^2\eta}{a^2}\right] \Omega_i = \frac{P_{ij}F_j}{4\pi a^5}. \quad (2)$$

Here, ρ_γ is the photon density, $\rho_{\rm b}$ the baryon density, and $\eta = (4/15)\rho_\gamma l_\gamma$ the shear viscosity coefficient associated with the damping caused by photons, the mean-free path of which is $l_\gamma = (n_{\rm e}\sigma_{\rm T})^{-1} \equiv L_\gamma a(t)$, where $n_{\rm e}$ is the electron density and $\sigma_{\rm T}$ the Thomson cross-section. For $(1 + z_*) \sim 1150$, we get $L_\gamma(\tau_*) \sim 1.8 f_{\rm b}^{-1}$ Mpc, where $f_{\rm b} = (\Omega_{\rm b}h^2/0.02)$. We have defined the Fourier transforms of the magnetic field by $\boldsymbol{b}_0(\boldsymbol{x}) = \sum_{\boldsymbol{k}} \boldsymbol{b}(\boldsymbol{k}) \exp({\rm i}\boldsymbol{k} \cdot \boldsymbol{x})$ and $\boldsymbol{F}(\boldsymbol{k}) = \sum_{\boldsymbol{p}}[\boldsymbol{b}(\boldsymbol{k} + \boldsymbol{p}) \cdot \boldsymbol{b}^*(\boldsymbol{p})]\boldsymbol{p} - [\boldsymbol{k} \cdot \boldsymbol{b}^*(\boldsymbol{p})]\boldsymbol{b}(\boldsymbol{k} + \boldsymbol{p})$. The projection tensor, $P_{ij}(\boldsymbol{k}) = [\delta_{ij} - k_i k_j/k^2]$, projects $\boldsymbol{F}$ on to its transverse components perpendicular to $\boldsymbol{k}$.

The comoving Silk damping scale at recombination, $L_{\rm S} = k_{\rm S}^{-1} \sim 10$ Mpc, separates scales on which the radiative viscosity is important ($kL_{\rm S} \gg 1$) from those on which it is negligible ($kL_{\rm S} \ll 1$). For $kL_{\rm S} \ll 1$, the damping arising from the photon viscosity can be neglected compared with the Lorentz force. Integrating the baryon Euler equation, assuming negligible initial vorticity perturbation, then gives $\Omega_i = G_i\tau/(1 + S_*)$, where $G_i = 3P_{ij}F_j/(16\pi\rho_0)$,

ρ_0 is the present-day value of ρ_γ, and $S_* = (3\rho_b/4\rho_\gamma)(\tau_*) \sim 0.53(\Omega_b h^2/0.02)$. In the other limit, with $kL_S \gg 1$, we can use the terminal-velocity approximation, neglecting the inertial terms in the Euler equation, to balance the Lorentz force by friction. This gives $\Omega_i = (G_i/k)(5/kL_\gamma)$, on scales where diffusion damping operates. The transition Silk scale can also be estimated by equating Ω_i in the two cases, to give $k_S \sim \{5(1+S_*)/[\tau L_\gamma(\tau)]\}^{1/2}$.

A new regime arises on very small scales that are well below the photon mean-free path. The radiative drag force is then no longer described by the diffusion approximation, but rather is described by the free-streaming drag given in equation (6.1) of SB98. Under strong damping, this term dominates the inertial terms in the Euler equation and the fluid reaches terminal velocity where friction arising from free-streaming photons balances driving by the Lorentz force, $\boldsymbol{F}_L$. From equation (6.2) of SB98, we get $4\rho_\gamma/(3\rho_b)n_e\sigma_T\boldsymbol{\Omega} = \boldsymbol{F}_L/\rho_b$, which gives $\Omega_i = G_i L_\gamma$. One can also estimate the transition scale, say k_{fs}^{-1}, below which free-streaming damping dominates, by equating the induced velocities in the diffusion and free-streaming damping regimes, to get $k_{fs} \sim \sqrt{5}/L_\gamma$.

In order to compute the C_ls we also need to specify the spectrum of the tangled magnetic field, say $M(k)$. We define $\langle b_i(\boldsymbol{k})b_j^*(\boldsymbol{q})\rangle = \delta_{\boldsymbol{k},\boldsymbol{q}}P_{ij}(\boldsymbol{k})M(k)$, where $\delta_{\boldsymbol{k},\boldsymbol{q}}$ is the Kronecker delta which is non-zero only for $\boldsymbol{k} = \boldsymbol{q}$. This gives $\langle \boldsymbol{b}_0^2\rangle = 2\int(\mathrm{d}k/k)\Delta_b^2(k)$, where $\Delta_b^2(k) = k^3M(k)/(2\pi^2)$ is the power per logarithmic interval in k-space residing in magnetic tangles, and we replace the summation over k-space by an integration. The ensemble average $\langle|v|^2\rangle$, and hence the C_ls, can be computed in terms of the magnetic spectrum $M(k)$. It is convenient to define a dimensionless spectrum, $h(k) = \Delta_b^2(k)/(B_0^2/2)$, where B_0 is a fiducial constant magnetic field. The Alfvén velocity, V_A, for this fiducial field is

$$V_A = \frac{B_0}{(16\pi\rho_0/3)^{1/2}} \approx 3.8\times 10^{-4}B_{-9}, \tag{3}$$

where $B_{-9} \equiv (B_0/10^{-9}\,\mathrm{G})$. We will also consider power-law magnetic spectra, $M(k) = Ak^n$ cut off at $k = k_c$, where k_c is the Alfvén-wave damping length-scale (SB98; Jedamzik et al. 1998). We fix A by demanding that the smoothed field strength over a 'galactic' scale, $k_G = 1\,h\,\mathrm{Mpc}^{-1}$ (using a sharp k-space filter), is B_0, giving a dimensionless spectrum for $n > -3$ of

$$h(k) = (n+3)(k/k_G)^{3+n}. \tag{4}$$

We can now put together the above results to derive analytic estimates for the CMBR anisotropy induced by tangled magnetic fields. As a measure of the anisotropy we define the quantity $\Delta T(l) \equiv [l(l+1)C_l/2\pi]^{1/2}T_0$, where $T_0 = 2.728$ K is the CMBR temperature. On large scales, such that $kL_S < 1$ and $k\sigma < 1$, the resulting CMBR anisotropy is (see Paper I)

$$\Delta T_B(l) = T_0\left(\frac{\pi}{32}\right)^{1/2} I(k)\frac{kV_A^2\tau_*}{(1+S_*)} \approx 5.8\,\mu\mathrm{K}\left(\frac{B_{-9}}{3}\right)^2\left(\frac{l}{500}\right) I\left(\frac{l}{R_*}\right). \tag{5}$$

Here, $l = kR_*$ and we have used cosmological parameters for the Λ-dominated model, with $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$ and $\Omega_b h^2 = 0.02$ (in Paper I, we used a purely matter-dominated $\Omega_m = 1$ model). We also use the fit given by Hu & White (1997b) to calculate $\tau_0 = 6000\,h^{-1}[(1+a_{eq})^{1/2} - a_{eq}^{1/2}][1 - 0.0841 ln(\Omega_m)]/\Omega_m^{1/2}$, valid for a flat universe.

On scales where $kL_S > 1$ and $k\sigma > 1$, but $kL_\gamma(\tau_*) < 1$, we get

$$\Delta T_B(l) = T_0\frac{\pi^{1/4}}{\sqrt{32}}I(k)\frac{5V_A^2}{kL_\gamma(\tau_*)(k\sigma)^{1/2}} \approx 13.0\,\mu\mathrm{K}\left(\frac{B_{-9}}{3}\right)^2\left(\frac{l}{2000}\right)^{-3/2} f_b h_{70}^{-1} I\left(\frac{l}{R_*}\right), \tag{6}$$

where $h_{70} \equiv (h/0.7)$ and $f_b = (\Omega_b h^2/0.02)$; there is also a weaker dependence of other parameters on f_b but the strongest dependence comes from the fact that $L_\gamma \propto f_b^{-1}$. In the free-streaming regime, the estimate for $\Delta T(l)$ is obtained by replacing $(kL_\gamma/5)^{-1}$ in equation (6) by (kL_γ). The CMBR anisotropy that results for scales so small that free-streaming damping dominates is

$$\Delta T_B(l) = T_0\frac{\pi^{1/4}}{\sqrt{32}}I(k)\frac{V_A^2 kL_\gamma(\tau_*)}{(k\sigma)^{1/2}} \approx 0.4\,\mu\mathrm{K}\left(\frac{B_{-9}}{3}\right)^2\left(\frac{l}{20000}\right)^{1/2} I\left(\frac{l}{R_*}\right)\frac{h}{f_b}. \tag{7}$$

The function $I^2(k)$ in equations (5)–(7) is a dimensionless mode-coupling integral given by

$$I^2(k) = \int_0^\infty \frac{\mathrm{d}q}{q}\int_{-1}^{1}\mathrm{d}\mu\,\frac{h(q)h[|(\boldsymbol{k}+\boldsymbol{q})|]k^3}{(k^2+q^2+2kq\mu)^{3/2}} \times (1-\mu^2)\left[1 + \frac{(k+2q\mu)(k+q\mu)}{(k^2+q^2+2kq\mu)}\right], \tag{8}$$

where $|(\boldsymbol{k}+\boldsymbol{q})| = (k^2+q^2+2kq\mu)^{1/2}$. In general, $I(k)$ can only be evaluated numerically, but for $h(k) = k\delta_D(k-k_0)$, where $\delta_D(x)$ is the Dirac delta function, it can be evaluated exactly. One gets $I(k) = (k/k_0)[1-(k/2k_0)^2]^{1/2}$ for $k < 2k_0$, and zero for larger k. So in this case $I(k)$ contributes a factor of the order of unity around $k \sim k_0$, with $I(k_0) = \sqrt{3}/2$.

We can also find an analytic approximation to the mode coupling integral for power-law magnetic spectra. The approximation is different for $n > -3/2$ and for $n < -3/2$. For $n > -3/2$ and for $k \ll k_c$ (which is relevant for $l \ll k_c R_*$), one gets (Seshadri & Subramanian 2001)

$$I^2(k) = \frac{28}{15}\frac{(n+3)^2}{(3+2n)}\left(\frac{k}{k_G}\right)^3\left(\frac{k_c}{k_G}\right)^{3+2n}. \tag{9}$$

The mode-coupling integral is dominated by the small-scale cut-off in this case. In the other limit $n < -3/2$, we find

$$I^2(k) = \frac{8}{3}(n+3)\left(\frac{k}{k_G}\right)^{6+2n}. \tag{10}$$

For a nearly scale-invariant spectrum, say with $n = -2.9$, we then get $\Delta T(l) \sim 4.7\,\mu\mathrm{K}\,(l/1000)^{1.1}$ for scales larger than the Silk scale, and $\Delta T(l) \sim 5.6\,\mu\mathrm{K}\,(l/2000)^{-1.4}$ for scales smaller than L_S but larger than L_γ. Larger signals will be expected for steeper spectra, $n > -2.9$ at the higher l end.

To complement these analytic results, we have also computed $\Delta T(l)$ for the above spectra, by evaluating the τ and k integrals in equation (1) numerically. We retain the analytic approximations to $I(k)$ and $\Omega_i(k)$, with transitions between the limiting forms at wavenumbers k_S and k_{fs}. The results are displayed in Figs 1 and 2. We see that for $B_0 \sim 3\times 10^{-9}$ G this leads to a predicted rms temperature anisotropy in the CMBR of the order of 10 μK for $1000 < l < 3000$, for a nearly scale-invariant power-law spectrum with $n = -2.9$. Larger signals result, at the high-l end, for a Λ-dominated universe, compared with a matter-dominated model (compare the solid and dotted curves), basically owing to an increase

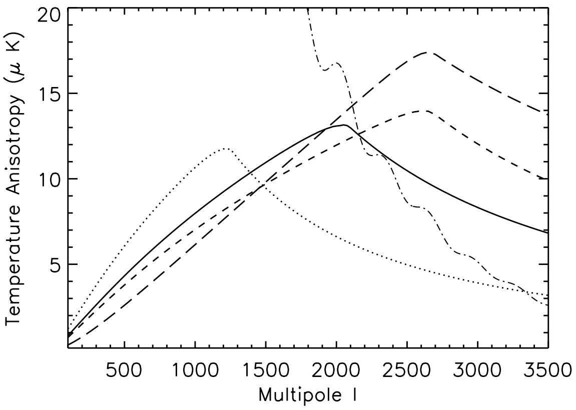

Figure 1. ΔT versus l predictions for different cosmological models and magnetic power spectrum $M(k) \propto k^n$, for $B_{-9} = 3$. The bold solid line is for a canonical flat, Λ-dominated model, with $\Omega_\Lambda = 0.7$, $\Omega_{\rm m} = 0.3$, $\Omega_{\rm b} h^2 = 0.02$, $h = 0.7$ and almost scale-invariant spectrum $n = -2.9$. The dotted curve is obtained when one changes to an $\Omega_{\rm m} = 1$ and $\Omega_\Lambda = 0$ model. The short-dashed curve is for the Λ-dominated model with a larger baryon density $\Omega_{\rm b} h^2 = 0.03$, while the long-dashed curve changes this model by adopting a magnetic spectral index of $n = -2.5$. We also show for qualitative comparison (dash–dotted curve) the temperature anisotropy in a 'standard' Λ-CDM model, computed using CMBFAST (Seljak & Zaldarriaga 1996), with cosmological parameters as for the first model described above. These curves show the build-up of power resulting from vortical perturbations from tangled magnetic fields that survive Silk damping at high $l \sim 1000$–3000. The eventual slow decline is due to the damping by photon viscosity, although this decline is only a mild decline as the magnetically sourced vortical mode is overdamped. By contrast, in the absence of magnetic tangles there is a sharp cut-off owing to Silk damping.

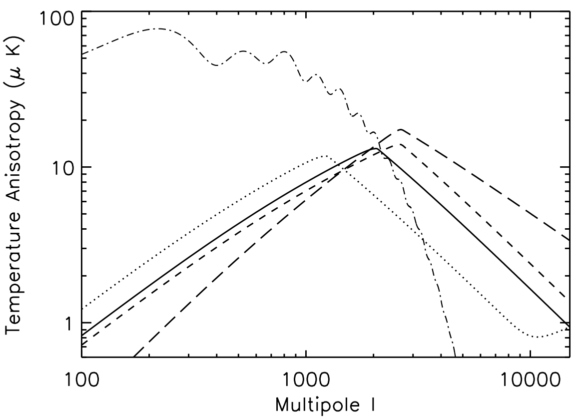

Figure 2. This plot shows $\log \Delta T$ versus $\log l$ predictions up to very large $l \sim 10^4$; the line styles are as in Fig. 1.

in R_* for this model. Larger signals also result for larger baryon density (compare the solid and short-dashed curves), owing to the decrease in L_γ and hence the damping effects of radiative viscosity. Also, a moderately steeper spectral index, with $n = -2.5$, which has more power on small scales, leads to an increased ΔT (compare the short-dashed and long-dashed curves). Much larger signals result from even steeper spectra, but we have not displayed these results, as spectra with $n > -2.5$ and $B_{-9} \sim 3$ are probably ruled out because of gravitational wave production, estimated by Caprini & Durrer (2002). They will also be significantly constrained by the high-l limits on the anisotropy by ATCA, given by Subrahmanyan et al. (2000). We find that our analytic approximation of the τ and k integrals in equation (1) tends to underestimate the amplitude of $\Delta T(l)$ by about a factor of ~ 2, although the analytically predicted l-dependences agree very well with the numerical integration at both small and large l. The numerical integration over the Bessel function and the visibility functions are of course expected to be more accurate. The new regime of free-streaming damping is only seen at very large $l \sim k_{\rm fs} R_*$, which is of the order of 10^4 even for the matter-dominated model, and is ~20 000 for the Λ-dominated model.

3 DISCUSSION

We have re-examined the expected small-angular-scale anisotropy induced by tangled magnetic fields, for different cosmological parameters and spectral indices and including a new regime at very small scales. We are motivated primarily by several ongoing and future experiments, which probe this large-l regime. Tangled magnetic fields give a distinctive contribution to the high-l signal, since they create vortical perturbations that survive Silk damping on much smaller scales than do compressional modes. Moreover, for small-scale rotational perturbations, the damping arising from the finite thickness of the LSS is also milder than for compressional modes. It should be emphasized that, by contrast, in the standard non-magnetic models the C_ls have a sharp cut-off for $l > k_{\rm S} R_*$, owing to Silk damping. As we see from Figs 1 and 2, it is precisely here that the magnetically induced signals begin to dominate. A scale-invariant spectrum of tangled fields that redshifts to a present value $B_0 = 3 \times 10^{-9}$ G produces temperature anisotropies at the 10-μK level between $l \sim 1000$ and 3000. Larger signals are produced in a Λ-dominated universe, in a universe with larger baryon density, or if the spectral index of magnetic tangles is steeper, $n > -3$. Note that the anisotropy in hot or cold spots could be several times larger, because the non-linear dependence of C_l on $M(k)$ will imply non-Gaussian statistics for the anisotropies. Indeed, as we pointed out in Paper I, the $\Delta T(l)$ will obey a modified form of the χ^2 distribution, the probability falling off almost exponentially, rather than as a Gaussian.

We see that the predicted signal is quite sensitive to both the cosmology and the baryonic content (the value and any clumping of $\Omega_{\rm b}$). It is also necessary in future work to calculate the damping effects more accurately, and also to take account of the effects of non-Gaussianity. The latter point is especially important in searches that cover limited regions of the sky. The interpretation of the data at these small angular scales will also be complicated by the need to understand the contribution from discrete sources.

The other potentially important source of temperature anisotropies at these small angular scales is the Sunyaev–Zeldovich signal. However, one can isolate this signal by its frequency dependence. An important means of distinguishing the effects of tangled magnetic fields is by looking at the polarization of the CMBR. Seshadri & Subramanian (2001) show that the vorticity induced by tangled magnetic fields produces significant and predominantly B-type polarization, at micro-kelvin levels, and this will also be dominant at large l. These should therefore be distinguishable from anisotropies produced by inflationary scalar and tensor perturbations. We hope to revisit this issue in the future.

Tangled magnetic fields also produce anisotropies on large angular scales, dominated by tensor metric perturbations induced by anisotropic magnetic stresses (Durrer, Ferreira & Kahniashvili 2000; Mack et al. 2002). Using the formalism described in these papers, we estimate a tensor contribution at small $l < 100$ of ~10.9 μK $(l/100)^{0.1}$ for the nearly scale-invariant spectrum with $n = -2.9$ and

$\Delta T(l) \sim 4.9\ \mu\mathrm{K}\,(l/100)^{0.5}$ for $n = -2.5$ and $B_{-9} \sim 3$. Since we have to add this power to the standard power produced by inflationary scalar perturbations, in quadrature, a tangled field with $B_{-9} \sim 3$ will produce of the order of a few to 10 per cent perturbation to the power in the standard CMBR anisotropy at small *l*s. So if they are indeed detected at large *l*, below the Silk damping scale, one will also have to consider their effects seriously at large angular scales, especially in cosmological parameter estimation.

A magnetic field that redshifts to a present-day value of the order of a nanogauss could impact significantly on the formation of galaxies (SB98; Rees & Reinhardt 1972; Wasserman 1978). Our results therefore provide a way of detecting small-scale magnetic inhomogeneities at a level that affects the formation of galaxies and clusters.

ACKNOWLEDGMENTS

KS thanks Simon White and Max Planck Institute for Astrophysics, Garching, for hospitality during the course of this work. We thank P. Ferreira, R. Crittenden, A. Lewis and A. Lasenby for discussions.

NOTE ADDED IN PRESS

After this paper was submitted, the Cosmic Background Imager (CBI) collaboration announced results (Mason et al. 2002) which identified significant power in the microwave background power spectrum up to $l = 3500$ (with 2σ limits of 14–31 μK at $l > 2010$). Tangled magnetic fields of strength $B_0 = 3$ nG and an almost scale-invariant spectrum, which we have discussed here, can contribute a significant fraction of this power. Other possible contributors to the excess high-*l* power that have been discussed include the Sunyaev–Zeldovich effect (Bond et al. 2002).

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