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Curves A, B, C and D are all within the reach of our existing technology at UWA. Thus a strain sensitivity $\sim 10^{-19}$, corresponding to a noise number $\sim 10^3$ can be confidently expected in the near future. However the achievement of a noise numbers $< 10^2$ will be more difficult, possibly involving the cooling of components below 1K, and use of amplifiers with noise temperature < 10K.

Conclusion

At the strain sensitivity expected in existing cryogenic gravitational radiation antennae in the near future, signal pulses of amplitude $\sim 10^{-19}$ will be detectable, using coincidence detection between widely spaced antennae. Likely events of galactic origin will be detectable, but it is not certain whether the event frequency will be sufficient to yield positive results. Further increases in sensitivity, needed to observe events in the Virgo cluster, will be difficult unless the conversion to gravitational radiation is much more efficient than currently expected.

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A NEW APPROACH TO QUANTUM COSMOLOGY

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ABSTRACT

It is argued that by quantizing the conformal degrees of freedom in the spacetime metric it is possible to throw new light on some of the outstanding problems of classical cosmology. By using the path integral method it is shown how to construct the Feynman propagator for the conformal fluctuations of the metric from the classical Einstein value. The problems of spacetime singularity, particle horizons and spatial flatness are reinterpreted and resolved with the help of this technique.

1. Introduction

In this talk I wish to present some results from an approach to quantum cosmology initiated by me six years ago [for reference, see Narlikar[1]] and further developed by me in collaboration with my student T.Padmanabhan[2].

At first sight quantum cosmology may seem a mismatch of two non-overlapping disciplines of physics, quantum theory dealing with small scale phenomena and cosmology dealing with the large scale structure of the universe. This impression is corrected, however, when one considers the presently popular big-bang model of the universe. According to this model, even though the present characteristic size of the universe is $\sim 10^{28}$ cm, there was an epoch in the past when the entire universe was created in a pointlike singularity. We will denote the cosmic time by t and the singular epoch by $t=0$. Sufficiently close to $t=0$ the universe was compact enough to be subject to the laws of quantum theory.

This characteristic size and the period of quantum domination may be roughly estimated as follows. Consider the Hilbert action for the curved spacetime subject to classical general relativity:

$$A_H = \frac{c^3}{16\pi G}\int_{\mathcal{V}} R\sqrt{-g}\, d^4x \qquad (1)$$

for a spacetime region $\mathcal{V}$ of characteristic spatial dimension L and temporal dimension L/c (c= speed of light and G= gravitational constant). The Riemannian curvature scalar R is estimated by the Einstein equations,

$$R = \frac{8\pi GT}{c^4} \tag{2}$$

where T is the trace of the energy tensor of matter.

To estimate T assume that the spatial section of $\mathcal{V}$ is a sphere of radius L and density ρ, so that

$$R = 3\pi G\rho/c^2, \quad A_H = cL^4\rho. \tag{3}$$

However, in the strong gravity situation the mass of the sphere is given by the black hole relation

$$L = 2GM/c^2, \text{ i.e., } A_H = 3c^3L^2/8\pi G. \tag{4}$$

From the quantum condition $A_H < \hbar$ we get

$$L < (8\pi/3)^{1/2}.L_P \tag{5}$$

where

$$L_P = (G\hbar/c^3)^{1/2} \sim 1.6.10^{-33}\text{cm} \tag{6}$$

is called the Planck length. The corresponding time is given by

$$T_P = L_P/c \sim 5.10^{-44}\text{s}. \tag{7}$$

For spacetime dimensions as small as these the laws of quantum theory cannot be ignored. Any discussion of the early big-bang epochs prior to T_P must incorporate quantum cosmology.

It must be emphasized that unlike electrodynamics which abounds with observed phenomena requiring quantum theory for their explanation, gravity does not present us with any unexplained laboratory or natural phenomena. Thus appeal to quantum gravity and cosmology is based solely on conceptual requirements. In particular, gravity like other basic forces of nature, has to be quantized if it is to form part of a single unified basic interaction.

However, we may make the need for quantum cosmology stronger by requiring it to solve the outstanding conceptual problems of classical cosmology. Three such instances will be described here.

2. The Problems of Singularity, Horizon and Flatness

The simplest cosmological spacetime describing the expanding universe is that given by the Robertson-Walker line element:

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{(1-kr^2)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]. \tag{8}$$

Here (r,θ,ϕ) are the constant coordinates of the so called 'fundamental observer' and t the observer's proper time is also the cosmic time. We have taken c= 1 and G= 1. In quantum cosmology we will also take $\hbar = 1$ so that all lengths are expressed in units of L_P. In practice galaxies may be considered to have constant (r,θ,ϕ) coordinates in the first approximation. The parameter k takes values 0,1 or -1 and it measures the sign of curvature of the spatial sections t= constant. The factor S(t) measures the scale factor for these sections which are homogeneous and isotropic.

Except for the special case $S(t)\propto t$, k=-1 when (8) is a coordinate transform of the flat Minkowski spacetime, the line element (8) describes a singular spacetime for S=0. We will assume without loss of generality, that S=0 occurs at t=0, provided S does become zero sometime. This epoch will be referred to as the singular epoch.

The first models of the expanding universe discovered by Friedmann[3] do have the singular epoch. In fact a singular epoch is now known to be inevitable in relativistic cosmology[4]. The spacetime singularity is thus universal and it prevents any discussion of physics for $t \leq 0$. The actual universe is assumed to have been created at t=0, and is subject to physical investigation only for t > 0.

The singular epoch leads to another difficulty which is identified with the existence of small particle horizons. An observer at r=0 at any time t > 0 has his past light cone extending upto $r=r_H$ where

$$\int_0^t \frac{du}{S(u)} = \int_0^{r_H} \frac{dr}{(1-kr^2)^{1/2}}. \tag{9}$$

For all big-bang models obtainable from Einstein's equations, $r_H \to 0$ as $t \to 0$ (cf. Narlikar[5]). Hence in the very early stages the range of physical communication between observers was severely limited. Under these circumstances it becomes difficult to understand the high degree of homogeneity of the microwave background radiation if it is to be considered as a relic of the early universe. If the particle horizon limited the range of communication, how did the observers located in different and widely separated parts of the universe 'know' the intensity of radiation at each other's location ? The only way to explain the observed homogeneity of the relic radiation is to assume that it was created that way. This need to resort to special initial conditions is clearly unsatisfactory.

The flatness problem can be understood in the following way. Let the Hubble constant and the density parameter at any epoch be defined by

$$H(t) = \dot{S}(t)/S(t), \tag{10}$$

$$\Omega(t) = 8\pi\rho/3H^2. \tag{11}$$

Then for the Friedmann models we have

$$k=0 \text{ for } \Omega = 1,\ k=1 \text{ for } \Omega > 1 \text{ and } k=-1 \text{ for } \Omega < 1. \tag{12}$$

Now the presently estimated range of Ω is over three orders of magnitude, from 0.01 to 10. However, the field equations tell us that

$$\Omega(t) - 1 = k/\dot{S}^2. \tag{13}$$

Hence the difference between Ω and 1 was smaller in the past than it is now. If we assume that the matter content of the universe was fixed very early on , say , when the grand unified theories played a major role, then it turns out that the expansion rate of the universe had to be very finely tuned for it to have reached the present condition with Ω lying in the above observed range. The fine-tuning has to be as good as one part in 10^{50}. This fine tuning near the flat ($k = 0$) value of Ω becomes even more precise if the epoch of fixing is pushed back to the Planck epoch, as was pointed out by Guth[7]. This need for fine tuning near the flat value is called the 'flatness problem' and it was first highlighted by Dicke and Peebles[6].

As mentioned earlier, there is no experimental or observational result in gravity that makes it necessary to quantize it. However, the three problems raised above do pose challenges to cosmology. Does our concept of the early universe get altered by quantum cosmology to such an extent as to remove these difficulties ?

3. Quantum Gravity via Path Integrals

There are several approaches to quantum gravity[8], but none of them can be said to be free from conceptual difficulties, nor have they taken the subject to a stage where it can work out specific problems.

The approach to be presented here has been described in detail elsewhere[2]. It is less ambitious and formally less satisfying than most other approaches, but it has the advantage of conceptual and practical simplicity.

Consider first the classical action which gives Einstein's field equations:

$$A = A_H + A_m \tag{14}$$

where A_H is as given by (1) and A_m is the matter action. In the geometrodynamics as formulated by Wheeler[9] we consider $\mathcal{V}$ as the thin sandwich region with three-geometries ^3G specified on its boundaries Σ_1 and Σ_2 which are spacelike hypersurfaces. However, as pointed out by Isenberg and Wheeler[10] the specification of three-geometries leads to ambiguities and the correct specification on the boundaries should be in the form of the conformal part of the three-geometries and the trace of the extrinsic curvature.

The corresponding problem in quantum geometrodynamics is given as the path integral over the three-geometries:

$$K[2;1] = \int \exp(iA/\hbar)\, \mathcal{D}^{(3)}G \tag{15}$$

where A is evaluated for any given evolution of 3-geometries from Σ_1 to Σ_2. The propagator K gives the probability amplitude of arriving at a given final geometry on Σ_2 for a given initial geometry on Σ_1.

Naturally the path integral in (15) is impossible to evaluate for arbitrary geometries. The situation is simplified considerably, however, if we restrict ourselves to conformal degrees of freedom only. This is done as follows. Let $\bar{g}_{ik}$ be the solution of the classical problem $\delta A = 0$ and write

$$g_{ik} = \Omega^2\, \bar{g}_{ik} \tag{16}$$

$$\Omega = 1 + \phi \tag{17}$$

for any general (non-classical) geometry conformal to the classical geometry. Ω is a $\mathcal{C}^2$ function of spacetime coordinates and ϕ denotes the conformal fluctuation from the classical geometry.

With (16) and (17) we can rewrite (14) as

$$A_H = \frac{1}{16\pi}\int_{\mathcal{V}} \{(1+\phi)^2 \bar{R} - 6\phi_{,i}\phi^{,i}\}(-\bar{g})^{-1/2} d^4x \qquad (18)$$

Notice that the variables ϕ, $\phi_{,i}$ occur in (18) at most in the quadratic. The evaluation of the path integrals in such cases is well defined and we can therefore define and evaluate K in (15) explicitly. (See for example, Feynman and Hibbs[11].)

What about A_m ? The simplest form for A_m is

$$A_m = \sum_a \int m_a\, ds_a \qquad (19)$$

where a,b,... are free particles of restmasses m_a, m_b, ... and proper time elements ds_a, ds_b, ...

In the early universe around the Planck epoch, the particles are expected to be moving with very high energies and under the principle of asymptotic freedom they are not expected to interact except through gravity. Thus (19) is a good approximation for the matter action. Notice also that

$$A_m = \bar{A}_m + \sum_a \int m_a \phi\, ds_a \qquad (20)$$

and the quadratic nature of A is not affected by the inclusion of A_m.

4. The Divergence of Quantum Uncertainty

Narlikar[12] has shown how to construct K under the above simple assumptions. The explicit form of K is as follows:

$$K[\phi_2,t_2;\ \phi_1,t_1] = F(t_2,t_1)\exp\frac{3i}{8\pi}\chi\ ,$$

$$\chi = \sum_{P,Q=1,2}\sum \int\int A_{PQ}(\underline{x}_P,t_P;\ \underline{x}'_Q,t_Q)\ \phi_P(\underline{x}_P)\phi_Q(\underline{x}'_Q)\ d^3\underline{x}_P\ d^3\underline{x}'_Q, \qquad (21)$$

where the coefficients A_{PQ} are determined in terms of the retarded

Green's function of the operator

$$\Box + \frac{1}{6}\bar{R} \qquad (22)$$

computed for the classical spacetime metric $\bar{g}_{ik}$. In particular, we have

$$A^{-1}_{12} = A^{-1}_{21} = \tilde{G}(\underline{x}_2\ ,\ t_2\ ;\ \underline{x}_1\ ,\ t_1) \qquad (23)$$

where $\tilde{G}(\underline{x}_2\ ,\ t_2\ ;\ \underline{x}_1,\ t_1)$ is the retarded Green's function.

Now consider the following situation. Let us turn the arrow of time backwards so that the big bang epoch $t = 0$ lies to the future. Thus, although t_2 follows t_1 the epoch t_2 is closer to $t = 0$ than t_1. In fact we will assume that $t_2 \lesssim t_P$ while $t_1 \gg t_P$.

At t_1 the universe is therefore describable by the classical solution. To allow for some quantum uncertainty suppose the state of the universe is described by a wavepacket

$$\Psi_1[\phi_1\ ,\ t_1\ ] = \frac{1}{(2\pi\Delta_1^2)^{1/4}}\exp-\frac{\phi_1^2}{4\Delta_1^2}\ . \qquad (24)$$

This wavepacket is centred on the classical value $\phi_1 = 0$ but has a small dispersion.

Applying the propagator to ψ_1 we get

$$\Psi_2[\phi_2,t_2] = \int K[\phi_2,t_2;\phi_1,t_1]\ \Psi_1\ [\phi_1\ ,\ t_1\ ]\mathcal{D}\phi_1\ . \qquad (25)$$

Notice, however, that K and ψ_1 have exponential forms which are quadratic. The integral (25) can therefore be performed and we find that ψ_2 is also a wavepacket centred on $\phi_2 = 0$ (the classical value) but with a dispersion given by

$$\Delta_2^2 = \Delta_1^2\ [\ A_{11}^2 + \frac{1}{16\Delta_1^4}\ ]\ A_{12}^{-2}\ . \qquad (26)$$

Let us now see what happens when $t_2 \to 0$.

It can be shown[12] that as $t_2 \to 0$ the Green's function G diverges and hence $A_{12} \to 0$. This means that $\Delta_2 \to \infty$. In other words the quantum uncertainty diverges as the classical singularity is approached.

We may interpret this result as follows. In the completely deterministic classical theory we can always trace the present state of the universe to a unique state in the past. In quantum cosmology, the Heisenberg uncertainty principle tells us that as we consider the past history of the universe it gets blurred so that our present state can be linked to a wider range of initial states in the past. What the above result demonstrates is that as the classical singular epoch is approached the blurring is so complete that it is not possible to identify even a wide range of states as being the likely initial states. This is also seen from (25) by the fact that as $A_{12} \to 0$ the ϕ_1-integral completely delinks ψ_1 from ψ_2. That is, no possible connection exists between the past and the future!

5. The Avoidance of Singularity and Horizons

The spacetime given by (16) is nonclassical for a general Ω. We denote it by M while the classical spacetime (with metric $\bar{g}_{ik}$) is denoted by $\bar{M}$. We know that $\bar{M}$ is singular at $t = 0$. What about M? Depending on how Ω behaves as $t \to 0$, M may be singular or nonsingular. Let us classify all singular M's together in a class C_S while the rest belong to class C_{NS}. Thus $\bar{M}$ also belongs to C_S.

What has been shown so far is that the present state of the universe could have evolved from any M, whether belonging to C_S or to C_{NS}. In a recent result derived by Narlikar[13] it was shown that the quantum mechanical probability that the universe came to be in its present state from M belonging to C_S is vanishingly small. It follows therefore that the classical singular state is an exception rather than the rule and very likely the universe had a nonsingular origin.

The elimination of singularity at $t = 0$ also eliminates finite and small particle horizons. For, now it is possible to extend the past light cone of any observer beyond $t = 0$. A classic example of this is the singular Friedmann model given by

$$ds^2 = dt^2 - \left(\frac{3Ht}{2}\right)^{4/3} (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2). \qquad (27)$$

The conformal transformation

$$\Omega \propto t^{-2/3} \qquad (28)$$

converts $\bar{M}$ to M, the nonsingular Minkowski spacetime. The epoch $t = 0$ is not singular in M and light cones can be extended

to $t < 0$.

6. Resolution of the Flatness Problem

Padmanabhan[14] has given the following arguments to resolve the flatness problem within the framework of quantum conformal fluctuations.

First note that the Robertson-Walker line element (8) describes a conformally flat spacetime. That is, it is possible to find a function Ω such that (8) is expressible in the manifestly conformally flat form:

$$ds^2 = \Omega^2\{d\tau^2 - d\rho^2 - \rho^2(d\theta^2 + \sin^2\theta \, d\phi^2)\} \qquad (29)$$

where (τ,ρ) are functions of (t,r). Ω is also a function of (τ,ρ). Only in the case $k = 0$ can we identify ρ with r and express τ as a function of t through the relations

$$\Omega = dt/d\tau, \; \Omega(\tau) = S(t). \qquad (30)$$

In this case Ω is a function of τ only.

Consider now conformal fluctuations about the empty Minkowski spacetime given by the line element

$$ds^2 = d\tau^2 - d\rho^2 - \rho^2(d\theta^2 + \sin^2\theta \, d\phi^2). \qquad (31)$$

Treating this as a solution of Einstein's equations, we can compute K by our previous methods. It is already known that flat empty spacetime is unstable to conformal fluctuations and matter creation[14-17]. Hence using K we now compute the probability that the universe made a transition from (31) to a conformally flat form. Following the detailed calculations of Padmanabhan (op. cit.) we find this probability to be

$$\mathcal{P} = A \exp\left[-\frac{3}{8\pi}\int\int \frac{\nabla_1\Omega(\rho_1)\cdot\nabla_2\Omega(\rho_2)}{|\rho_1 - \rho_2|^2} d^3\rho_1 \, d^3\rho_2\right] \qquad (32)$$

where A is a normalizing constant and Ω is the conformal function in the final state.

Notice that $\mathcal{P}$ is largest when $\nabla\Omega = 0$, i.e., when Ω does not depend on ρ, θ, ϕ. At most Ω then depends on t and we arrive at the Robertson-Walker spacetimes with $k = 0$. For $\nabla\Omega \neq 0$, the double integral is positive definite and in general large compared to unity. Thus $\mathcal{P}$ is negligibly small for all such transitions which include the models with $k \neq 0$.

Thus, starting from the simplest possible initial conditions, quantum cosmology leads the universe uniquely to the k = 0 models.

7. Future Outlook

The simplistic approach of limiting ourselves to quantizing conformal degrees of freedom has thus yielded quick dividends in providing solutions to the three outstanding problems of quantum cosmology. Our defence of this simple approach rests on its usefulness and on the historic analogy one may draw with the development of quantum mechanics. It is well known that quantum mechanics developed from a pragmatic outlook starting from Bohr's quantization rule for the hydrogen atom, and progressing through De Broglie's wave mechanics and Heisenberg's uncertainty condition. It took many years for a formal structure to be erected, and judging by the present ongoing discussions of foundations of quantum mechanics, we may well argue that a completely satisfactory formal structure is not yet available. Nevertheless, the subject developed so far has proved indispensable for theoretical physics.

Likewise we feel that while the search for a satisfactory formal framework for quantum gravity is being made, simple but pragmatic approaches like ours have a useful role to play. For, such forays bring valuable insights into the complex problem of quantum gravity.

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