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DYNAMICAL SCENARIOS WITH UNSTABLE NEUTRINOS

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ABSTRACT

We describe a cosmological scenario with an unstable warm dark matter candidate (a heavy neutrino ν_H with mass of ~ 120 eV) which decays into a "hot" particle (a light neutrino ν_L with mass of ~ 6 eV) and a relativistic boson. Theoretical and observational constraints on such a model are discussed. We note that (1) decay of ν_H disrupts the condensates made of primordial ν_L, lowering their mass to acceptable values of $\sim 10^{12}\ M_\odot$. (2) The relativistic boson can contribute nearly 0.25 to the total density in a "recent" decay. The model predicts two prominent scales in dark matter distribution: (i) a mass of $\sim 4 \times 10^{12}\ M_\odot$ around the galaxies, distributed over ~ 200 kpc and (ii) a smoother density of $\sim 10^{-27}$ g cm^{-3} distributed over ~ 1 Mpc. We argue that the model agrees with observations at all scales.

Subject headings: cosmology — elementary particles — neutrinos

I. INTRODUCTION

The majority of astrophysicists believe that a large amount of dark matter exists in our universe. The flat rotation curves of spirals (Rubin 1979; Rubin *et al.* 1982; Rubin, Thonnard, and Ford 1982), virial mass estimates (Rood 1981; Faber and Gallagher 1979) of large clusters, and density determinations from Virgo infall (Davis and Peebles 1983) are taken to indicate the existence of dark matter. One popular hypothesis has been that the density of the universe is equal to the critical density. There are no observations which *demand* this hypothesis. (At the same time it is probably fair to say that observations are not in clear contradiction with this hypothesis.) Measurements at largest scales only require (Faber and Gallagher 1979)

$$\Omega h_0^2 \geq 0.2\ , \tag{1}$$

where Ω is the ratio between the density of matter and the critical density

$$\rho_c = 3H^2/8\pi G = 1.88 \times 10^{-29} h_0^2 \text{ g cm}^{-3}\ . \tag{2}$$

(Here and in what follows, h_0 denotes the Hubble constant in units of 100 km s^{-1} Mpc^{-1}.)

Can dark matter satisfying equation (1) be completely made up of baryons? The conventional answer (which we shall take as correct in this paper) is "no." Big bang nucleosynthesis imposes the following constraint on baryonic part of the Ω (Yang *et al.* 1984):

$$0.01 \leq \Omega_B h_0^2 \leq 0.049\ . \tag{3}$$

Comparing equations (1) and (3), one may conclude that major part of the dark matter must be nonbaryonic.

It is convenient to divide the nonbaryonic candidates for dark matter as "hot" and "cold" (Bond and Szalay 1983; Blumenthal *et al.* 1984). The simplest hypothesis, viz. that the same particle constitutes the dark matter at all scales (from dwarf spheroidals to clusters) leads to severe constraints on dark matter modeling.

Hot dark matter (e.g. neutrinos with masses of ~ 10 eV) cannot cluster effectively at small scales. The first structures that form in hot dark matter scenario will have masses of the order of

$$M_J \approx 3 \times 10^{16} M_\odot (m_\nu/10 \text{ eV})^{-2} \tag{4}$$

(Bond, Efsthathiou and Silk 1980; Doroshkevich *et al.* 1981). Since collisionless dark matter evolves without dissipation, it is impossible to make hot dark matter fragment and condense on smaller, galactic scales. It is also known that galaxies form very late in these models (Frenk, White, and Davies 1983; Dekel and Aarseth 1984; Hut and White 1984; Kaiser 1983; Mellot 1983; Faber 1984).

Cold dark matter (with $m > 200$ keV) will form small structures ($M_J \sim 10^6\ M_\odot$) first. Larger structures have to be produced by hierarchical clustering. The problem with this scenario is that since cold dark matter is so good at clustering, galaxies and clusters would consist of primordial fractions of dark and baryonic matter. Since mass densities at these scales correspond to Ω between 0.1 and 0.5 (at most), cold dark matter will be incompatible with the $\Omega = 1$ universe. Detailed cold dark matter modeling runs into trouble with the age of the universe as well (Peebles 1982, 1984).

If both "hot" and "cold" versions run into problems, then what could be an alternative?

One simple solution would be to assume that both hot and cold dark matter candidates are present simultaneously. With an increased number of parameters at our disposal, it should not be very difficult to satisfy all the constraints. It must be, however, admitted that such an approach appears aesthetically unsatisfactory.

An indirect way of achieving the same end would be to invoke a dark matter candidate which is unstable. Such models involving cold dark matter scenario (Turner, Steigman, and Krauss 1984; Turner 1985; Suto *et al.* 1985; Olive, Sekel, and Vishniac 1985) have attracted considerable attention recently. These papers aim at constraining the properties of the dark matter candidates based on cosmological requirements.

In this paper, we perform a similar analysis with unstable neutrinos. The role of unstable neutrinos in cosmology has been investigated previously by many authors (Gelmini, Schramm, and Valle 1984; Turner, Steigman, and Krauss 1984; Davis *et al.* 1981; Fukugita and Yanagida 1984; Dicus, Kolb, and Teplitz 1978). The present work differs from the previous analysis (mainly) in the following aspects:

1. We attempt to estimate the effect of heavy particle decay in detail. This allows us to calculate the density contribution due to dark matter at various length scales.

2. The disruptive effect of the decay allows us to reduce the mass of initial condensates to acceptable values.

3. The model is based on a decay scheme ($\nu_H \rightarrow \nu_L b$), where ν_H and ν_L are the heavy and light neutrinos and b is a very light boson. This boson b is relativistic even at the present epoch and provides a smooth, subdominant contribution to Ω. (As far as we know, this contribution was not taken into account in the previous discussions.)

The details of the model are presented in § II, and the conclusions are discussed in § III.

The main results of the paper may be summarized as follows (see Fig. 1 for the overall evolution). In the very early universe both ν_H and ν_L will be present in equilibrium concentrations. Suppose $m_H \approx 200$ eV and $m_L \approx 6$ eV (the choice of these parameters as well as the range allowed for them are discussed in § II*c*), The ν_H becomes nonrelativistic and dominates the dynamics of the universe when the temperature drops below $T_{\rm eq} \approx 1.3 \times 10^5$ K. Structures with masses $M_J \approx 10^{14}$ $M_\odot$ start growing and will contain primordial ν_L and ν_H. On recombination, baryons fall into these potential wells. The decay of ν_H to (ν_L, b) occurs around ~ 51 K. The decay partially disrupts the condensates already formed and disperses a fraction of primordial ν_L. The surviving dark matter structures containing primordial ν_L will have masses of $\sim 3 \times 10^{12}$ $M_\odot$ and will have scale lengths of ~ 200 kpc today. The dispersed component of primordial ν_L condenses back at scales ~ 1 Mpc. The decay products become nonrelativistic at ~ 4 K. It is possible to have $\Omega = 1$ in this model with $\Omega_L \approx 0.51$ (due to two species of primordial ν_L), $\Omega_D \approx 0.26$ (due to ν_L produced in the decay), and Ω_R ~ 0.23 (due to the relativistic boson that is produced in the decay). The main source of constraint in the model appears to be the age of the universe. We get ~ 13 billion years $\times$ $(h_0/0.5)^{-1}$ for the model values described above. Clearly, h_0 cannot be much above 0.5 if the model has to work. It is, of course, possible to produce a variety of models by changing the parameters in the above discussion. These details are presented in § II*c*.

II. COSMOLOGY WITH UNSTABLE NEUTRINOS

The model uses three "active" parameters—masses, m_H and m_L of the heavy and light neutrinos and the lifetime τ of the unstable neutrino. (In order to illustrate the ideas in the simplest possible manner we have assumed that the third species of neutrino [ν_3] contributes an amount equal to m_L. This can

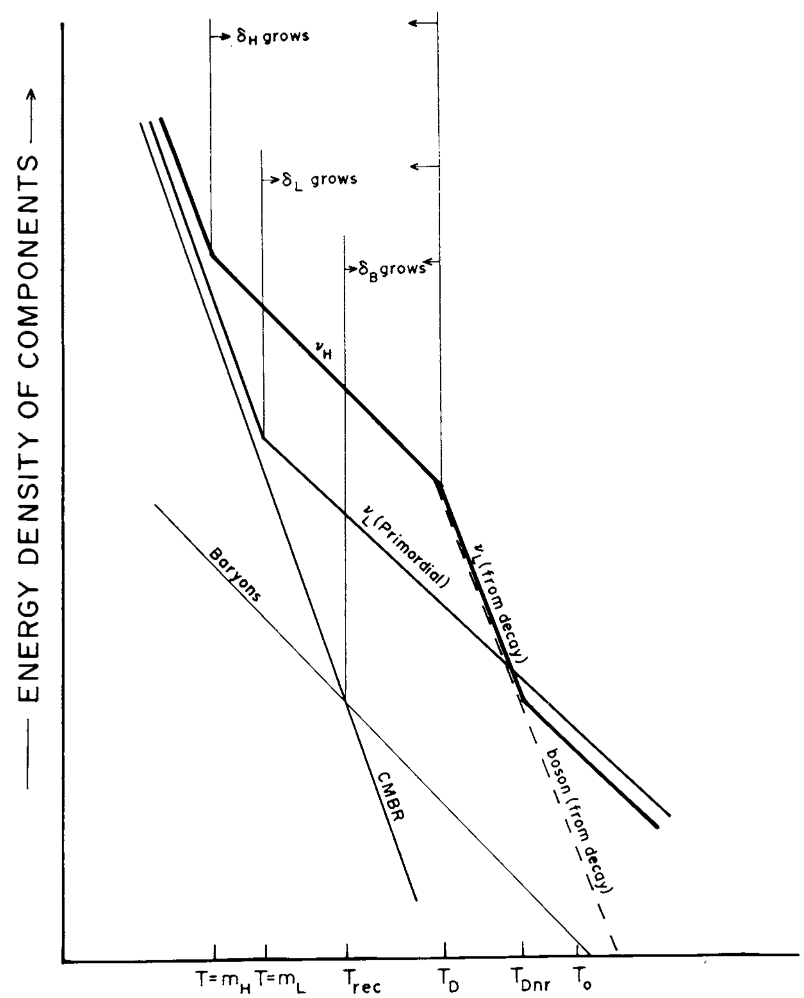

FIG. 1.—Evolution of energy density of various components in the model

arise for example because ν_3 is very heavy, because it has decayed very early into ν_L, or simply because there are two species with mass m_L. Our conclusions are insensitive to the specific assumptions about ν_3; the parameters can be readjusted to accommodate other possibilities.) We take the decay scheme to be $\nu_H \to \nu_L b$, where b is a massless (or very light) boson. At the level of this paper, we have not bothered to present a particle physics model justifying the assumption. It should be, of course, obvious that any particle with sufficiently weak interaction and the same values of m_L, m_H, and τ will do as well. We shall continue to call them "neutrinos" keeping this provisio in mind.

It turns out to be more convenient to express our formulae in terms of the three parameters Ω_L, Ω_R, and T_D (rather than the original m_L, m_H, and τ). Here Ω_L is the fraction of density contributed by the primordial component of ν_L, which will be nonrelativistic today; Ω_R is the contribution from the relativistic boson b, produced in the decay; T_D is the temperature of CMBR at the epoch of ν_H decay.

a) *Kinematic Details*

The evolution of the energy density contributed by various components is shown schematically in Figure 1. We shall begin with the present epoch and proceed backward. The contribution to Ω due to primordial ν_L is

$$\Omega_L = \rho_L/\rho_c = m_L n_{\gamma 0} r_L/\rho_c \,, \tag{5}$$

where r_L is the ratio of number densities of ν_L to the CMBR photons, which is $(3/4 \times 4/11 \times 2 = 6/11)$ when there are two species of ν_L; $n_{\gamma 0}$ is the number density of photons today. Clearly,

$$m_L = (\Omega_L/r_L)(\rho_c/n_{\gamma 0}) = 48.6 \text{ eV } (\Omega_L h_0^2) \,. \tag{6}$$

At the time of decay ($t = \tau$, $T = T_D$), the rest energy m_H of the heavy neutrino is shared almost equally (we shall see that $m_L \ll m_H$) between m_L and the relativistic boson b. These bosons are relativistic even today and contribute to Ω the amount (where $r_H = n_H/n_\gamma = 3/11 = \frac{1}{2} r_L$),

$$\Omega_R = \frac{\rho_R}{\rho_c} = \frac{1}{\rho_c}\left(\frac{1}{2} m_H\right)[r_H n_\gamma(T_D)]\left(\frac{T_0}{T_D}\right)^4 = \frac{1}{2} m_H r_H \left(\frac{n_{\gamma 0}}{\rho_c}\right)\left(\frac{T_0}{T_D}\right) . \tag{7}$$

Therefore,

$$m_H = (\Omega_R/r_H)(2\rho_c/n_{\gamma 0})(T_D/T_0) \,. \tag{8}$$

The other half of the energy ($\frac{1}{2} m_H$) is carried by the ν_L produced in the decay. Clearly these ν_{LD} produced in the decay are extremely relativistic at the time of production. As the expansion proceeds, their random velocities will decrease, and they will become nonrelativistic at some temperature, $T_{\rm Dnr}$ ("Dnr" signifies "decay product ν_L is nonrelativistic") given by

$$T_{\rm Dnr} = (2m_L/m_H) T_D \,. \tag{9}$$

Substituting equations (6) and (8) in equation (9), we get

$$T_{\rm Dnr} = (r_H/r_L)(\Omega_L/\Omega_R) T_0 \,. \tag{10}$$

We shall assume that $T_{\rm Dnr} > T_0$; i.e., these neutrinos have already become nonrelativistic sometime in the past, (The Virgo infall of our Galaxy provides some constraints on the unstable dark matter scenarios. These constraints are less severe and can be satisfied if the present epoch is matter dominated; see for example, Efsthathiou 1985.) These ν_L from decay will, therefore, contribute to the amount

$$\Omega_D = (r_H/r_L)\Omega_L = \tfrac{1}{2}\Omega_L \,. \tag{11}$$

In making the above estimates we have approximated the decay as an instantaneous process. Taking into account the exponential law for decay will modify the expressions slightly (e.g., eq. [7] will be multiplied by [0.88] on the right-hand side). This aspect has been discussed in detail by Turner (1985). For our purposes the approximation of instantaneous decay is adequate.

For temperatures $T > T_D$, the universe was dominated by ν_H. The ν_H becomes nonrelativistic at the temperature $T_{\rm NR} = m_H$. Thus,

$$T_{\rm NR} = (\Omega_R/r_H)(2\rho_c/n_{\gamma 0})(T_D/T_0) \,. \tag{12}$$

At a slightly lower temperature $T = T_{\rm eq}$, the energy density of ν_H

$$\rho_H = m_H n_{\gamma 0} r_H (T_{\rm eq}/T_0)^3 \tag{13}$$

becomes equal to the energy density of the relativistic species of particles (two species of ν_L and γ),

$$\rho_{\rm rel} = \rho_\gamma(1 + a) = (1 + a)\rho_{\gamma 0}(T_{\rm eq}/T_0)^4 \,, \tag{14}$$

where

$$a = (\rho_L/\rho_\gamma) = \tfrac{7}{8} \times (\tfrac{4}{11})^{4/3} \times 2 \approx 0.45 \,. \tag{15}$$

(The value of a also depends on whether the neutrino is treated as Dirac or Majorana type. These differences change a by a factor of 2. Most of the following discussion is independent of the precise choice made for a above.)

Equating equations (13) and (14), we determine $T_{\rm eq}$ to be

$$T_{\rm eq} = T_0\left[\frac{m_H r_H}{1 + a}\right]\left(\frac{n_{\gamma 0}}{\rho_{\gamma 0}}\right) = \frac{2}{1 + a}\left(\frac{\Omega_R}{\Omega_\gamma}\right)(T_D) \,, \tag{16}$$

where

$$\Omega_\gamma = \frac{\rho_\gamma}{\rho_c} = 2.38 \times 10^{-5} h_0^{-2} \,. \tag{17}$$

At some lower temperature $T_{\rm nr}$, the primordial ν_L becomes nonrelativistic; we have

$$T_{\rm nr} = m_L = (\Omega_L/r_L)(\rho_c/n_{\gamma 0}) \,. \tag{18}$$

To avoid any possible confusion we stress the notation that is being adopted. The symbol $T_{\rm nr}$ refers to the temperature at which primordial ν_L becomes nonrelativistic; "$T_{\rm NR}$" refers to the temperature at which (primordial) ν_H becomes nonrelativistic; "$T_{\rm Dnr}$" refers to the temperature at which the ν_L produced out of the ν_H decay becomes nonrelativistic. If T_D refers to the decay epoch and T_0 to the present-day CMBR temperature, then the ordering of temperatures is the following: $T_{\rm NR} > T_{\rm nr} > T_D > T_{\rm Dnr} > T_0$. We hope the notation does not confuse the reader.

In the radiation-dominated early universe, we have the following relationship between time and temperature (Narlikar 1983):

$$(t/1 \text{ s}) = 2.4(T/1 \text{ MeV})^{-2}[2(1 + a)]^{-1/2} \,. \tag{19}$$

Evolving the universe as (i) matter dominated from $T_{\rm eq}$ to T_D, (ii) radiation dominated from T_D to $T_{\rm Dnr}$, and (iii) matter dominated from $T_{\rm Dnr}$ to T_0, we can estimate the age of the universe t_u to be

$$t_u = t_{\rm eq}(T_{\rm eq}/T_D)^{3/2}(T_D/T_{\rm Dnr})^2(T_{\rm Dnr}/T_0)^{3/2} \ . \tag{20}$$

Expressing everything in terms of Ω_L, Ω_R, and T_D, and using the fact that $r_L = 2r_H$, we get

$$t_u = (4.94 \times 10^9 \text{ yr})h_0^{-1}\Omega_L^{-1/2} \ . \tag{21}$$

Clearly, we will obtain the strongest constraint on Ω_L and h_0 from this expression for the age.

The contribution to the present-day energy density from the primordial ν_L, the decay product ν_L, and from the relativistic boson all differ from each other only by phase space factors (e.g., $1 + a$, etc.). This is due to the fact that the decoupling temperature of the neutrinos is reasonably insensitive to its mass (when the mass is in the range of a few eV to a few MeV). Because of this fact, the original number densities all match with each other except for phase space factors. We have confined ourselves to this conventional picture in this paper, even though it is possible to think of more exotic scenarios involving unstable neutrinos.

b) *Kinematic Constraints*

The scenario in Figure 1 is based on the following ordering:

$$T_{\rm eq} > T_{\rm nr} > T_D > T_{\rm Dnr} > T_0 \ . \tag{22}$$

It is easy to show that the first inequality implies the third one. The other inequalities reduce to

$$T_D/T_0 < (2.1 \times 10^5)(\Omega_L h_0^2) \ , \tag{23}$$

$$T_D/T_0 > 3.6(\Omega_L/\Omega_R) \ , \tag{24}$$

$$\Omega_R < 0.5\Omega_L \ . \tag{25}$$

From equations (25) and (11) it follows that

$$\Omega \equiv \Omega_L + \Omega_D + \Omega_R < 2\Omega_L \ . \tag{26}$$

The age determination of globular clusters is still not free from controversy. It is usual to cite the age as $t_g = 15 \pm 3$ billion yr (Iben 1974; Sandage 1982; Schramm 1983), where "± 3" represents the range of opinions of the workers in the field. Combining equation (26) with (21), we obtain the constraint

$$\Omega h_0^2 < [t_g/(6.99 \times 10^9 \text{ yr})]^{-2} \ . \tag{27}$$

This constrains the value of h_0. We see that it is certainly possible to obtain $t_g \approx 13$ billion yr and $\Omega = 1$ by keeping $h_0 \lesssim 0.5$. Thus these kinematic constraints do not pose serious threats to the model. (We emphasize that the relativistic boson b will be providing $\Omega_R \approx 0.25$, toward $\Omega = 1$; see § IIe[i]. This is crucial–although equation [27] does not make this fact explicit—in satisfying the constraints.)

Constraints (23)–(25) and (27) can be cast into various other forms. For example, we can write equation (20) as

$$\Omega_L h_0^2 < 0.144(t_g/13 \times 10^9 \text{ yr})^{-2} \ . \tag{28}$$

Comparing with equation (6), we get

$$m_L < 6.8 \text{ eV}(t_g/13 \times 10^9 \text{ yr})^{-2} \tag{29}$$

Similarly, we get from equations (6), (8), and (24)

$$m_H > 14.4 \ m_L \ . \tag{30}$$

More stringent constraints can be obtained by analyzing the growth of fluctuations in the model, as we shall now see.

c) *Growth of Fluctuations*

In the linear regime, the perturbations can be decomposed into separate modes, each of which evolves independently. The behavior of those modes for which the physical wavelength (which is proportional to the expansion factor) is smaller than the horizon size can be understood rather easily. Modes for which the wavelength is larger than the horizon size require a careful study based on gauge invariant parameters. In this paper we shall confine our attention to the growth of perturbations after the relevent scale has entered the horizon. We shall parameterize the modes by the value of their amplitude when they "enter the horizon." Because of this parameterization, we need not bother about the modes longer than the horizon size.

The specific form of the growth can be determined from the evolution equations. During the ν_H–dominated (matter-dominated) phase, the density contrast grows proportional to $S(t)$, the expansion factor (Peebles 1980); i.e.,

$$\delta(t) = (\delta\rho/\rho) \propto S(t) \ . \tag{31}$$

Let us consider fluctuations at a scale λ_0 chosen in such a way that it entered the horizon in the past at $t = t_{\rm eq}$. In other words, we consider the fluctuation for which the physical wavelength $\lambda(t_{\rm eq})$ is equal to the horizon size $(ct_{\rm eq})$ at $t = t_{\rm eq}$. Such a fluctuation can grow by a factor $[S(\tau)/S(t_{\rm eq})]$ during the ν_H–dominated phase:

$$\begin{aligned} A^0_{\nu_H \text{epoch}} &= S(\tau)/S(t_{\rm eq}) = T_{\rm eq}/T_D \\ &= [2/(1 + a)](\Omega_R/\Omega_\gamma) = 5.8 \times 10^4 \Omega_R h_0^2 \ . \end{aligned} \tag{32}$$

The fluctuations also pick up some residual growth at two other epochs (Primack and Blumenthal 1984; also see Bond, Szalay, and Turner 1982; Turner, Wilczek, and Zee 1983). First, note that ν_H stops free streaming at $T = T_{\rm NR} > T_{\rm eq}$ itself. The universe is radiation dominated for $T > T_{\rm NR}$ and is matter dominated for $T < T_{\rm eq}$. For $T_{\rm NR} > T > T_{\rm eq}$. the universe is in a transition stage. Let us denote by the subscript 1, the scale λ_1 that enters the horizon at $T = T_{\rm NR}$. If the universe were matter dominated right from $T_{\rm NR}$, then these scales could have grown by a factor $(T_{\rm NR}/T_D)$ before ν_H decay. We see that the extra gain is by the factor

$$A^1_{\rm extra} = T_{\rm NR}/T_{\rm eq} \approx 14.4 \ . \tag{33}$$

It is safe to say that fluctuations that enter the horizon between $T_{\rm NR}$ and $T_{\rm eq}$ can pick up an extra growth factor of (5–10) easily in the pre–ν_H-dominant phase itself.

Second, the growth does not cease abruptly at $t = \tau$ because $\dot{\delta}(\tau) = 0$. The fact that the actual decay is not instantaneous but has a tail adds to this feature.

With all these considerations in mind, we shall estimate the *total* growth factor for fluctuations that enter the horizon somewhere between $T_{\rm NR}$ and $T_{\rm eq}$ to be

$$A_{\rm tot} = O(10)A_{\nu_H \text{ epoch}} \approx 5.8 \times 10^5 \Omega_R h_0^2 \ . \tag{34}$$

The value for the initial amplitude for the perturbations is constrained by the limits on the anisotropy of microwave background. For $(\Delta T/T) < 6 \times 10^{-5}$, this constraint is equivalent to $\delta_i < 1.2 \times 10^{-4}$ (Fixen, Cheng, and Wilkenson 1982). From equation (34) we see that these perturbations would have grown to

$$\delta_f \approx 70\Omega_R h_0^2 \tag{35}$$

at the time of decay. We shall demand that these fluctuations have reached near nonlinearity at the time of decay ($\delta_f > 1$), leading to the constraint:

$$\Omega_R h_0^2 > 0.014 . \tag{36}$$

We shall next estimate the size of these structures just before the decay. The perturbation which had a physical wavelength (ct_{eq}) at $t = t_{eq}$ will correspond to a size

$$\begin{aligned}\lambda_0 &= (ct_{eq})(T_{eq}/T_D) \\ &= 1.36 \times 10^{25}(\Omega_R h_0^2)^{-1}(T_D/T_0)^{-2} \text{ cm}\end{aligned} \tag{37}$$

at the time of decay ($t = \tau$, $T = T_D$). The perturbation which entered the horizon at $t = t_{NR}$ will have a smaller size:

$$\begin{aligned}\lambda_1 &= (ct_{NR})(T_{NR}/T_D) = (T_{eq}/T_{NR})\lambda_0 \\ &= 0.94 \times 10^{24} \text{ cm } (\Omega_R h_0^2)^{-1}(T_D/T_0)^{-2} .\end{aligned} \tag{38}$$

The masses contained within these perturbations

$$M = (4\pi/3)\rho\lambda^3 \tag{39}$$

remain constant during the cosmic expansion (since $\rho \propto S^{-3}$ and $\lambda \propto S$). Initially, for $T_{NR} > T > T_{nr}$, this mass is entirely due to ν_H. At $T < T_{nr}$, the primordial ν_L becomes nonrelativistic and "falls into" the ν_H potential "well," equalizing the density contrasts ν_L and ν_H. (This is similar to the behavior of baryonic perturbations in a stable neutrino-dominated universe discussed by many authors. For example, see Doroshkevich *et al.* 1981.) From then onward, the perturbation will contain a mass M_L in the form of primordial ν_L and a mass M_H in the form of ν_H. Clearly,

$$M_L(\lambda_0) = (4\pi/3)[m_L r_L n_{\gamma 0}(T_D/T_0)^3]\lambda_0^3 , \tag{40}$$

$$M_L(\lambda_0) = (2.97 \times 10^{14} M_\odot)\frac{\Omega_L h_0^2}{(\Omega_R h_0^2)^3}\left(\frac{T_D}{T_0}\right)^{-3} , \tag{41}$$

and

$$\begin{aligned}M_L(\lambda_1) &= (T_{eq}/T_{NR})^3 M_L(\lambda_0) \\ &= (9.95 \times 10^{10} M_\odot)[\Omega_L h_0^2/(\Omega_R h_0^2)^3](T_D/T_0)^{-3} .\end{aligned} \tag{42}$$

The corresponding M_H values can be found by multiplying by $(m_H r_H)/(m_L r_L)$; i.e.,

$$\begin{aligned}M_H(\lambda) &= (m_H r_H/m_L r_L)M_L(\lambda) \\ &= (2\Omega_R/\Omega_L)(T_D/T_0)M_L(\lambda) .\end{aligned}$$

Since free streaming of ν_H ends at $t = t_{NR}$ (corresponding to the scale λ_1) and matter domination begins at $t = t_{eq}$ (corresponding to the scale λ_0), we expect the power spectrum of the perturbations to be reasonably flat between the values $M(\lambda_1)$ and $M(\lambda_0)$ (This is similar to the cold dark matter scenario in which the spectrum remains flat from 10^6 to 10^{10} $M_\odot$.)

In the above analysis we picked out the largest structures that can form in this scenario. It is easy to estimate what size λ_0 would correspond to at the present epoch. This will be $\lambda_0(T_0)$:

$$\begin{aligned}\lambda_0(T_0) &= \lambda_0(T_D)(T_D/T_0) \\ &= 1.36 \times 10^{25}(\Omega_R h_0^2)^{-1}(T_D/T_0)^{-1} \text{ cm} .\end{aligned} \tag{43}$$

(This allows one to have a feel for the concerned scale. Physically, of course, equation [43] is of dubious value if this scale has gone nonlinear at $T = T_D$ itself and has stopped participating in cosmic expansion.) We shall now study the effect of decay on these structures.

d) Estimate of the Effect of Decay

Let us consider the perturbation in the wavelength scale λ_0 just before the decay of ν_H ($T > T_D$). This perturbation entered the horizon at T_{eq} and will be carrying a mass $M(\lambda_0)$. As we saw before, this scale would have gone nonlinear at $T > T_D$. Therefore, we shall assume that these structures are distributed as self-gravitating isothermal spheres with a cutoff radii R which is of the same order as λ_0. Since these structures originate essentially by the violent relaxation of collisionless ν_H, they will approximate an isothermal sphere reasonably well (Lynden-Bell 1967).

Consider one such isothermal sphere with the mass distribution

$$M(r) = M(r/R) . \tag{44}$$

Part of this mass (M_H) is due to ν_H and part (M_L) is due to ν_L. Once the ν_H decays, the mass M_H will be converted into relativistic species and will escape from the gravitational potential of the sphere. This, in turn, will reduce the mass available for the gravitational binding of the system. Consequently we may expect a considerable fraction, $\bar{f}$, of M_L also to be dispersed away. The surviving amount $(1 - \bar{f})M_L$ will respond adiabatically to the decay and expand, maintaining the virial equilibrium. To estimate the fraction that disperses, we proceed as follows (this calculation is similar to the one in Olive, Sekel, and Vishniac 1985).

Consider a typical ν_L originally bound to the condensate. If the (average) orbital size and velocity of this ν_L are r and σ, then the characteristic time scale for the orbit is (r/σ). As long as this time scale (r/σ) is small compared to the decay time scale τ (which is the time scale over which the gravitational potential changes), the orbit can respond adiabatically to the decay. On the other hand, neutrinos for which $(r/\sigma) > \tau$ cannot readjust their orbits and hence will disperse from this structure. (This claim that all neutrinos exterior to $r = \sigma\tau$ will disperse from the system is strictly true only if $\bar{f} > \frac{1}{2}$. This condition will be very well satisfied in our models: see eq. [75] and eqs. [85], [87] below.)

For isothermal spheres, the velocity

$$\sigma = [GM(r)/r]^{1/2} = (GM/R)^{1/2} \tag{45}$$

is independent of r. Since neutrinos with orbits that satisfy $r > \sigma\tau$ will escape from the system, it is convenient to define a critical radius r_c:

$$r_c = \sigma\tau = (GM\tau^2/R)^{1/2} . \tag{46}$$

We will assume that the amount of mass equivalent to that within r_c survives the decay. Therefore,

$$M_{fin} \approx M_L(r_c/R) , \tag{47}$$

leading to

$$(M_f/M_L) \approx r_c^2/R^2 = (4\pi/3)G\rho_L(T_D)\tau^2 . \tag{48}$$

Using the Einstein's equation and the fact that $S \propto t^{2/3}$, we get

$$H^2 = (\dot{S}^2/S^2) = 4/9\tau^2 = (8\pi G/3)\rho_H(T_D) . \tag{49}$$

Combining equations (48) and (49), we get, after some algebra,

$$(M_f/M_L) = (\tfrac{1}{3})(\Omega_L/\Omega_R)^{1/2}(T_0/T_D)^{1/2} \equiv k \; . \tag{50}$$

Using equation (40) we can estimate M_f. We find that

$$M_{\max} = M_f(\lambda_0) = (9.9 \times 10^{13}\ M_\odot)\left(\frac{T_D}{T_0}\right)^{-7/2} \frac{(\Omega_L h_0^2)^{3/2}}{(\Omega_R h_0^2)^{7/2}} \; . \tag{51}$$

The surviving matter will expand in size adiabatically. Under spherically symmetric adiabatic mass loss (which conserves the angular momentum), average orbital size $r(t)$ and velocity $v(t)$ will change satisfying the constraints

$$v^2(t)/r(t) = GM(t)/r^2(t) ;$$

$$v(t)r(t) = \text{constant} \; .$$

This leads to the scaling law (see also Olive *et al.* 1985, who reach the same conclusion from a more detailed analysis)

$$r_f M_f = r_i M_i \; .$$

Material within a characteristic radius r_c would have now bloated up to a final radius,

$$R_f = r_c(M_i/M_f) = R \; ,$$

where we have used, equation (47). In other words, the material expands to the original size $R \approx \lambda_0$. Thus we may consider the characteristic size of the objects to be again

$$R_f \approx \lambda_0 \; . \tag{52}$$

The fraction of mass $(1 - k)M_L$, which was dispersed at the decay, will free stream for some time in space. The initial velocity σ will decrease due to cosmic expansion, and these v_L will get recaptured by the potential wells. The free streaming length until the time of capture t_{cap} is given by

$$\lambda_{\text{fs}}(t_{\text{cap}}) = S(t_{\text{cap}}) \int_\tau^{t_{\text{cap}}} [v(x)/S(x)]dx \; . \tag{53}$$

This scale $\lambda_{\text{fs}}(t_{\text{cap}})$ will correspond to a wavelength $\lambda_{\text{fs}}(t_u)$ today, where

$$\lambda_{\text{fs}}(t_u) = [S(t_u)/S(t_{\text{cap}})]\lambda_{\text{fs}}(t_{\text{cap}}) \; . \tag{54}$$

Using

$$v(x) = \sigma[T_D/T(x)] \; ; \qquad S(x) = S_D(T_D/T) = S_D(t/\tau)^{1/2} \; , \tag{55}$$

we find

$$\lambda_{\text{fs}}(t_u) = 2(\sigma\tau)(T_D/T_0) \ln{(T_D/T_{\text{cap}})}$$
$$\approx 2(\sigma\tau)(T_D/T_0) \; . \tag{56}$$

We have neglected the logarithmic factor, which is of order unity (since $T_D > T_{\text{cap}} > T_0$ and, as we shall see, $T_D \approx 20T_0$). Equation (56) can be rewritten as

$$(\lambda_{\text{fs}}/R) \approx (\tfrac{2}{3})(\Omega_L/\Omega_R)^{1/2}(T_D/T_0)^{1/2} \; . \tag{57}$$

Physically λ_{fs} characterizes the scales at which the dispersed v_L will recondense.

Lastly, the v_L produced in the decay of v_H will be free streaming until $t = t_{\text{Dnr}}$. The characteristic length scales for these v_L will be very large:

$$\lambda_{\text{fsD}} = 9.4 \times 10^{27}\ \text{cm}\ (\Omega_L h_0^2)^{-1/2}(\Omega_R/\Omega_L) \; . \tag{58}$$

Even at the level of clusters, the v_L from the decay will only produce a smooth density. Most of the dark matter manifests at the other two scales: a mass of kM_L distributed over sizes R and a mass of $(1 - k)M_L$ distributed over sizes λ_{fs}. We shall now impose constraints on the parameters of the theory from dark matter observations.

e) Constraints from the Dynamics

As we have already seen, the fluctuations can reach nonlinearity by $T = T_D$, only if

$$\Omega_R h_0^2 > 0.014 \; . \tag{59}$$

For a universe with an age of 13×10^9 yr, the constraints discussed in § III*c* give the following bounds:

$$0.014 < \Omega_R h_0^2 < 0.072 \; , \tag{60}$$

$$0.5\Omega h_0^2 < \Omega_L h_0^2 < 0.144 \; . \tag{61}$$

These bounds also imply

$$h_0 \leq 0.53 \; . \tag{62}$$

Out of the three parameters Ω_R, Ω_L, and T_D, we have constrained two. Constraints on T_D arises from observations of dark matter distribution, in the following way.

The study of rotation curves of spiral galaxies suggests the existence of a dark matter halo with an r^{-2} density profile (which is well approximated by an isothermal sphere). Previous studies (Padmanabhan and Vasanthi 1985) have shown that the mass-radius relation for the spirals are very well approximated by the relation

$$M(r) = (1\text{–}5) \times 10^{10}\ M_\odot(r/1\ \text{kpc}) \; . \tag{63}$$

It is necessary that the isothermal spheres that survive the decay in our model satisfy, equation (63) reasonably well. We know that, these isothermal spheres satisfy,

$$M(r) \approx (M_f/R) \cdot r \tag{64}$$

or, using (51) and (52)

$$M(r)/r = 2.18 \times 10^{10}(M_\odot/\text{kpc})(T_D/T_0)^{-3/2}(\Omega_R h_0^2)^{-1}(\Omega_L/\Omega_R)^{3/2} \tag{65}$$

Comparing, equation (65) with equation (63), we can set the bounds

$$0.58 < (T_D/T_0)(\Omega_R/\Omega_L)(\Omega_R h_0^2)^{2/3} < 1.68 \; . \tag{66}$$

This constrains our last parameter T_D. We shall now discuss two explicit examples in detail, bringing out the nature of the constraints.

i) *Case 1: Theoretician's Model*

Many physicists exhibit great affinity toward the assumption "$\Omega = 1$." This—almost religious—faith continues in spite of the absence of (i) any unambiguous observational evidence for $\Omega = 1$ and (ii) any natural, inconsistency-free inflationary model. In deference to the sentiments of such physicists we shall first explore an $\Omega = 1$ universe.

We make the following choices: (i) $\Omega = 1$, (ii) age of globular clusters is 13×10^9 yr, and (iii) $h_0 = 0.53$. Then equations (28) and (26) imply (remember that $\Omega = 1.5\Omega_L + \Omega_R$):

$$\Omega_L = 0.514 \; , \quad \Omega_R = 0.23 \; . \tag{67}$$

Equation (66) now gives

$$8.05 < (T_D/T_0) < 23.3 \tag{68}$$

For the sake of simplicity, we shall adopt a fiducial value $(T_D/T_0) \approx 18$. The size of these structures are then of the order of (see eq. [52]),

$$R \approx 200 \text{ kpc} , \qquad (69)$$

and total mass carried inside R is about (eq. [51])

$$M \approx 3.1 \times 10^{12} \, M_\odot . \qquad (70)$$

The density of dark matter at $r < R$ follows the r^{-2} profile, with

$$\rho(r) = [M/(4\pi R^3)](R/r)^2$$
$$= 0.9 \times 10^{-24} \text{ gm cm}^{-3} \, (r/10 \text{ kpc})^{-2} . \qquad (71)$$

The next larger condensation will occur at scales of the order of λ_{fs} (eq. [57]). For the parameters we have chosen

$$\lambda_{fs} = 890 \text{ kpc} , \qquad (72)$$

which is slightly smaller than the size of cluster. The mass M_{dis} contained within this size is equal to the fraction of M_L which was dispersed in the decay of ν_H:

$$M_{dis} = (1 - k)M_L = [(1 - k)/k]M_{fin} . \qquad (73)$$

Using equation (50), we get

$$M_{dis} \approx 7.3 M_{fin} \approx 2.3 \times 10^{13} \, M_\odot . \qquad (74)$$

We emphasize that M_{dis} is a smooth distribution which exists *over and above* the isothermal clumps discussed before. The smooth density provided by M_{dis} is

$$\bar{\rho}_{dis} = [3M_{dis}/(4\pi\lambda_{fs}^3)] \approx 6 \times 10^{-28} \text{ g cm}^{-3} . \qquad (75)$$

Finally, we express the constraints in terms of m_L and m_H. From equations (6) and (8) we get

$$m_L \approx 7 \text{ ev} , \qquad (76)$$

and

$$100 \text{ eV} < m_H < 290 \text{ eV} . \qquad (72)$$

The overall picture which emerges from these considerations is shown in Figure 2. The dark matter is essentially lumped around galaxies, with an extended halo of size $R \approx 170$ kpc or so. In addition, there is a smoother distribution of dark matter with density of $\sim 10^{-27}$ g cm^{-3} spread over sizes of ~ 0.8 Mpc. It is easy to estimate the length scale L at which the smooth component contributes as much as the clumped part. We need

$$(4\pi/3)\rho_{dis} \, L^3 \approx (M/R)L , \qquad (78)$$

giving

$$L \approx [3M/(4\pi R\rho_{dis})]^{1/2} \approx 700 \text{ kpc} . \qquad (79)$$

Therefore we expect an $M \propto r$ behavior for $r < 500$ kpc and $M \propto r^3$ behavior for $r > 1$ Mpc. Such a trend is indeed observed in the M-r fits for dynamical systems (Padmanabhan and Vasanthi 1986).

ii) *Case 2: Observer's Model*

If the theoretical prejudice for $\Omega = 1$ is given up, then somewhat wider class of models are possible. Since all large-scale density determinations are consistent with $\Omega \approx 0.2$, we shall describe briefly a model with the following parameters:

$$\Omega = 0.5 \; ; \quad t_{glob} = 15 \times 10^9 \text{ yr} \; ; \quad h_0 = 0.65 . \qquad (80)$$

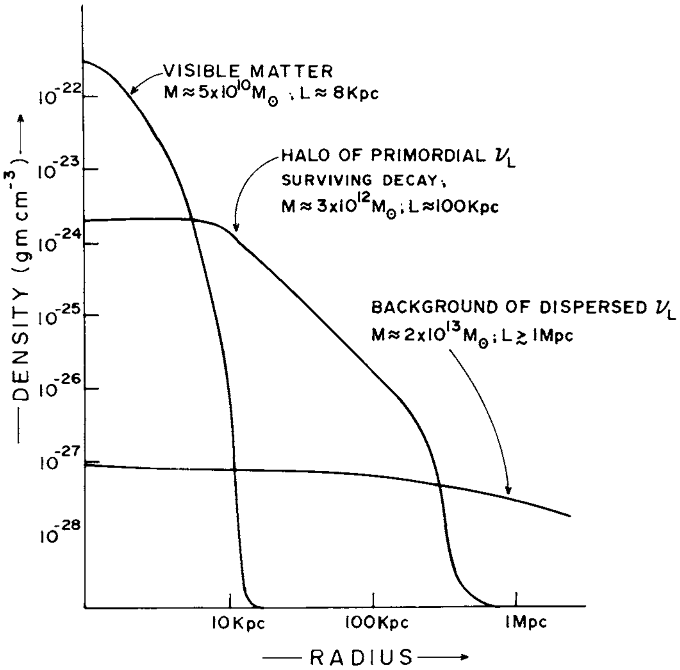

FIG. 2.—Schematic description of the distribution of dark matter arising from the model.

Analysis similar to that in previous case results in the following values:

$$\Omega_L = 0.26 , \quad \Omega_R = 0.12 , \qquad (81)$$

$$9.2 < T_D/T_0 < 26.6 . \qquad (82)$$

We shall take, say, $T_D/T_0 = 20$. Then

$$\lambda \approx 223 \text{ kpc} , \qquad (83)$$

$$M \approx 3.43 \times 10^{12} \, M_\odot , \qquad (84)$$

$$\lambda_{fs} \approx 978 \text{ kpc} , \qquad (85)$$

$$M_{dis} \approx 2.78 \times 10^{13} \, M_\odot , \qquad (86)$$

$$\rho_{dis} \approx 5.24 \times 10^{-28} \text{ g cm}^{-3} , \qquad (87)$$

$$L \approx 761 \text{ kpc} . \qquad (88)$$

The masses of neutrinos are

$$m_L \approx 5.3 \text{ eV} , \qquad (89)$$

$$90 \text{ eV} < m_H < 260 \text{ eV} . \qquad (90)$$

Comparison of cases (1) and (2) show that the overall pattern of dark matter distribution is not drastically altered; neither is the range of values allowed for m_H or T_D.

We emphasize that the need for unstable neutrinos is not motivated by the desire to have $\Omega = 1$. Observations clearly show that structures like spiral galaxies possess individual dark matter halos. It will be difficult to make a stable hot DM candidate (like stable ν) to condense on to such small scales. Cold DM candidates, admittedly, provides an alternative to the unstable scenario if $\Omega < 1$. However, these cold dark matter candidates have trouble in reproducing the large-scale structures. On this account we feel that unstable neutrino scenarios are relevent even if $\Omega < 1$. The discussion in this section was motivated by the above point of view.

f) *Comment on Smaller Structures*

We noted at the end of § II*c* that we expect a relatively flat perturbation spectrum between the mass scales $M_L(\lambda_1)$ and $M_L(\lambda_0)$. In the later sections we concentrated on $M_L(\lambda_0)$ because they correspond to the largest structures. It is clear that the mass surviving out of $M_L(\lambda_0)$ is completely capable of explaining dark matter at galactic scales.

There have been claims in the literature that dwarf spheroidals may have dark matter halos around them (Aaronson 1983; Faber and Lin 1983; Lin and Faber 1983). Since cold dark matter scenarios have a flat spectrum between (10^6–10^{10} $M_\odot$), these scenarios identify the dwarf spheroidals with the lower end of the mass spectrum. In our case, the mass condensate that will form out of $M_L(\lambda_1)$ after the decay, will have a mass of (see eqs. [40], [42], and [50]),

$$M_{\rm sur}(\lambda_1) = 3.3 \times 10^{-4} M_{\rm sur}(\lambda_0) \approx 10^9 \; M_\odot$$

and size (see eqs. [37], [38]),

$$R_1 \approx \lambda_1 = 6.9 \times 10^{-2} R \approx 14 \text{ kpc} .$$

(The numerical values are for the $\Omega = 1$ described earlier) *We have shown elsewhere* (Padmanabhan and Vasanthi 1985) *that dwarf spheroidals with such an extended halo are quite consistent with observations.* Thus our parameters can also account for dark matter at smaller scales.

It is, however, doubtful whether dwarf spheroidals *really* have dark matter in them. The existing observations are still somewhat tentative.

III. DISCUSSION OF RESULTS

It should be clear from the previous analysis that the model is physically acceptable and is free of internal contradictions. Clearly, the comparative advantages and disadvantages of such a model will be largely based on aesthetic criteria and, hence, will be somewhat subjective. We shall take up the strong and weak points of the model separately.

a) *The Positive Aspects of the Model*

1. The model can explain dark matter observations at all scales from dwarf spheroidals to clusters using the same candidate particle.

2. The observed rotation curve of spirals [which indicate $M(r) \propto r$] is explained in a natural fashion.

3. The parameters Ω_R, Ω_L, and T_D can be so chosen that both the age ($t \approx 13 \times 10^9$ yr) and $\Omega = 1$ requirements can be satisfied.

4. The model leads—in a natural fashion—to the $M \propto r$ behavior at small scales and $M \propto r^3$ at large scales. The systematics of dark matter indicates such a dependence.

5. Lastly, neutrinos are known to exist, and there is an (unconfirmed) experimental claim indicating mass for the electron neutrino! *This is better than what can be said for any other dark matter candidate.*

b) *The Problems with the Model*

1. The model constraints h_0 and t_u toward the lower side of the permissible range, if $\Omega = 1$.

2. It demands the existence of unstable and stable neutrino species with preassigned range of values. For example, m_H and T_D are constrained within factors of 2 and 3, respectively. (Note that nonradiative, flavor changing decays for neutrinos are not "natural" in standard electroweak theory. They have to be invoked *a priori*.)

3. The analysis performed above is based on a series of approximations (e.g., instantaneous decay, qualitative estimate of the effects of decay).

In defense against these objections, we point out the following features: (*a*) while the model does constrain h_0 and t_u, it definitely does *not* push these parameters *outside* the permitted range. Any model with $\Omega = 1$, $t_u \geq 13 \times 10^9$ yr *will* push h_0 to the lower end. (*b*) Considering the range of observations that is explained—about four orders in mass scale—it is debatable whether the constraint on parameters is "too severe." (*c*) Particle physics models describing neutrinos have fluctuated wildly in the recent years. (A prime example would be the number of models which were invoked to "explain" the recent 17 keV observations! Glashow and Manohar 1985; Dugan *et al.* 1985.) Cosmology has done consistently better in the field of neutrino physics. It may not be proper to take any of the theoretical neutrino models too seriously at this stage. (*d*) The approximations made are no worse than what is usually accepted in preliminary modeling. We hope to do a numerical analysis in a future publication. (*e*) Lastly, we emphasize that the only other popular alternative—cold dark matter with stable or unstable components—involves similar fine tunings and constraints.

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