Astron. Astrophys. 313, 703–712 (1996)

ASTRONOMY AND ASTROPHYSICS

The quasi-steady state cosmology: analytical solutions of field equations and their relationship to observations

Rainer Sachs[1 *], **Jayant V. Narlikar**[1], **and Fred Hoyle**[2]

[1] Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Pune 411 007, India
[2] 102 Admirals Walk, West Cliff Road, West Cliff, Bournemouth, Dosset BH2 5HF, UK

Received 1 July 1995 / Accepted 2 March 1996

Abstract. We solve the cosmological equations obtained by Hoyle, Burbidge and Narlikar (1995a) from a Machian theory of gravity in the case where the universe satisfies the Weyl postulate and the cosmological principle. The equations in effect are the Einstein equations of general relativity together with a negative cosmological constant and a trace-free zero rest-mass scalar field. We find a wide range of solutions for spatial sections of zero, positive and negative curvature. The solution for the quasi-steady state cosmology used by Hoyle, Burbidge and Narlikar (1994 a,b) is shown to be an approximation to the simplest of the above solutions.

We apply the simplest solution to work out the redshift-magnitude relation and the radio source count. We show that there are marginal differences from the results obtained by Hoyle et al (1994a), although the present exact solution provides a better rationale for the parameters of the model.

Key words: cosmology: theory; observations

1. Introduction

The quasi-steady state cosmology (QSSC) has been recently developed in a series of papers by Hoyle, Burbidge and Narlikar (1993, 1994a,b, 1995a), with the intention of offering the theory as a viable alternative to the standard hot big bang cosmology. The hot big bang cosmology (HBBC) has two versions: *(i)* the orthodox version in which the universe expands from a singularity in a radiation dominated phase which changes over to a matter dominated one and *(ii)* the more recent, post-1981 version in which there is a brief interlude of inflation very early in the radiation dominated phase. The latter version, the so-called inflationary big bang cosmology (IBBC) was proposed to get rid of some of the conceptual and practical defects of the orthodox HBBC. While it has been partially successful in this enterprise, it has problems of its own. (For a review, see Narlikar and Padmanabhan, 1991) We will refer to some later in this paper while contrasting the IBBC from the QSSC.

The QSSC as developed so far can explain the temperature, spectrum and anisotropies of the cosmic microwave background, offers a different theory for the origin of light nuclei, and is consistent with the large scale observations of discrete sources. The QSSC is also able to account for the very old as well as the very young galactic systems, is consistent with the baryonic option for dark matter and relates the observed activity of galactic nuclei to mini creation events where matter and energy pour out in an explosive fashion.

Indeed the motivation for the QSSC was to replace the singular event of big bang cosmology which has no formal mathematical description within conventional physics by a rigorous formalism that describes creation of matter. It turns out that when a simple scalar field version is used for the creation phenomenon the simplest model to emerge consistent with the Weyl postulate and the cosmological principle is the steady state model. This formalism, known as the C-field formalism was inspired by M.H.L. Pryce (private communication, 1960) and was used by two of us in the sixties (Hoyle and Narlikar, 1962,1966 a,b). This was revived in the context of the QSSC by Hoyle et al (1993).

Subsequently Hoyle et al (1995a) found a more elegant way of describing the idea in a framework that is conformally invariant, incorporates Mach's principle and which in the many particle approximation leads to Einstein-like equations of general relativity including additional terms incorporating the creation of matter and a negative cosmological constant. The dynamics of the quasi-steady state cosmology was expected to follow from these field equations.

These equations are, however, more complicated to solve and to work out the observable details of the model. A similar situation exists in general relativity where the Einstein field equations cannot be solved for a realistic universe with all its observed inhomogeneity and anisotropy and for the large scale motions of its galaxies. To make any progress with model building in relativistic cosmology, one has to simplify the problem to

* *Permanent address:* Physik-Department T30, TU München, James-Frank-Strasse, D-85747 Garching, Germany

be solved, i.e., to assume a large scale regularity in the structure and dynamics of the universe.

In standard relativistic cosmology this is done by the so-called Weyl Postulate and the Cosmological Principle. The former assumes that the motions of galaxies are streamlined, being described by non-intersecting trajectories thereby allowing the definition of a cosmic comoving time t to be possible. The latter tells us that the hypersurfaces t = constant are homogeneous and isotropic. A rigorous analysis then leads us to the standard Robertson-Walker line element.

A similar approach was used for the QSSC and a rather simple scale factor was assumed to approximate to the exact solution. Thus with the usual Robertson-Walker line element given by

$$ds^2 = c^2dt^2 - S^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right\}, \qquad (1)$$

where (r, θ, ϕ) are the comoving coordinates of the fundamental Weyl observers, t is the cosmic time and $k = 0, \pm 1$ represents the three possible curvature signatures for the spaces t = constant, the scale factor for the QSSC was given as

$$S(t) = \exp\left(\frac{t}{P}\right)\left(1 + \eta\cos\frac{2\pi t}{Q}\right). \qquad (2)$$

Here P is the longer time scale for the steady state expansion and Q is the period of a typical oscillatory fluctuation from the steady state. η is a parameter whose magnitude lies between 0 and 1. As explained in the earlier papers (Hoyle et al, 1993,1994 a,b) these fluctuations arise because the creation activity follows a stop-go pattern summarized briefly below.

Imagine the universe as having a smoothed out background of the creation field whose intensity falls short of the threshold required for the creation of a typical primary particle. The threshold corresponds to the equality of the energy-momentum of the creation field quantum with the 4-momentum of the created particle. The intensity of the field may rise above the average background and above the threshold near a collapsed massive object which therefore becomes a typical site for a creation event. Since the creation field has negative stresses it blows outwards not only the matter created but the space itself.

The long term steady state with time scale P thus represents an overall expansion of space produced by several of such minicreation events acting in unison. The ups and downs occur because the creation activity around a typical object rises and falls as the gap between the threshold and the background decreases or increases. If the gap narrows, the creation activity picks up thereby increasing the expansion rate above the steady rate. This leads to a lowering of the background level of the creation field which makes the gap wider. This in turn slows down the creation of matter and the expansion of the universe. At this stage the negative cosmological constant steps in to bring about the contraction of the universe. As the universe contracts the background level of the creation field rises and creation activity picks up. This converts the expansion into contraction. The time scale represents these physical processes.

Since the heuristic expansion factor seems to describe the cosmological picture reasonably well, it is now opportune to attempt a more exact solution of the field equations. This is what we shall do in this paper. In the following sections we will outline the field equations and then find the solutions. As expected the solutions will be more complicated but it is possible to pick the one that closely resembles the approximate solution (2) assumed in earlier papers on the QSSC. We will then work out two large scale features of the exact solution and compare them with those of (2). Future work will explore other aspects of the QSSC in terms of the exact solution.

2. The field equations

To fix ideas,we summarize the work of Hoyle et al (1995a) concerning the basic theoretical framework of the QSSC as follows:

(i) The starting point of the theory takes note of two important considerations. First, in line with electrodynamics the theory is required to be conformally invariant. That is, if the spacetime metric is changed from g_{ik} to $\Omega^2 g_{ik}$ for any well-behaved nonzero function Ω of spacetime variables, the theory should not change physically. This means that even if the local standard of length is changed arbitrarily from one spacetime point to another,the basic equations of the theory do not change in form. Just as Lorentz invariance guarantees the invariance of light cones locally, conformal invariance guarantees global invariance of light cones. Thus any physical theory based on propagation of causal influences across space and time along light cones should respect conformal invariance.

The second important point is that the theory is Machian in origin; that is in the theory the inertia of a particle is expected to arise from the presence of all other particles in the universe. Thus a typical particle a has an inertial mass m_a at a world point A on its worldline through individual and additive contributions from all other particles b,c, etc., in the universe. Writing da as the element of proper time along the world line of a at A, the action describing the theory is given by

$$\mathscr{A} = \sum_a \int m_a(A)da. \qquad (3)$$

Note that this is the entire action for the theory ! There is no other gravitational term such as the Hilbert action term of general relativity. The difference from the Newtonian concept of inertia is that the mass is dependent on spacetime position of the particle and its source lies in other particles.

Conformal invariance therefore requires that under the above conformal transformation the mass function should be divided by the function Ω. Such a variation makes even the Dirac equation for a massive spinorial particle conformally invariant. If we assume that the Mach effect on each particle is communicated through a scalar wave equation with sources in other particles, then the form of the wave equation is well determined as we will soon see.

(ii) In the QSSC, the typical particle to be created is the so called Planck particle which is short-lived and decays into more stable particles like the baryons. The contributions to inertia of a typical particle come of course from all particles, stable or otherwise. The details are as below.

A scalar conformally invariant field $M(X)$ describes the inertia at a typical spacetime point X with coordinates x^i where, in general, $i = 1, 2, 3$ describe the three spatial coordinates and x^0 denotes the timelike coordinate. The contribution at X is made up of two components:

$$M(X) = c(X) + m(X) \tag{4}$$

of which $c(X)$ is that due to short-lived primary particles, the 'Planck particles' with mass

$$m_0 = (3\hbar c/4\pi G)^{1/2}. \tag{5}$$

Thus if the Planck particles are labelled by a, b, c, etc. then we write $c(X)$ as the sum of contributions of all such particles:

$$c(X) = \sum_a c^{(a)}(X). \tag{6}$$

These particles are, however, unstable and decay in a time scale τ of the order of 10^{-44}s, a typical particle a producing $n \cong 6 \times 10^{19}$ baryons (and radiation) as decay products. Labelling these secondary baryonic products of a as $a_1, a_2, a_3, \ldots, a_r, \ldots$, write their contributions to inertia at X as

$$m^{(a)}(X) = \sum_{r=1}^{n} m^{(a_r)}(X). \tag{7}$$

$m(X)$ is the sum of all such contributions $m^{(a)}(X)$. The wave equations describing these contributions are the most general conformally invariant ones:

$$\Box\, c + \frac{1}{6} Rc + \Lambda c^3 = \sum_a \int_{A_0}^{A_0+\delta A_0} \frac{\delta_4(X, A)}{\sqrt{-g(A)}} da \tag{8}$$

and,

$$\Box\, m + \frac{1}{6} Rm + \Lambda m^3 = \sum_a \sum_{r=1}^{n} \int_{A_0+\delta A_0} \frac{\delta_4(X, A_r)}{\sqrt{-g(A_r)}} da_r. \tag{9}$$

Here we assume that the Planckian source is in existence for a brief time span $\tau \sim 10^{-44}$s from world point A_0 to $A_0 + \delta A_0$. The resulting decay products start at the world point $A_0 + \delta A_0$ and then continue onwards, but each one contributes to inertia a fraction of $1/n$ compared to the contribution of the parent Planck particle. Because of the very short duration of the worldline of the parent particle, however, the total primary contribution $c(X)$ is small compared to the total secondary contribution $m(X)$. The constant Λ is the square of the reciprocal of the number of primary Planck particles from which the secondaries present in the observable universe were derived.

(iii) The field equations being conformally invariant one can simplify them by the choice of a specific conformal frame in which the part $m(X)$ is made constant. The contribution $c(X)$ is negligible in view of the reason given in *(ii)*, but the derivatives of the function are not negligible and they survive in the final field equations:

$$R_{ik} - \frac{1}{2} g_{ik} R + \lambda g_{ik} = -8\pi G \left[T_{ik} - \frac{2}{3}\left(c_i c_k - \frac{1}{4} g_{ik} c^l c_l \right)\right]. \tag{10}$$

Notice that we have set the speed of light $c = 1$ and $\hbar = 1$ in the above equations. Notice also that these equations are supplemented by the creation equation (8) which operates wherever the Planck particles are created.

The constant value of the mass field at X due to the secondaries is set equal to m_0 from (5) above. The constant of gravitation G as interpreted from (10) is positive and its value is related to the mass m_0 by the relation

$$G = \frac{3}{4\pi m_0^2}. \tag{11}$$

Thus the proper way of interpreting the relation is the Machian one which tells us that the constant of gravitation is related to the basic property of inertia. Likewise, the cosmological constant λ arises from the cubic terms of all the secondary wave equations put together,

$$\lambda = -3 \Lambda m_0^2. \tag{12}$$

Notice the negative sign of the cosmological constant whose magnitude is of the order $\sim 10^{-56}$ cm^{-2} in conventional units.

(iv) The energy momentum tensor of all matter particles (the stable secondaries) is given by

$$T^{ik}(X) = \sum_a \sum_{r=1}^{n} \frac{m_0}{n} \int \frac{\delta_4(X, A_r)}{\sqrt{-g(A_r)}} \frac{da_r^i}{da_r} \frac{da_r^k}{da_r} da_r \tag{13}$$

with each secondary having a mass m_0/n, i.e., of the order of the mass of a typical baryon.

The classical condition for the creation of a primary particle at a typical world point X is

$$c(X) = m_0. \tag{14}$$

The sign of the energy tensor of the c-field in (9) is negative and hence it implies that the c-field has negative energy and stresses. If we restore the physical constants c and $\hbar$ to their typical values in conventional units then the field equations become

$$R_{ik} - \frac{1}{2} g_{ik} R + \lambda g_{ik} = -\frac{8\pi G}{c^4} \left[T_{ik} - f\left\{ c_i c_k - \frac{1}{4} g_{ik} c^l c_l \right\}\right], \tag{15}$$

6ith $f = 2\hbar c/3$. We will refer to f as the coupling constant of the c-field to gravity in conventional units. Notice that both the sign and magnitude of the coupling constant of the c-field to gravity are completely determined by the basic theory. Just as the sign of $G\,(> 0)$ is important in relating the theory to the ordinary properties of gravity, so is the sign of f (>0) crucial in relating the theory to the expanding universe. It must be pointed out that the c-field derivatives that appear in Eq. (15) are those that survive in a spacetime averaging of large $c_i^{(a)}$ terms from different directions and over time scales $\gg \tau$.

It is of course the sharpness of the time derivatives of $c(X)$ at the ray conoid of each Planck particle that produces this effect. But their full sharpness is not maintained when a Planck particle decays into $\sim 6\times 10^{18}$ secondaries, whose ray conoids naturally separate from one another. Then because terms involving derivatives appear squared, like $\dot{c}^2$, one gets $\sum_r \dot{c}^{(a_r)\,2}$ instead of $(\sum_r \dot{c}^{(a_r)})^2$ in the gravitational equations, the terms involving cross-products averaging out to zero. The lack of coherence causes the influence after decay to fall by $\sim (6\times 10^{18})^{-1}$. This is the reason for the limits on the integral (8).

Alternatively, we can see creation of matter as coming out of the divergence of (15):

$$T^{ik}_{;k} = f\{c^i c^k_{;k} + \frac{1}{4} c^i_{;k} c^k\}. \tag{16}$$

Creation of matter implies that the terms on both sides of this relation are nonzero. It may of course happen that both sides are zero, in which case we have the non-creative mode.

There is a simple way of understanding the significance of the c-field in the above form. If we consider the de Sitter spacetime with the event horizon size as R then in a sphere of this radius there are $\mathscr{N}_P$ Planck particles that produce $n\mathscr{N}_P$ baryons. For $\tau \sim 10^{-43}$s, $\mathscr{N}_P \sim 10^{61}$ and $R \sim 10^{28}$ cm, we get

$$\mathscr{N}_P c\tau \approx R. \tag{17}$$

In other words, if we set together end-to-end all the tiny intervals τ of duration of Planck particles in this horizon sphere we will get the cosmological length R. That is, at a typical point X the contribution to $c(X)$ is that due to only one worldline of cosmological length. The relation (17) is another way of looking at the large numbers hypothesis of Dirac (1938). The determination of the magnitude of the cosmological constant by Hoyle et al. (1995a) is another instance of relating the magnitudes of cosmological coupling constants to the number of particles in the horizon sphere.

The QSSC and the IBBC are superficially similar in that both contain scalar fields leading to negative pressure in the gravitational equations, and both satisfy a wave equation with a cubic term. However, as seen in Eqs. (8) and (9) the typical wave equation in the QSSC has sources on the right hand side whereas the wave equation of IBBC has no sources. The latter therefore has a solution which is entirely specified by the initial conditions, i. e. the initial values of the field and its derivatives.

The situation in the QSSC is fundamentally different in that the fields are related to sources. In fact the situation here is similar to the more familiar case of electrodynamics where there is a close relationship between the electric charges and the electromagnetic fields. This analogy can indeed be pushed further to appreciate that just as in electrodynamics there can be several self consistent distributions of electric charges and the electromagnetic fields, the creation mechanism can operate in several different self consistent modes giving different cosmological models for different creative modes.

The observed homogeneity and isotropy of the universe suggests that over a very long time scale the loop

cosmological expansion $\leftrightarrow$ creation sources

has settled with a long term steady state condition. This is the SS part of the QSSC. However, a steady solution, being stable, is expected to have bounded fluctuations, leading to a variety of different cosmological behaviour on the short term. As outlined in the introduction this oscillatory behaviour is an indication of the mutual feedback between the creation events and the cosmological expansion. We are thus not stuck with one fixed behaviour of the IBBC (which requires very special and finely tuned initial parameters) any more than there is just one possible arrangement of charges and currents in electrodynamics.

(v) The above theory so far is entirely classical and in a sense incomplete in telling us the circumstances in which creation takes place. Once Eq. (14) is satisfied, then at the quantum level there should be a probability of creation of a particle, just as for an atom with an electron in the excited state there is a probability of spontaneous transition downwards. The computation of this probability is a nontrivial problem that has not been solved so far because, quantizing this theory in curved spacetime is formidable especially when we note that the back reaction of the creation process on the spacetime geometry cannot be ignored. Because of its negative energy and negative stresses, the c-field causes the spacetime to expand and thereby leads to the creation of additional phase space. Quantizing the field in such a dynamic scenario is indeed complicated and beyond the currently avaiable techniques.

Under these circumstances we will work within the classical framework developed so far and use a phenomenological approach. As we shall discuss in detail in the following section we can identify two different scenarios; one with creation and the other without. In the former, there is interchange of energy and momentum between the c-field and matter whereas in the latter the two are independent of each other. Even though we do not yet understand the microscopic quantum mechanical details of the creation process, the equations given above allow us to work out the details in either of the two situations. We will next solve the field equations for homogeneous and isotropic cosmological models to illustrate these possibilities.

3. Solutions to the field equations

We write the field equations (15) for the Robertson-Walker line element (1) and for matter in the form of dust, when they reduce to essentially two independent equations:

$$2\frac{\ddot{S}}{S} + \frac{\dot{S}^2 + k}{S^2} = 3\lambda + 2\pi G f \dot{c}^2 \tag{18}$$

$$\frac{3(\dot{S}^2+k)}{S^2} = 3\lambda + 8\pi G\rho - 6\pi G f \dot{c}^2, \quad (19)$$

where we have set the speed of light $c = 1$ and the density of dust is given by ρ. From these equations we get the conservation law in the form of an identity:

$$\frac{d}{dS}\{S^3(3\lambda + 8\pi G\rho - 6\pi G f \dot{c}^2)\} = 3S^2\{3\lambda + 3\pi G f \dot{c}^2\}. \quad (20)$$

This law incorporates "creative" as well as "non-creative" modes. We will discuss both in that order.

(i) The creative mode: This has

$$T^{ik}_{;k} \neq 0 \quad (21)$$

which, in terms of our simplified model becomes

$$\frac{d}{dS}(S^3\rho) \neq 0. \quad (22)$$

For the case $k = 0$, we get a simple steady-state deSitter type solution with

$$\dot{c} = m, \qquad S = \exp(t/P) \quad (23)$$

and from (18) and (19) we get

$$\rho = f m^2, \qquad \frac{1}{P^2} = \frac{2\pi G\rho}{3} + \lambda. \quad (24)$$

Since $\lambda < 0$, we expect that

$$\lambda \approx -\frac{2\pi G\rho}{3}, \qquad \left|\frac{1}{P^2}\right| \ll \lambda, \quad (25)$$

but will defer the determination of P to after we have looked at the non-creative solutions.

For the sake of completeness we may mention the cases $k = \pm 1$, for which the scale factor is different, although the rest of the quantities remain the same. Thus we have

$$S = \frac{1}{P}\cosh\left(\frac{t}{P}\right), \quad \text{for } k = 1 \ ;$$
$$S = \frac{1}{P}\sinh\left(\frac{t}{P}\right), \quad \text{for } k = -1. \quad (26)$$

Both these are in fact variations on the deSitter metric. In both cases timelike Killing vectors exist, corresponding to the "steady state postulate". However, we shall take the $k = 0$ case further.

The rate of creation of matter is given by

$$J = \frac{3\rho}{P}. \quad (27)$$

As explained in the quasi-steady state case, this rate of creation is an overall average made of a large number of small events. Further, since the creation activity has ups and downs, we expect J to denote some sort of temporal average. This will become clearer after we consider the non-creative mode and then link it to the creative one.

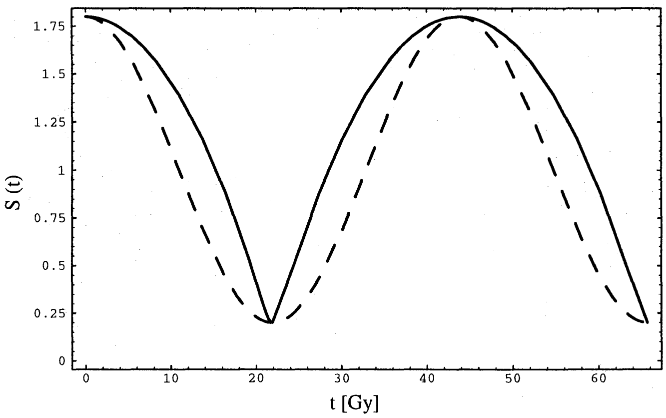

Fig. 1. The continuous line (——) shows the scale factor of the exact solution, the dashed line (– – – –) of the simpler but approximate solution.

(ii) The non-creative mode: In this case $T^{ik}_{;k} = 0$ and we get a different set of solutions.

The conservation of matter alone gives

$$\rho \propto \frac{1}{S^3}, \quad (28)$$

while for (28) and a constant λ, (20) leads to

$$\dot{c} \propto \frac{1}{S^2}. \quad (29)$$

Therefore, Eq. (19) gives

$$\frac{\dot{S}^2+k}{S^2} = \lambda + \frac{A}{S^3} - \frac{B}{S^4}, \quad (30)$$

where A and B are positive constants arising from the constants of proportionality in (28) and (29).

We will next consider the different solutions of (30) for the cases $k = 0$, $k = \pm 1$, taking the former first.

3.1. Solutions for $k = 0$

In the absence of an exact analytical solution of this differential equation the earlier work on the QSSC had taken an approximate solution as

$$S \approx 1 + \eta \cos\frac{2\pi t}{Q}. \quad (31)$$

We now find that the exact solution of (30) in the case $k = 0$, is given by

$$S = \bar{S}[1 + \eta\cos\theta(t)] \quad (32)$$

where η is a parameter and the function $\theta(t)$ is given by

$$\dot{\theta}^2 = -\lambda(1 + \eta\cos\theta)^{-2}\{6 + 4\eta\cos\theta + \eta^2(1+\cos^2\theta)\}. \quad (33)$$

$\bar{S}$ is a constant and the parameter η satisfies the condition. Thus the scale factor never becomes zero and the model oscillates between finite scale limits

$$S_{\min} \equiv \bar{S}(1-\eta) \leq S \leq \bar{S}(1+\eta) \equiv S_{\max}, \tag{34}$$

The density of matter and the c-field energy density are given by

$$\bar{\rho} = -\frac{3\lambda}{2\pi G}(1+\eta^2), \tag{35}$$

$$f\dot{c}^2 = -\frac{\lambda}{2\pi G}(1-\eta^2)(3+\eta^2), \tag{36}$$

while the period of oscillation is given by

$$Q = \frac{1}{\sqrt{-\lambda}}\int_0^{2\pi} \frac{(1+\eta\cos\theta)d\theta}{\{6+4\eta\cos\theta+\eta^2(1+\cos^2\theta)\}^{1/2}} \equiv \frac{1}{\sqrt{-\lambda}}\xi(\eta). \tag{37}$$

Fig. 1 shows the typical oscillatory cycle which is shown by a continuous curve while the same figure shows by a dotted curve the typical oscillatory cycle for the approximate solution (31), adjusted to have the same period and amplitude. Notice that the exact solution has flatter crests and narrower troughs compared to the approximate solution which, otherwise simulates the former well.

We expect that the time scale of oscillation will be short compared to the time scale of the exponential exapansion as in the approximate solution of Hoyle, et al. This expectation is quantified by the inequality:

$$P^2 \gg (-\lambda)^{-1}\xi^2(\eta). \tag{38}$$

However, before we explore this issue further let us look at the other solutions in the non-creative mode which come from $k = \pm 1$.

3.2. *Solutions for $k = \pm 1$:*

In this case we have (30) given by

$$\dot{S}^2 = \pm 1 + \lambda S^2 + \frac{A}{S} - \frac{B}{S^2}, \tag{39}$$

where the plus sign corresponds to $k = -1$ and the minus sign to $k = +1$.

The different dynamical possibilities for the scale factor are shown in Fig. 2 by the curves for the right hand side of (39) as the parameters take different values. Since the right hand side of (39) must be non-negative these curves tell us that the scale factor never becomes zero. Likewise, because of the negative cosmological constant there is a finite upper bound on S. Hence the models all oscillate between finite radii. The generic solution is again of the kind given by (32) but with $\theta(t)$ satisfying the equation:

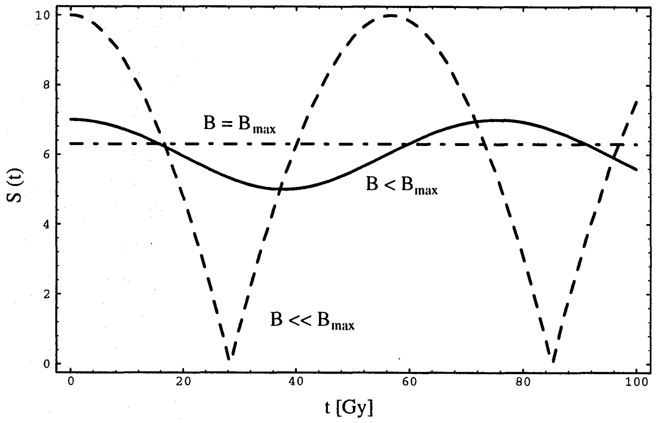

Fig. 2. Typical scale factors for different parameters. The horizontal line denotes a static universe with $B = B_{\max}$ (– · – · –). We used $B < B_{\max}$ to get small oscillations in the scale factor (——) and $B \ll B_{\max}$ for large oscillations (– – –).

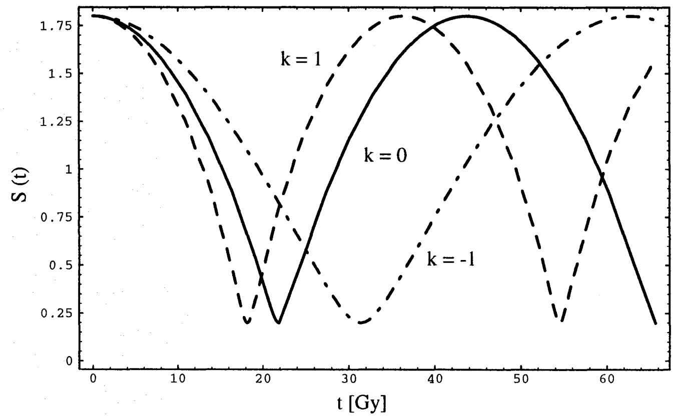

Fig. 3. The influence of k. The continuous line is the $k = 0$ case, the dashed line for $k = +1$, the dot-dashed line for $k = -1$.

$$\dot{\theta}^2 = \pm \frac{1}{\bar{S}^2(1+\eta\cos\theta)^2} - \frac{\lambda\{6+4\eta\cos\theta+\eta^2(1+\cos^2\theta)\}}{(1+\eta\cos\theta)^2}. \tag{40}$$

In Fig. 3 we show typical scale factors as functions of time for the cases $k = 0, \pm 1$. As expected, in the positive curvature case the contracting phase is shorter and faster than for the zero or negative curvature case. Also, the time period is shorter in the $k = \pm 1$ case compared to the $k = 0, -1$ cases.

It is interesting to consider the family of solutions in the phase-space diagram. We illustrate their behaviour with the constants A, B and $|\lambda|$ in the ratio

$$A \; : \; B \; : \; |\lambda| \cong 10 \; : \; 1 \; : \; 1, \tag{41}$$

and $k = 0, \pm 1$ as usual. To obtain the trajectories we made use of the Eq. (30) only. Fig. 4 shows the result for different values

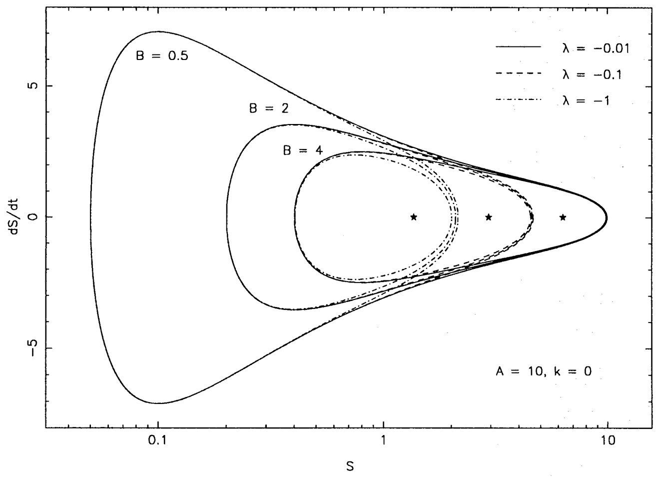

Fig. 4. Phase space diagram for the scale factor for $k = 0$ and different values of B and λ

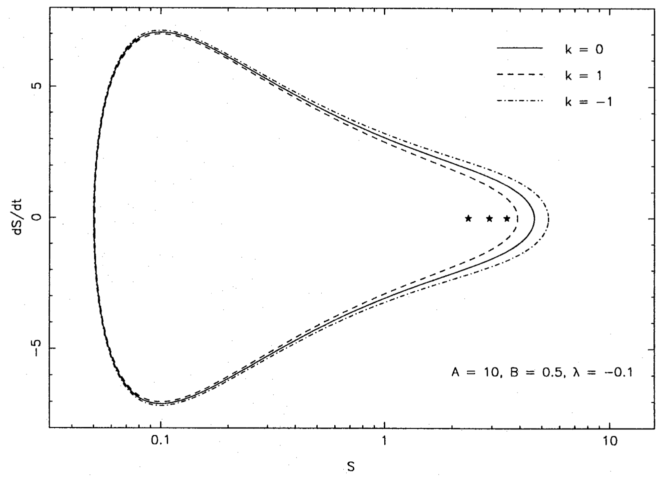

Fig. 5. The influence of k in the phase space diagram. From left to right the "$\star$"s denote the static universe for $k = +1, 0, -1$.

of B and λ when $k = 0$. One can clearly see, that there are always closed trajectories, provided B and λ are non-zero. For B becoming smaller and smaller, the trajectory approaches $S = 0$ more and more closely. Finally, for $B = 0$, there is no closed loop anymore, but the scale factor begins at zero with infinite "velocity", slows down, reaches a maximum S and decreases again. This is, of course, the case of the big bang with a cosmological constant. The influence of the latter is quite lucid of larger S. Whereas the c-field takes care of avoiding a singularity, the cosmological constant prevents the scale factor from increasing for ever. Decreasing λ leads to larger and larger S, until the latter reaches a maximum and turns back. The oscillation starts again. It can clearly be seen, that the approximate solution of Hoyle et al (1994a,b) would have given a perfect elliptical trajectory, whereas our present solutions lead to a modest deviation from this simple picture.

The next Fig. 5 shows the influence of k. As expected, the oscillation period is shorter for $k = \pm 1$, and vice versa. Clearly, there is no difference in principle between these three cases.

The "$\star$"s in the figures denote a static universe, where the parameters A, B, λ, k are fine-tuned to give $\dot{S} \equiv 0$, $\ddot{S} \equiv 0$ for Eqs. (18) and (19). One can "reach" such a solution by keeping A and C fixed and increasing B. The scalefactor S_{static} is then given by

$$S_{\text{static}} = \left(\frac{-A}{8\lambda} + \sqrt{\left(\frac{-k}{6\lambda}\right)^3 + \left(\frac{A}{8\lambda}\right)^2}\right)^{\frac{1}{3}} + \frac{k}{6\lambda}\left(\frac{-A}{8\lambda} + \sqrt{\left(\frac{-k}{6\lambda}\right)^3 + \left(\frac{A}{8\lambda}\right)^2}\right)^{-\frac{1}{3}} \quad (42)$$

A simple analysis will show that small distortions in the scale factor cause the transition from a static into an oscillating universe with a very small period and amplitude. The universe behaves like a pendulum, which gets slightly pushed from a position of rest. However, B has to satisfy a very specific relation to A and λ. For $k = 0$ we have

$$B_{\max} = \frac{3}{4}\left(\frac{-A^4}{4\lambda}\right)^{1/3}. \quad (43)$$

Small deviations from this value lead to oscillations, namely if $B < B_{\max}$, or to no solution at all for $B > B_{\max}$. Thus the static model is stable.

4. The composite solution

The quasi-steady state cosmology is based on a combination of the creative and the non-creative modes. For this the general procedure to be followed is to look for a solution of the form

$$S(t) = \exp\left(\frac{t}{P}\right)\{1 + \eta\cos\theta(t)\} \quad (44)$$

wherein $P \gg Q$. Thus over a period Q as given by (37), the universe is essentially in a non-creative mode. However, at regular instances separated by the period Q it has injection of new matter at such a rate as to preserve an average rate of creation over period P as given by J in (27). It is most likely that these epochs of creation are those of the minimum value of the scale factor during oscillation when the level of the c-field background is the highest.

Eliminating λ between Eqs. (24) and (37) we get

$$\rho = \frac{3}{2\pi G}\left\{\frac{1}{P^2} + \frac{\xi^2(\eta)}{Q^2}\right\}. \quad (45)$$

Comparing ρ above with $\bar{\rho}$ as given by (35) we get

$$\rho = \bar{\rho}\left(1 + \frac{Q^2}{\xi^2 P^2}\right)(1 + \eta^2)^{-1}. \quad (46)$$

It is easy to verify that

$$\rho_{\min} \equiv \bar{\rho}(1 + \eta)^{-3} < \rho < \bar{\rho}(1 - \eta)^{-3} \equiv \rho_{\max}, \quad (47)$$

so that it is possible to identify ρ with an intermediate density during a typical oscillation.

Suppose that matter creation takes place at the minimum value of $S = S_{\min}$, and that N particles are created per unit volume with mass m_0. Then the extra density added at this epoch is

$$\Delta\rho = m_0 N. \tag{48}$$

After one cycle the volume of the space expands by a factor $\exp(3Q/P)$ and to restore the density to its original value we should have

$$(\rho + \Delta\rho)e^{-3Q/P} = \rho, \quad \text{i.e.,} \quad \Delta\rho/\rho = 3Q/P. \tag{49}$$

The c-field strength likewise takes a jump at creation and declines over the following cycle by the factor $\exp(-4Q/P)$. Thus the requirement of "steady state" from cycle to cycle tells us that the change in the strength of $\dot{c}^2$ must be

$$\Delta\dot{c}^2 = \frac{4Q}{P}\dot{c}^2. \tag{50}$$

The above result is seen to be consistent with (49) when we take note of the conservation law (20). A little manipulation of this equation gives us

$$\frac{3}{4}\frac{1}{S^4}\frac{d}{dS}(f\dot{c}^2 S^4) = \frac{1}{S^3}\frac{d}{dS}(\rho S^3). \tag{51}$$

However, the right hand side is the rate of creation of matter per unit volume. Since from (49) and (50) we have

$$\frac{\Delta\dot{c}^2}{\dot{c}^2} = \frac{4}{3}\frac{\Delta\rho}{\rho}, \tag{52}$$

and from (23) and (24) we have $\rho = f\dot{c}^2$, we see that (52) is deducible from (49) and (50).

To summarize, we find that the composite solution properly reflects the quasi-steady state character of the cosmology in that while each cycle of duration Q is exactly a repeat of the preceding one, over a long time scale the universe expands with the de Sitter expansion factor $\exp(t/P)$. The two time scales P and Q of the model thus turn out to be related to the coupling constants and the parameters λ, f, G, η of the field equations. Further progress in the theoretical problem can be made after we understand the quantum theory of creation by the c-field.

These solutions contain sufficient number of arbitrary constants to assure us that they are generic, once we make the simplification that the universe obeys the Weyl postulate and the cosmological principle. The composite solution can be seen as an illustration of how a non-creative mode can be joined with the creative mode and more possibilities may exist of combining the two within the given framework. We have, however, followed the simplicity argument (also used in the standard big bang cosmology) to limit our present choice to the composite solution described here. This solution is the closest to the approximate solution used to describe the QSSC by Hoyle et al (1994 a).

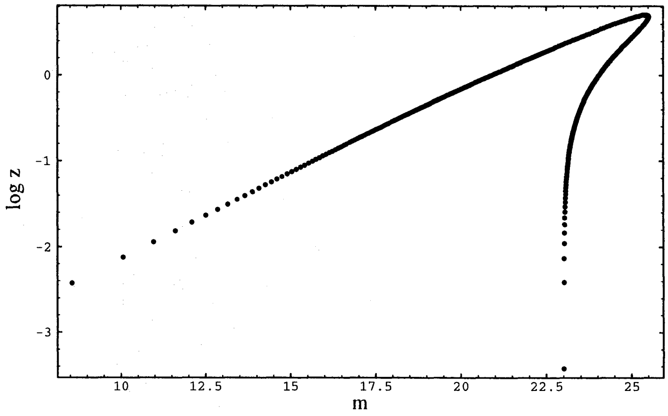

Fig. 6. Magnitude redshift relation for $M = -22.44$

5. Relation to observations

We now relate the parameters P and Q to the observable features of the QSSC. Hoyle et al (1994 a,b) had taken the following set of values for the approximate solution (32):

$$P = 20Q, \quad Q = 4 \times 10^{10}\ \text{yrs.},$$
$$\eta = 0.75, \quad t_0 \equiv \text{present epoch} = 0.85Q \tag{53}$$

Since they were linked to the approximate solution we reconsider them in the light of the present solution:

$$P = 20Q, \quad Q = 4.38 \times 10^{10}\ \text{yrs.},$$
$$\lambda = -0.29 \times 10^{-56}\ \text{cm}^{-2}, \quad \eta = 0.8, \quad t_0 = 0.7Q \tag{54}$$

For these parameters we have the well known cosmological parameters as:

$$\text{Hubble parameter } h_0 = 0.75,$$
$$\text{Deceleration parameter } q_0 = 0.99. \tag{55}$$

Note that we have taken the Hubble constant to be slightly higher than the value chosen by Hoyle et al (op. cit.) at $h_0 \sim 0.65$. These parameters are not uniquely chosen but their values may be considered as indicative. We now use them to work out two well known cosmological observations.

(i) The magnitude redshift relation: The standard formula is

$$m = 5\log D - 5 + M, \tag{56}$$

with D being the luminosity distance in parsecs. Here

$$D = S_0 r(1+z), \qquad S_0 = S(t_0). \tag{57}$$

As in HBN (1994a) we used a standard absolute magnitude $M = -22.44$ to plot Fig. 6. Whereas there is no difference from the well known standard big bang results for bright sources, the situation changes dramatically when one goes to fainter and

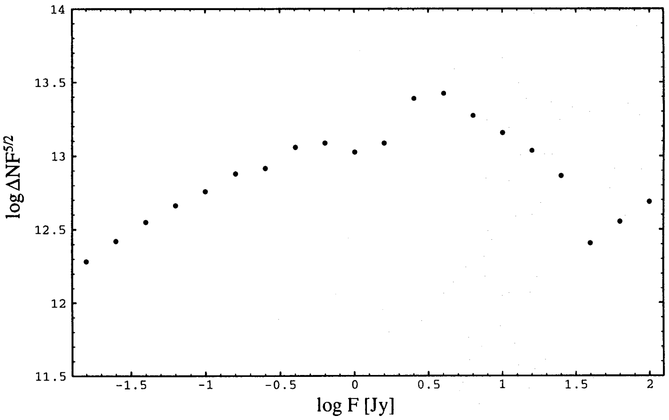

Fig. 7. Differential source counts for luminosity range 3×10^{28} — 3×10^{29}W Hz^{-1}

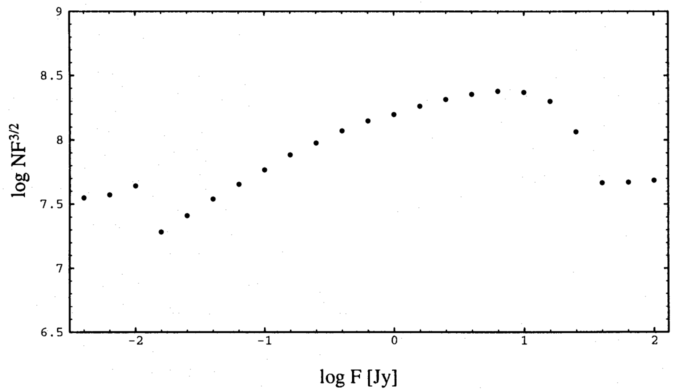

Fig. 8. Integral source counts for the mixture of sources as given in Table 1

fainter objects. At a magnitude of about 25.5 the curve turns back to smaller apparent magnitudes and smaller redshifts. In fact, at a magnitude of $\simeq 23$ objects with small redshift or even blueshift are expected. Of course, the figure does not explicitly show any blueshifted objects, as we plotted the results on the usual logarithmic scale. In practice, absorption near $S = S_{\rm min}$ will increase the apparent magnitudes by $\Delta m \sim 2$. Thus blueshifted objects may be detected if one went to $m \gtrsim 25$.

(ii) The counting of radio sources: We have repeated the calculations of Hoyle et al (1994a) for the exact composite solution. As in *(i)* above we have followed the same method of numerical computation for a radio luminosity function $\propto L^{-2.1}$ for the radio luminosity L in the range $3.10^{28} \ldots 3.10^{29}\ W\ Hz^{-1}$. Fig. 7 shows the plot of $\Delta NF^{5/2}$ against F on a logarithmic scale.

The most obvious features of this figure are:

- the decrease down to fluxes of 50 Jy,
- the sharp rise that starts at about 30 Jy,
- the flattening at around 5 Jy,
- the plateau between 0.4 and 2 Jy,
- the gradual decline for even fainter sources.

As in the calculation done with the approximate solution, we determined the ratio P/Q to differ not much from 20, and we got a good matching with the observational data by Kellermann and Wall (1987). There are only minor differences from the result of Hoyle et. al. (1994a), so that we regard the discussion given there to be reasonably accurate.

However, to demonstrate the range of possibilities in this cosmology we illustrate another way of obtaining the observed source count curve consistent with what is known of the radio sources from relatively local studies so far. Fig. 8 shows the source count curve for a different mixture of radio sources as given in Table 1. Notice that *(i)* low luminosity sources dominate in this solution and *(ii)* the class II & III sources switch off towards the low density part of each cycle. Imagine, in dimensionless time units a cycle is expressed by $0 \leq t \leq 1$, with $t = 0, 1$ describing maxima of S. The part of every cycle during which the different classes are present is then given by $t_{\rm min}$ and $t_{\rm max}$ in units of Q. This alternative was suggested by Hoyle, Burbidge and Narlikar (1995b) to highlight the fact that in contrast to the first solution in this alternative one would not expect any blueshifted sources in the optical identifications of sources $\gtrsim 1 - 2$ Jy. Clearly observational studies will play a crucial role in distinguishing between the two solutions.

Table 1. Properties of radio sources. n denotes a relative number density, T is the interval during which the sources are present

class	$\frac{L}{\text{W Hz}^{-1}}$	n	$t_{\rm min}$	$t_{\rm max}$
I	3×10^{25}	5000	0	1
II	3×10^{26}	1000	0.32	0.68
III	3×10^{28}	1	0.40	0.60

In both these examples we have not included any evolutionary effects. In general these are inferred from the fits of theory to the observations: that is, if a non-evolutionary model does not fit the data one tries epicycles of evolution such as luminosity evolution or density evolution. The fits given by the present model to the $m - z$ relation or to source counts is so good that no strong evolutionary effects are needed. To what extent they are needed for very faint sources will no doubt be determined by future observations.

6. Conclusions

We therefore find that unlike the various versions of the big bang cosmology (HBBC and IBBC), which require finely tuned initial conditions, the cosmology with a physically defined creation mechanism offers a range of solutions in which the creation process and the expansion (or contraction) of the universe are closely linked in a self consistent way. The quasi-steady state solution represents the state the universe would eventually settle down to.

The exact solutions of QSSC equation therefore tell us that the generic property of solutions with creation of matter is an exponential (de Sitter type) expansion while solutions without creation of matter indicate oscillations. The latter range of solutions include static models for all values of the curvature parameter.

The composite of the two solutions describes the quasi-steady state model in which creation of matter takes place close to the oscillatory minima of the scale factor which epochs are spaced apart at intervals Q. The creation drives the main solution on a de Sitter track with a characteristic scale $P \gg Q$.

Although the solution obtained here differs in functional form from the approximate solution used by Hoyle et. al. (1994a,b, 1995a), the observational differences between the two appear to be marginal. We will explore the applications of the exact model to other observations like microwave background, ages of galaxies, structure formation etc. in later investigations.

Acknowledgements. RS thanks the Inter-University Centre for Astronomy and Astrophysics for a fellowship. JVN thanks the Institute of Astronomy, Cambridge for hospitality when part of this work was initiated.

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