

GRAVITATIONAL COLLAPSE OF RADIATING DYON SOLUTION AND COSMIC CENSORSHIP HYPOTHESIS

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Abstract

We investigate the possibility of cosmic censorship violation in the gravitational collapse of radiating dyon solution. We show that the final outcome of the collapse depends sensitively on the electric and magnetic charge parameters. The graphs of the outer apparent horizon, inner Cauchy horizon for different values of parameters have been drawn. It is found that the electric and magnetic components push the apparent horizon towards the advanced time coordinate axis, which in turn reduces the radius of the apparent horizon in Vaidya space-time. In the present paper we extend the earlier work of A. Chamorro and K. S. Virbhadra.

Keywords: Cosmic censorship; naked singularity; gravitational collapse; dyon.

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1 Introduction

The final outcome of complete gravitational collapse in Einstein's theory is an open problem. Study of exact solutions of Einstein equations have shown that a large

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number of them contain singularities. In this respect one would like to understand the meaning of singularity. Singularity is a region where some invariants, like Kretschman scalar, scalar curvature diverge. These singularities are mathematical in the sense that they are present in the solution of the corresponding field equations. Over the last 35 years, there has been considerable interest in the formation of naked singularities and cosmic censorship hypothesis(CCH).¹ Roughly speaking naked singularities are singularities that may be seen by physically allowed observers. CCH states that for physically reasonable initial data, the gravitational collapse of the spacetime cannot yield a naked singularity. That is, if a singularity forms, it must be covered by an event horizon of the gravity. Now a days this hypothesis has become one of the most challenging open problems in classical general relativity. A rigorous formulation and proof for CCH is not available so far, hence examples showing the occurrence of naked singularities remain important to arrive at a provable formulation for the hypothesis. Important cases of naked singularities analyzed so far include dust collapse,²⁻⁷ radiation collapse,⁸⁻¹⁵ collapse of perfect fluid,¹⁶⁻¹⁹ and strange quark matter.²⁰⁻²¹ It has been shown in Refs. 2-5 that naked singularities can be produced with pressureless matter and CCH is thus easily violated. Considering the radiating solution which completely radiate away all of the mass of a star, Lake and Hellaby have shown that a class of radiating fluid sphere violets the CCH.²²

A. Chamorro and K. S. Virbhadra have obtained an exact solution of the Einstein-Maxwell equations which is a magnetic charge generalization to the Bonnor-Vaidya solution and describes the gravitational and electromagnetic fields of a non-radiating massive radiating dyon.²³ The paper is based on composite charges i.e. an electric charge and a magnetic charge bound together by their gravitational interaction. Hence it would be interesting to study the nature of the singularities formed in the gravitational collapse of such composite spacetime. One could even imagine a scenario in which the electric and magnetic charges are created independently. In the present work we show that the presence of magnetic components can in principle change the boundary of the trapped region.

2 Radiating Dyon Solution

The metric, which describes the gravitational field of non-rotating massive radiating dyon as found by Chamorro and Virbhadra²³ is

$$ds^2 = - \left(1 - \frac{2m(u, r)}{r} \right) du^2 + 2dudr + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where

$$m(u, r) = f(u) - \frac{q_e^2(u) + q_m^2(u)}{2r}. \quad (2)$$

Here $f(u)$ is the standard Vaidya mass and $q_e(u)$, $q_m(u)$ are electric and magnetic charge parameters respectively. These parameters depend on the Eddington advanced time coordinate u .

The model considered in this paper is obtained from an energy-momentum tensor of the form

$$G_i^k = R_i^k - \frac{1}{2}Rg_i^k = 8\pi (E_i^k + N_i^k), \quad (3)$$

where

$$\{x^\mu\} = \{u, r, \theta, \phi\}, (\mu = 0, 1, 2, 3).$$

E_i^k is related to the electromagnetic tensor F_{ki} in the familiar way

$$E_i^k = \frac{1}{4\pi} \left[-F_{im}F^{km} + \frac{1}{4}g_i^k F_{mn}F^{mn} \right]. \quad (4)$$

$$N_i^k = V_i V^k, \quad (5)$$

is the energy-momentum tensor of the null fluid. V^k is the null fluid current vector satisfying $g_{ik}V^iV^k = 0$.

Electric current vector $J_{(e)}^i$ and magnetic current vector $J_{(m)}^i$ are given by

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^k} (\sqrt{-g} F^{ik}) = 4\pi J_{(e)}^i, \quad (6)$$

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^k} (\sqrt{-g} {}^*F^{ik}) = 4\pi J_{(m)}^i, \quad (7)$$

where ${}^*F^{ik}$ is the dual of the electromagnetic field tensor F^{ik} and is given by

$${}^*F^{ik} = \frac{1}{2\sqrt{-g}} \epsilon^{iklm} F_{lm}. \quad (8)$$

ϵ^{iklm} is the Levi-Civita tensor density.

The non-vanishing components of the Einstein tensor for the above metric are given by

$$G_0^0 = G_1^1 = -G_2^2 = -G_3^3 = \frac{q_e^2 + q_m^2}{r^4}, \quad (9)$$

$$G_0^1 = K^2, \quad (10)$$

where

$$K^2 = \frac{2(q_e \dot{q}_e + q_m \dot{q}_m - \dot{f}r)}{r^3}. \quad (11)$$

Here $(\dot{})$ denotes the derivative with respect to the retarded coordinate u .

Energy-momentum tensor of the electromagnetic field and null fluids are given by

$$E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = \frac{q_e^2 + q_m^2}{8\pi r^4}, \quad (12)$$

$$N_0^1 = \frac{K^2}{8\pi}. \quad (13)$$

3 Nature of the Singularity

The physical situation is for $u < 0$. The spacetime becomes flat with $f(u) = 0$, $q_e(u) = 0$, $q_m(u) = 0$. At $u = T$, say, the radiation is turned off. For $u > T$, $\dot{f}(u) = \dot{q}_e(u) = \dot{q}_m(u) = 0$ i.e. $f(u)$, $q_e^2(u)$ and $q_m^2(u)$ are positive definite. Thus the metric for $u = 0$ to $u = T$ is radiating dyon solution, and for $u > T$ it becomes a static dyon solution.²⁴

To investigate the structure of the collapse, we need to consider the radial null geodesics defined by $ds^2 = 0$, ($K^\theta = K^\phi = 0$). The equation for outgoing radial null geodesics for metric (1) is given by

$$\frac{dr}{du} = \frac{1}{2} \left(1 - \frac{2f(u)}{r} + \frac{q_e^2(u) + q_m^2(u)}{r^2} \right) \quad (14)$$

In general, Eq. (14) does not yield an analytic solution. However, if $f(u) \propto u$, $q_e^2(u) \propto u^2$ and $q_m^2(u) \propto u^2$, Eq. (14) becomes homogeneous and can be solved in terms of elementary functions.¹¹

In particular, we take

$$f(u) = \begin{cases} 0, & u \leq 0 \\ \frac{\lambda u}{2}, & 0 < u \leq T \\ M_0(\text{constant}), & u > T \end{cases} \quad (15)$$

and

$$q_e^2(u) = \begin{cases} 0, & u \leq 0 \\ \mu^2 u^2, & 0 < u \leq T \\ \mu^2 T^2(\text{constant}), & u > T, \end{cases} \quad (16)$$

$$q_m^2(u) = \begin{cases} 0, & u \leq 0 \\ \delta^2 u^2, & 0 < u \leq T \\ \delta^2 T^2(\text{constant}), & u > T, \end{cases} \quad (17)$$

where λ , μ^2 and δ^2 are some positive constants. Inserting the expressions for $f(u)$, $q_e^2(u)$ and $q_m^2(u)$ into Eq. (2) we obtain the mass function for radiating dyon solution as

$$m(u, r) = \frac{\lambda u}{2} - \left(\frac{\mu^2 u^2 + \delta^2 u^2}{2r} \right) \quad (18)$$

It follows that with the choice of above mass function, the metric (1) becomes self-similar¹ admitting a homothetic killing vector ξ^a given by

$$\xi^a = u \frac{\partial}{\partial u} + r \frac{\partial}{\partial r}, \quad (19)$$

and satisfies

$$L_\xi g_{ab} = \xi_{a;b} + \xi_{b;a} = 2g_{ab}, \quad (20)$$

where L denotes the Lie derivative.

Defining $K^a = dx^a/dk$ as a tangent to radial null geodesics, where k is an affine parameter, it follows that $\xi^a K_a$ is constant along radial null geodesics.

Thus

$$\xi^a K_a = u K_u + r K_r = C, \quad (21)$$

where C is a constant. Radial null geodesic equations of metric (1), on using the null condition $K^a K_a = 0$, takes the simple form

$$\frac{dK^u}{dk} - \left(\frac{m'}{r} - \frac{m}{r^2} \right) (K^u)^2 = 0, \quad (22)$$

$$\frac{dK^r}{dk} + \left(\frac{\dot{m}}{r} - \frac{m'}{r} + \frac{m}{r^2} + \frac{2mm'}{r^2} - \frac{2m^2}{r^3} \right) (K^u)^2 + 2 \left(\frac{m'}{r} - \frac{m}{r^2} \right) K^u K^r = 0. \quad (23)$$

Let

$$K^u = \frac{du}{dk} = \frac{P(u, r)}{r}. \quad (24)$$

Then from the null condition $K^a K_a = 0$ we obtain

$$K^r = \left(1 - \frac{2m}{r} \right) \frac{P}{2r}, \quad (25)$$

where P satisfies the differential equation

$$\frac{dP}{dk} - \frac{P^2}{2r^2} \left(1 - \frac{4m}{r} + 2m' \right) = 0. \quad (26)$$

Eq. (21), because of the Eqs. (18), (24) and (25) yields

$$P = \frac{2C}{2 - X + \lambda X^2 - (\mu^2 + \delta^2) X^3}, \quad (27)$$

where X is a self-similarity variable defined by $X = u/r$. The singularity occurring at $r = 0$ is naked if the outgoing radial null geodesic equation has atleast one real positive root.²⁵ In the case of pure Vaidya spacetime it has been shown that for a mass function $m(u) = \lambda u/2$, the central singularity is naked for $\lambda \leq 1/8$, and the collapse ends into a black hole if $\lambda > 1/8$.²⁶ Hence it would be interesting to investigate whether the gravitational collapse of Vaidya spacetime could yield a

¹A spherically symmetric spacetime is self-similar if $g_{tt}(ct, cr) = g_{tt}(t, r)$ and $g_{rr}(ct, cr) = g_{rr}(t, r)$ for every $c > 0$.

naked singularity under the influence of the composite field produced by electric and magnetic charges.

With the help of Eq. (18), the equations of the outgoing radial null geodesics for the metric (1) are given by

$$\frac{du}{dr} = \frac{2}{1 - \frac{\lambda u}{r} + \frac{(\mu^2 + \delta^2)u^2}{r^2}}. \quad (28)$$

Let

$$X_0 = \lim_{\substack{u \rightarrow 0 \\ r \rightarrow 0}} \frac{u}{r} = \lim_{\substack{u \rightarrow 0 \\ r \rightarrow 0}} \frac{du}{dr}. \quad (29)$$

Hence Eq. (28) can be written as

$$X_0 = \lim_{\substack{u \rightarrow 0 \\ r \rightarrow 0}} \frac{du}{dr} = \frac{2}{1 - \lambda X_0 + (\mu^2 + \delta^2)X_0^2}. \quad (30)$$

i.e.

$$(\mu^2 + \delta^2)X_0^3 - \lambda X_0^2 + X_0 - 2 = 0. \quad (31)$$

The above equation governs the nature of the singularity. If this equation has at least one real and positive root, then the singularity will be naked. If the equation has no positive root, then the collapse ends into a black hole.

In particular, for $\lambda = 0.1$, $\mu^2 = 0.0001$, $\delta^2 = 0.0004$, one of the roots of Eq. (31) is $X_0 = 2.2410$; indicating that the gravitational collapse in this case ends into a naked singularity.

3.1 Strength of the naked singularity

Having seen the nakedness of the singularity, we now turn towards the strength of the singularity. The strength of the singularity is an important issue because there have been attempts to relate it to the stability.²⁷ If the naked singularity is not strong then it can not be considered as a physically reliable singularity and hence such naked singularities may not be considered as counter examples to CCH. A naked singularity is said to be strong if at least along one radial null geodesic with affine parameter k , with $k = 0$ at the singularity, one should have²⁸

$$\psi = \lim_{k \rightarrow 0} k^2 R_{ab} K^a K^b > 0, \quad (32)$$

where K^a is the tangent to the null geodesics and R_{ab} is the Ricci tensor.

In the present case we find that

$$\psi = \lim_{k \rightarrow 0} k^2 R_{ab} K^a K^b = \lim_{k \rightarrow 0} k^2 \frac{2\dot{m}}{r^2} (K^u)^2 \quad (33)$$

$$= [\lambda - 2X_0(\mu^2 + \delta^2)] \lim_{k \rightarrow 0} \left(\frac{kP}{r^2} \right)^2. \quad (34)$$

Using L-Hospital's rule, Eqs. (24)–(27) and the fact that at the central singularity $X \rightarrow X_0$, we obtain

$$\psi = \frac{4 [\lambda - 2X_0(\mu^2 + \delta^2)]}{[1 - (\mu^2 + \delta^2)X_0^2]^2}. \quad (35)$$

For our particular case (i.e. $\lambda = 0.1$, $\mu^2 = 0.0001$, $\delta^2 = 0.0004$, $X_0 = 2.7410$) we have $\lambda - 2X_0(\mu^2 + \delta^2) > 0$, Thus the naked singularity arising in the radiating dyon solution is a strong curvature singularity.

4 Analysis of the Apparent Horizons

When a large amount of mass is contained in a small region of a spacetime, a trapped surface forms around it. Therefore as the matter collapses under the influence of a gravitational force, there is a possibility that a trapped surface will form as the collapse proceeds. If this happens then on a sufficiently late time spatial surface, there will be a boundary that separates the trapped region from the normal region. This boundary is known as the apparent horizon. Using the mass function (18) the apparent horizon for the radiating dyon solution (1) are given by

$$r^2 - \lambda ur + (\mu^2 + \delta^2)u^2 = 0 \quad (36)$$

i.e.

$$r_{\pm} = \frac{\lambda u \pm \sqrt{\lambda^2 u^2 - 4(\mu^2 + \delta^2)u^2}}{2} \quad (37)$$

i.e.

$$r_{\pm} = \frac{\lambda u}{2} \pm \sqrt{\left(\frac{\lambda u}{2}\right)^2 - (\mu^2 + \delta^2)u^2}, \quad (38)$$

where r_+ and r_- denote respectively the *outer apparent horizon* and the *inner Cauchy horizon*. We will assume that $\lambda^2 > 4(\mu^2 + \delta^2)$ as an initial condition. At the equality these two horizons coincide, and for $\lambda^2 < 4(\mu^2 + \delta^2)$ they are absent and the singularity is visible to an external observer. In other words, if one chooses the electric and magnetic charge parameters (μ and δ) in such a way that the total charge, $(\mu^2 + \delta^2)u^2$ exceeds the square of pure Vaidya mass, $(\frac{\lambda u}{2})^2$ then the CCH would be violated (because in this case there will be no horizons). Thus the final outcome of the collapse depends sensitively on the electric and magnetic charge parameters.

For the particular case $\lambda = 0.1$, $\mu^2 = 0.0001$, $\delta^2 = 0.0004$ the equations for the outer apparent horizon and inner Cauchy horizon for the radiating dyon solution are given by $r_+ = (1/10.5573)u$ and $r_- = (1/189.4427)u$ respectively. If we remove the electric and magnetic fields (i.e. $\mu = \delta = 0$) then the solution reduces to the pure Vaidya solution. In this case the equations for the apparent horizon is given by $r = (1/10)u$. Figures 1. (a), (b), (c) and (d) show the the apparent horizons for radiating dyon and Vaidya solution for different values of λ, μ^2 and δ^2 . From the nature of the graphs it can be seen that the presence of electric and magnetic

components can in principle change the boundary of the trapped region. In fact, the electric and magnetic charges push the apparent horizon in Vaidya spacetime towards the u -axis which in turn reduces the radius of the apparent horizon.

It has been observed that the radius of the apparent horizon decreases as we increase the total charge.

5 Discussion

Perhaps the most important open problem of the classical general relativity is to prove (or disprove) the CCH. In the absence of the proof, finding an acceptable counter example is very important, as it would resolve the issue, one way or the other. Here we have presented a scenario for the gravitational collapse of radiating dyon solution.

Depending on the choice of the parameters, either a blackhole or a naked singularity is formed. The values of the parameters in the solution (18) determine which of these possibilities occurs. Here we would increase the black hole charge, $(\mu^2 + \delta^2)u^2$ faster than its mass by sending a lightlike flux of electric and magnetic charged scalar matters to this black hole from the exterior. As a result if the blackhole would become to have charge larger than square of its mass i.e. if

$$(\mu^2 + \delta^2)u^2 > \left(\frac{\lambda u}{2}\right)^2,$$

then CCH would be violated, since in this case there is no more horizon, so that a singularity is visible to an external observer. Thus one can argue that composite charged field (electric and magnetic charges taken together) effect to gravity cannot prevent a naked singularity from forming completely, so that CCH actually violates.

Further, using the Clarke and Krolak Criterion²⁸ the strength of singularities has been analyzed and shown that these naked singularities are gravitationally strong.

The graphs of the trapped surfaces in radiating dyon and Vaidya solutions have also been analyzed. These graphs show that, the presence of electric and magnetic charges modify the boundary of the trapped surfaces. In fact, they push trapped surface towards the advanced time coordinate axis (i.e. u -axis).

In the limits $\mu \rightarrow 0, \delta \rightarrow 0$ our results reduce to those obtained previously.¹⁰

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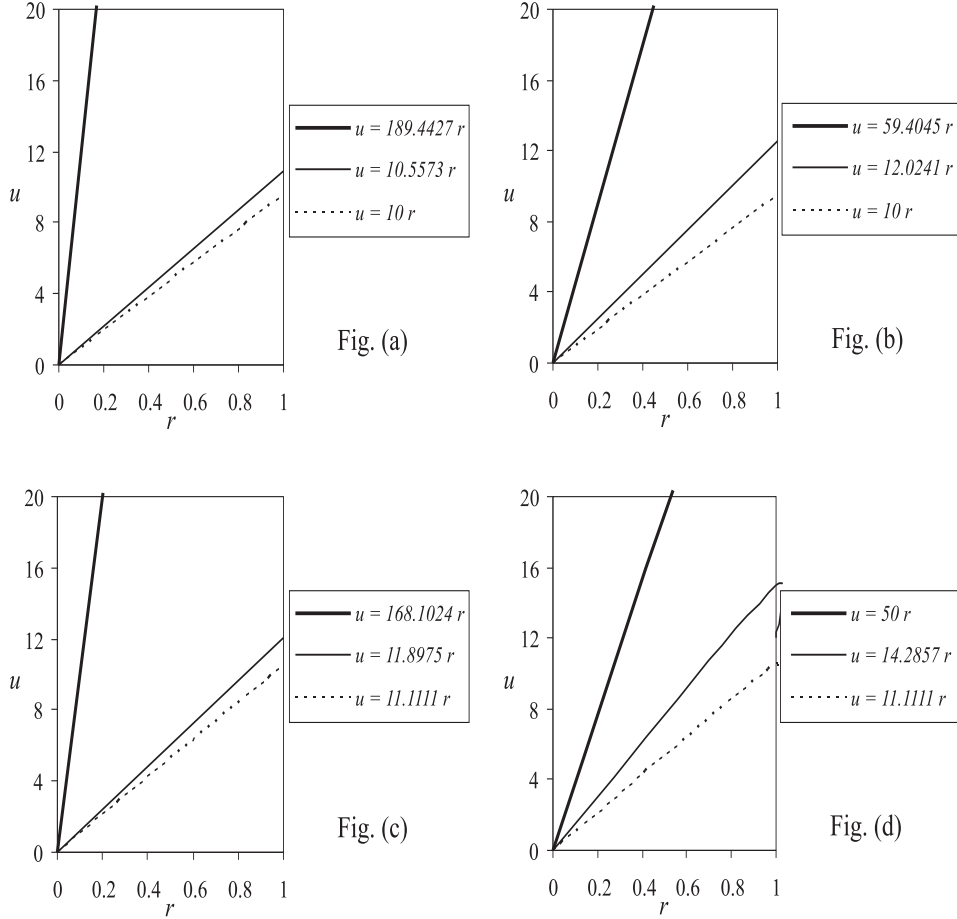


Figure 1: Figs. (a), (b), (c) and (d) show the inner Cauchy horizon, the outer apparent horizon for radiating dyon solution, and the apparent horizon for Vaidya solution respectively by a thick line, a thin line, and a dotted line for different values of λ , μ and δ :

For Fig. (a) $\lambda = 0.1$, $\mu^2 = 0.0001$, $\delta^2 = 0.0004$. For Fig. (b) $\lambda = 0.1$, $\mu^2 = 0.001$, $\delta^2 = 0.0004$. For Fig. (c) $\lambda = 0.09$, $\mu^2 = 0.0001$, $\delta^2 = 0.0004$. For Fig. (d) $\lambda = 0.09$, $\mu^2 = 0.001$, $\delta^2 = 0.0004$.

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