

Time In Quantum Gravity

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Abstract

The Wheeler-DeWitt equation in quantum gravity is timeless in character. In order to discuss quantum to classical transition of the universe, one uses a time prescription in quantum gravity to obtain a time contained description starting from Wheeler-DeWitt equation and WKB ansatz for the WD wavefunction. The approach has some drawbacks. In this work, we obtain the time-contained Schroedinger-Wheeler-DeWitt equation without using the WD equation and the WKB ansatz for the wavefunction. We further show that a Gaussian ansatz for SWD wavefunction is consistent with the Hartle-Hawking or wormhole dominance proposal boundary condition. We thus find an answer to the small scale boundary conditions.

1 Introduction

A basic feature of quantum cosmology is that the universe starts with a quantum character being dominated by quantum uncertainty and eventually it then becomes large and completely classical. In quantum cosmology the universe is described by a wavefunction Ψ which satisfies the equation

$$\hat{H}\Psi = 0 \quad (1)$$

where \hat{H} is the Hamiltonian operator. The equation (1) is known as the Wheeler-DeWitt (WD) equation. Equation (1) when compared with the Schroedinger equation in quantum mechanics

$$i\hbar \frac{\partial}{\partial t} = \hat{H}\Psi, \quad (2)$$

reveals that there is no "time" in quantum gravity and this is commonly referred to as 'the problem of time' in quantum gravity. It is now accepted as a broad consensus [1,2,3,4] that the time in quantum gravity has an intrinsic character. Recent trends suggest that one achieves an equation like (2) through a prescription of time. The problem with equation (1) is not to find solutions but to find a proper boundary condition that will not disturb the basic aspect of inflationary cosmology. At present we have three boundary condition proposals. These are : (i). Hartle-Hawking no boundary proposal [5] (ii). quantum tunneling proposal [6] and less commonly known (iii). wormhole dominance proposal [7]. The third boundary condition is more general in the sense that the proposals (i) and (ii) follow from (iii) when the respective boundary conditions are introduced in it. The first two proposals produce wavefunctions that are not normalized and have to rely on the concept of conditional probability [8] for an interpretation of the wavefunction. The proposal (iii) obtains wavefunction which is normalisable and the probabilistic interpretation remains quite sensible, and workable as in ordinary quantum mechanics. The problem with (2) is also to choose suitable initial conditions and to obtain a reasonable connection with the three boundary conditions. In most works an equation like (2) is derived from (1) and the equation is called Schroedinger-Wheeler-DeWitt (SWD) equation [1,2,3,4].

As mentioned the major problem in quantum gravity is not to find solutions of the WD and SWD equations but to obtain suitable initial conditions such that the inflationary scenario for the early universe and fruits emerging out of it are not changed. It is now accepted that the inflation provides a natural mechanism for structure formation and its origin is traced back to the quantum fluctuations in early universe. These quantum fluctuations are related to a scalar field ϕ in phase transition model and to the geometry itself in Starobinsky's spontaneous transition model [9]. The idea of quantum universe necessitates an interpretation of the wavefunction. For the orthodox "Copenhagen interpretation" one requires an external classical observer but for a universe there is no observer external to it. The success of classical Einstein equation along with the classical spacetime is a reality,

so we need along with an interpretation of the wavefunction, also a mechanism from microscopic to macroscopic reduction. More specifically, we need a mechanism from quantum to classical transitions. There have been many discussions for a unified dynamics for microscopic and macroscopic systems [10]. Now it is known that classical properties emerges through the interactions of the variables describing a quantum system such that the configuration variables are divided in some way into macroscopic variables M and microscopic variables Q and quantum interference effects are suppressed by averaging out the microscopic variations not distinguished by the associated observable. This process is known as decoherence [11,12].

In the context of quantum gravity, the Hamiltonian constraint leads to the time-less WD equation and recovery of semiclassical time is carried out using two main approaches. In one approach [13,14] a variable t (depending on the original position and momenta) whose conjugate momenta occurs linearly in the Hamiltonian H is brought to a form

$$H = H_r + p_t, \quad (3)$$

through a canonical transformation. The quantization $p_t \rightarrow -i\hbar \frac{\partial}{\partial t}$ then brings (3) to the form (2) and obtains SWD equation from the Hamiltonian constraint $H = 0$. This approach succeeds in some cases like cylindrical gravitational waves or eternal black holes but its general viability is far from clear though the standard Hilbert space of quantum theory can be employed in such an approach.

The other approach starts from the WD equation (1) and treats all variables in the same footing and tries to identify a sensible concept of time after quantization. In this approach (i) the choice for an appropriate Hilbert space structure is obscure, (ii) normalization of the wavefunction and probabilistic interpretation remain awkward in absence of time and (iii) whether the prescription of time parameter is an artifact and is related to Minskowskian time are not clear. Though the concept of 'conditional probabilities' is enforced for an interpretation of the wavefunction, the driving quantum force guaranting the validity of superposition principle in early universe and subsequently enforcing decoherence remain unclear in the picture.

The motivation of the present work is to investigate the role of time in quantum gravity especially to understand the initial conditions of both the WD and SWD equations and to obtain an inter-relation between them. In our approach we do not enforce any canonical transformation to obtain an equation like (3) and do not consider the Wheeler-DeWitt equation to obtain the SWD equation. If we look at classical Einstein equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = kT_{\mu\nu}, \quad (4)$$

we observe that 'geometry and matter' get coupled through (4). It is also a well known fact that the matter field is quantized and for that reason in equation (4) one writes $\langle T_{\mu\nu} \rangle$ on the right hand side of (4) and treats $g_{\mu\nu}$ as classical background. Keeping this spirit of (4) in mind we introduce Minskowski time t through

Schroedinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}_m \psi, \quad (5)$$

where \hat{H}_m is now the matter field Hamiltonian. This t now serves as an external label. Without having the gravitational field quantized, we formulate a time parameter $\sigma(x)$ such that (5) becomes equivalent to Einstein equation with $\sigma(x) = t = \text{const.}$ acting as a global parameter. The geometry itself acts as a generator of time and manifest only through matter field. We discuss this recovery of semiclassical time in section II. In section III we discuss the initial conditions for solution of SWD equation and its connection to the three boundary condition proposals for the timeless Wheeler-DeWitt equation mentioned earlier in the introduction. Section IV contains a discussion of 'quantum force' originated in the geometry through wormhole picture.

2 Semiclassical Time in Quantum Gravity

We consider a gravitational action with a minimally coupled scalar field ϕ in a Friedman-Robertson-Walker (FRW) background

$$I = M \int dt \left[-\frac{1}{2} a \dot{a}^2 + \frac{ka}{2} + \frac{1}{M} \left\{ \frac{1}{2} \dot{\phi}^2 - V(\phi) \right\} a^3 \right], \quad (6)$$

where $M = \frac{3\pi}{2G} = \frac{3\pi m_p^2}{2}$, m_p being the Planck mass and $k = 0, \pm 1$ for flat, closed and open models respectively. The $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ component of Einstein equation is

$$-\frac{M}{2} \left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) + \frac{1}{2} \dot{\phi}^2 + V(\phi) = 0. \quad (7)$$

Identifying

$$P_a = -Ma\dot{a}, \quad P_\phi = a^3\dot{\phi}, \quad (8)$$

(7) gives the Hamiltonian constraint

$$-\frac{1}{2M} \left(\frac{P_a^2}{a} \right) + \frac{P_\phi^2}{2a^3} - \frac{M}{2} ka + a^3 V(\phi) = 0. \quad (9)$$

The dynamical equations are

$$-\frac{\ddot{a}}{a} = \frac{\dot{a}^2}{2a^2} + \frac{k}{2a^2} + \frac{3}{M} \left\{ \frac{1}{2} \dot{\phi}^2 - V(\phi) \right\}, \quad (10)$$

$$-\ddot{\phi} = 3 \frac{\dot{a}}{a} \dot{\phi} + \frac{\partial V}{\partial \phi}. \quad (11)$$

The matter field Hamiltonian H_m for the scalar field is

$$H_m = \frac{1}{2a^3} P_\phi^2 + a^3 V(\phi) \quad (12)$$

as if $a^3(\frac{1}{2}\dot{\phi}^2 + V(\phi)) = E$ is the energy of the scalar field. We now define an action $S(a, \phi)$ such that

$$H_m = -\frac{\partial S}{\partial t} \quad (13)$$

and (12) reduces to

$$-\frac{\partial S}{\partial t} = \frac{1}{2a^3} P_\phi^2 + a^3 V(\phi). \quad (14)$$

This t is obviously a Newtonian time and acts as an external label. Seemingly it appears that (14) has no connection with the gravitational field. Using the Hamiltonian constraint (9), we write (14) as

$$-\frac{\partial S}{\partial t} = +\frac{1}{2M} \frac{P_a^2}{a} + \frac{k}{2} M a. \quad (15)$$

If we quantize in standard way with $P_i = -i\hbar \frac{\partial}{\partial q_i}$, (14) and (15) are added we get the Wheeler-DeWitt equation and the time disappears from the equation and this is the problem of time in quantum gravity. Our view is that quantization is permitted in (14) but not in (15) as if (15) represent the classical Einstein equation whereas (14) with $p_t = \frac{\partial S}{\partial t} = -i\hbar \frac{\partial}{\partial t}$ and $P_\phi = -i\hbar \frac{\partial}{\partial \phi}$ acting as quantum equation. It seems as if (14) and (15) have no dynamical content.

We define therefore a time evolution parameter σ

$$\frac{\partial}{\partial \sigma} = \frac{\partial H}{\partial P_a} \frac{\partial}{\partial a} + \frac{\partial H}{\partial P_\phi} \frac{\partial}{\partial \phi} - \frac{\partial H}{\partial a} \frac{\partial}{\partial P_a} - \frac{\partial H}{\partial \phi} \frac{\partial}{\partial P_\phi}. \quad (16)$$

Using (9) and (16) one finds

$$\begin{aligned} \frac{\partial}{\partial \sigma} &= -\frac{1}{Ma} \left(\frac{\partial S}{\partial a} \right) \frac{\partial}{\partial a} + \frac{1}{a^3} \left(\frac{\partial S}{\partial \phi} \right) \frac{\partial}{\partial \phi} \\ &+ \left[\frac{kM}{2} - \frac{(\frac{\partial S}{\partial a})^2}{2Ma^2} + 3 \frac{(\frac{\partial S}{\partial \phi})^2}{2a^4} - 3a^2 V(\phi) \right] \frac{\partial}{\partial P_a} \\ &- a^3 \left(\frac{\partial V}{\partial \phi} \right) \frac{\partial}{\partial P_\phi}. \end{aligned} \quad (17)$$

In view of (14) and (15), we demand that σ depends only upon geometry (i.e., on a). This necessitates the co-efficients of $\frac{\partial}{\partial P_a}$ and $\frac{\partial}{\partial P_\phi}$ to become zero in (17). This gives $V(\phi) = 0$, and S is a function of a only with

$$\frac{(\frac{\partial S_a}{\partial a})^2}{2Ma^2} - \frac{kM}{2} = 0. \quad (18)$$

The second term vanishes identically. We henceforth denote $S_o = S(a)$. Thus we have

$$\frac{\partial}{\partial \sigma} = -\frac{1}{Ma} \frac{\partial S_o}{\partial a} \frac{\partial}{\partial a}. \quad (19)$$

Let us suppose that S defined in (14), (15) and (17) is related to $S_o(a)$ by the relation

$$S(a, \phi) = S_o(a) + S_1(\phi) \quad (20)$$

with $S_1(\phi) \ll S_o(a)$. The reason for such an assumption will be clear as we proceed through the text. From (19) we write

$$\frac{\partial S}{\partial \sigma} = -\frac{1}{Ma} \frac{\partial S_o}{\partial a} \frac{\partial S}{\partial a}, \quad (21)$$

and using (20) and (21), we find

$$\frac{\partial S}{\partial \sigma} = -\frac{1}{Ma} \left(\frac{\partial S}{\partial a} \right)^2 + \frac{1}{Ma} \frac{\partial S_1}{\partial a} \frac{\partial S}{\partial a}. \quad (22)$$

Because of $S_1(\phi) \ll S_o(a)$, we write the second term in (22), using (18) as

$$\begin{aligned} \frac{1}{Ma} \frac{\partial S_1}{\partial a} \frac{\partial S_o}{\partial a} &= \frac{1}{Ma} \left(\frac{\partial S_o}{\partial a} \right) \left(\frac{\partial S}{\partial a} \right) - \frac{1}{Ma} \left(\frac{\partial S_o}{\partial a} \right)^2, \\ &= -\frac{\partial S}{\partial \sigma} - \frac{1}{Ma} kM^2 a^2, \end{aligned} \quad (23)$$

so that (22) reduces to

$$\frac{\partial S}{\partial \sigma} = -\frac{1}{2Ma} P_a^2 - \frac{kMa}{2}. \quad (24)$$

In arriving at (24) we have neglected $\frac{1}{Ma} \left(\frac{\partial S_1}{\partial a} \right)^2$ term in (23) and is quite obvious in large a region. Comparing (24) with (15) we find that

$$\frac{\partial S}{\partial \sigma} = \frac{\partial S}{\partial t}. \quad (25)$$

The prescription (19) and the condition (25) implants the geometry in the quantum structure provided $\sigma = t$ is identified as time. Thus we have avoided the quantization of gravitational field. This does not imply that the gravitational field is not quantized. It manifests its quantum character only through the matter field which is quantized. If we quantize the gravitational field through the replacement $P_a = -i\hbar \frac{\partial}{\partial a}$, time will disappear and this is why we get timeless character of the Wheeler-DeWitt equation. Now upon quantization with $P_i = i\hbar \frac{\partial}{\partial q_i}$ with $q_i = t, \phi$, we get from (14)

$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2a^3} \partial_\phi^2 + a^3 V(\phi) \right] \psi. \quad (26)$$

This is the desired Schroedinger-Wheeler-DeWitt equation of quantum gravity and works with the standard Hilbert space structure. The derivative $\frac{\partial}{\partial t}$ is a directional

3 Initial Conditions

The Wheeler-DeWitt equation is a quantum equation and the SWD equation is a semiclassical equation. It is therefore necessary to prescribe an initial condition for (26) so that decoherence can be effective in the framework and investigate the nature of initial condition for WD equation that results from such a choice, provided the inflation is guaranteed in the description. The usual assumption [4] is that at an early time, the modes are in their adiabatic ground state and the initial adiabatic ground state is a Gaussian state for the wavefunction since the Gaussian ansatz preserves the Gaussian form during time evolution. We will now show that this initial condition is not sufficient and requires a condition for the normalization of the wavefunctions. For this purpose we start with equation (26) with a form of the potential

$$V(\phi) = \frac{\lambda}{2}(1 + m^2\phi^2), \quad (32)$$

λ and m^2 being constants, and m^2 can also be chosen as negative. We take for ψ , the Gaussian ansatz

$$\psi = N(t) \exp\left[-\frac{1}{2}\Omega(t)\right]\phi^2 \quad (33)$$

for the ground state of the wavefunction. Inserting (33) in (26) one finds the set of equations

$$i\frac{d}{dt}\ln N = \frac{\Omega}{a^3} + \lambda a^3, \quad (34)$$

$$-i\dot{\Omega} = \frac{\omega^2 - \Omega^2}{a^3}, \quad (35)$$

with

$$\omega^2 = m^2\lambda. \quad (36)$$

It is worthwhile to point out that in (34) and (35)

$$\dot{\Omega} = \frac{\partial\Omega}{\partial t}, \quad (37)$$

$$\frac{1}{2}\frac{d}{dt}\ln N = \frac{\partial}{\partial t}\ln N \quad (38)$$

because of (25) since we would evaluate N considering multiple reflections at the turning points arising out of the WD equation such that $N = N(t, \sigma)$. Identification of 'many fingered' time σ as a global parameter leads to the choice (38). We now introduce conformal time η through the relation $dt = a d\eta$ to reduce (37) to the form

$$y'' + 2\frac{a'}{a}y' + m^2\lambda a^2 y = 0. \quad (39)$$

In obtaining (39) we have taken

$$\Omega = -ia^2\frac{y'}{y}, \quad (40)$$

derivative along each of the classical spacetime and is viewed as classical ‘trajectories’ in gravitational configuration space. In conformity with classical Einstein equation, (26) describes quantized matter field in a classical background of spacetime. In other works one starts from the WD equation with the ansatz

$$\Psi(G, \phi) \simeq C[G] e^{\frac{iS_0[G]}{\hbar}} \psi[G, \phi], \quad (27)$$

where G denotes the gravitational field on a three dimensional space, ϕ stands for non-gravitational field and C is a slowly varying prefactor. In obtaining (26) from the Wheeler-DeWitt equation, a WKB form has to be assumed for Ψ in (1) and expand $S(a, \phi)$ as

$$S = MS_0 + M^{-1}S_2 + \dots \quad (28)$$

Substituting all these in (1) and equating co-efficient of different orders of M to zero, one finds for M^2 order $\frac{\partial S_2}{\partial \phi} = 0$, M^1 order gives source free Hamilton-Jacobi equation

$$\left(\frac{a^2}{2}\right)\left(\frac{\partial S_0}{\partial a}\right)^2 + \frac{ka^4}{2} = 0$$

and the prefactor $C(G)$ is determined through a condition such that

$$f(a, \phi) = C(a) \exp\left(\frac{iS_1}{\hbar}\right) \quad (29)$$

satisfies (26). As the fundamental equation (1) is linear, it allows arbitrary superposition of states like (27) and would thus forbid the derivation of (26). Our approach remains free from all these defects. Our derivation itself suggests, (26) is valid in semiclassical region because taking

$$\psi = \exp\left[\frac{i}{\hbar}S(a, \phi)\right], \quad (30)$$

in (26), we find

$$-\frac{\partial S}{\partial t} = \frac{1}{2a^3}P_\phi^2 + a^3V(\phi) - \frac{i\hbar}{2a^3}\frac{\partial^2 S}{\partial \phi^2}, \quad (31)$$

which in the limit $\hbar \rightarrow 0$ gives back the classical equation (15). Thus we conclude that the WD equation remains valid in high curvature region (small a) and the SWD equation is effective in small curvature region (large a). The emergence of (31) also implies that somehow the superposition has been wiped out during the evolution and this mechanism is known as decoherence. It is argued that the seed of this decoherence i.e., the non-occurrence of interference terms lies in the early universe. More precisely, initial conditions at early time (i.e., near the onset of inflation) somehow regulates the behavioural pattern of the wavefunction necessitating decoherence. We now discuss this initial condition for the SWD equation (26). The present trend of investigation concentrates on the quantum to classical transition of the universe especially in the light of decoherence mechanism. We discuss it in the next section.

in which $y' = \frac{\partial y}{\partial \eta}$. As we require an exponential expansion, we solve (39) with $a(\eta) = -\frac{1}{\sqrt{\lambda}\eta}$. In this case (39) reads

$$y(\eta) = \eta^{\frac{3}{2} \pm \sqrt{\frac{9}{4} - m^2}}. \quad (41)$$

In the limit $m^2 < \frac{9}{4}$ (which is usually assumed to be satisfied in inflationary model), where the exponent in (41) can be approximated (taking negative sign) as $\frac{1}{3m^2}$ so that

$$\Omega \approx \frac{im^2 a^3 \sqrt{\lambda}}{3}. \quad (42)$$

As Ω is imaginary, the state (33) will not be normalizable. Usually, higher order modes of the scalar field are considered to obtain a real part in Ω , but here we take a different procedure. To be consistent with standard notation we take $m^2 \lambda = m_o^2$, the mass of the scalar field, and $\lambda = H^2$ so that (42) reduces to

$$\Omega = \frac{im_o^2 a^3}{3H}, \quad (43)$$

where H is now the Hubble constant. The imaginary Ω that contains now the mass describes back reaction. Substituting (43) in (34) and taking

$$\frac{d}{dt} = \sqrt{\lambda} a \frac{d}{da} \equiv H a \frac{d}{da}, \quad (44)$$

we find from (34)

$$N = N_o a^{\frac{m_o^2}{3H}} \exp \left[\frac{-ia^3 H}{3} \right], \quad (45)$$

so that

$$\psi = N_o a^{\frac{m_o^2}{3H}} \exp \left[\frac{-ia^3 H}{3} \left(1 + \frac{1}{2} m^2 \phi^2 \right) \right]. \quad (46)$$

Since $m^2 \phi^2 \ll 1$, we write (46) absorbing a factor 2 in $V(\phi)$ to make comparison with the standard result [6,7] as

$$\psi \simeq N_o a^{\frac{m_o^2}{3H}} \exp \left[\frac{-i}{3V} (a^2 V - 1)^{\frac{3}{2}} \right], \quad (47)$$

where we have taken $a^2 V \gg 1$ as expected and $\sqrt{V(\phi)} = H(1 + \frac{1}{2} m^2 \phi^2)$. From the WD equation, we know that $a^2 V \gg 1$ and $a^2 V \ll 1$ regions are respectively termed as classically allowed and classically forbidden region. The points $a = 0$ and $a = \frac{1}{\sqrt{V}}$ are the turning points. According to wormhole dominance proposal [7], the normalization constant N_o is given by multiple reflections such that

$$N_o = \frac{\exp[S(a_o, 0)]}{1 - \exp[2S(a_o, 0)]}, \quad (48)$$

where

$$S(a_2, a_1) = \left[\frac{-i(a^2V - 1)^{\frac{3}{2}}}{3V} \right]_{a_1}^{a_2}. \quad (49)$$

Evaluating (48) we find

$$N_o = \frac{\exp\left[\frac{1}{3V}\right]}{(1 - \exp\left[\frac{2}{3V}\right])}. \quad (50)$$

The wavefunction (47) now reads

$$\psi = C_1 a^{\frac{m_o^2}{3H}} \exp\left[\frac{1}{3V}(1 - i(a^2V - 1)^{\frac{3}{2}})\right], \quad (51)$$

where C_1 now refers to $(denominator)^{-1}$ in (50). Continuing in classical forbidden region, we get from (51)

$$\psi(a^2V < 1) = C_1 a^{\frac{m_o^2}{3H}} \exp\left[\frac{1}{3V}(1 - (1 - a^2V)^{\frac{3}{2}})\right]. \quad (52)$$

We see from (52) that as $a \rightarrow 0$, the wavefunction behaves as

$$\psi \sim \exp\left(\frac{a^2}{2}\right). \quad (53)$$

Thus we find that Eqns.(51)-(53) all represent Hartle-Hawking wavefunction. Thus we find that Gaussian ansatz at $a^2V \gg 1$ correctly reproduce the wavefunction corresponding to the wormhole-dominance proposal, at least the Hartle-Hawking proposal. Not only the Gaussian ansatz correctly describes the inflation, its seed lie in the wormhole-dominance proposal.

4 Discussion

With the inception of quantum cosmology and description through the Wheeler-DeWitt equation attempts have been made to interpret the wavefunction of the universe in terms of classical dynamics (i.e., Einstein equation) and probabilistic concept. We have been able to show that a Gaussian ansatz for SWD wavefunction correctly simulates the Hartle-Hawking wavefunction and is normalized according to our prescription. The prescription obtains the normalization constant through the contribution of wormholes to the wavefunction and it contributes mainly around the zero scale factor region. In otherwords, repeated reflections between the turning points and superposition of states like $\exp(iS)$ and $\exp(-iS)$ are basically the quantum character and owe its origin to the wormholes. Our work shows that the times ' t ' and σ become equal at the semiclassical region and begin to differ as we approach more and more to the classically forbidden region. We observe that the quantization becomes worthy only through the matter Lagrangian i.e., the Newtonian time

emerges through the matter field Hamiltonian in SWD equation. This is in conformity with classical Einstein equation where in the righthand side we use $\langle T_{\mu\nu} \rangle$ to obtain the classical description i.e., the geometry acts as a “guidance field” for matter. Akin to Madelung [15] and Bohm [16] in our approach the initial positions are random and the quantum force generating the randomness arises from wormhole contribution. In view of some apathy towards the wormhole philosophy, we like to mention that the quantum force generated at the initial stage (i.e., repeated reflections at the turning points and the superposition) finds an interpretation through wormhole contributions. This also guarantees the universal validity of superposition principle. In otherwords, the geometry in the early universe initiates the quantum randomness and with the beginning of nucleation and inflation, the Newtonian time emerges through the SWD equation such that $S_1 \ll S_o$ and $S_o = S_o(a)$, an aspect relevant to decoherence. The scale factor a of a Friedmann universe then becomes the relevant variable and attains the classical character through continuous measurement. In the literature a question arises that if the states $\exp(iS)$ and $\exp(-iS)$ describing an expanding and collapsing universe decohere, can one recover an approximate time-dependent Schroedinger equation from the timeless Wheeler-DeWitt equation and what are the boundary conditions at small scales [17] that lead to quantum effects in the vicinity of the turning point. In this work we answer all three questions. We show that (i) an approximate time dependent Schroedinger equations follows irrespective of the Wheeler-DeWitt equation, (ii) the adiabatic Gaussian wavefunction for the SWD equation is consistent with the Hartle-Hawking criterion plus the normalization prescription or the wormhole dominance proposal. This solves the problem of small scale boundary condition. (iii). Through the wormhole dominance proposal it has been explicitly shown that the wormholes lead to quantum effects in the vicinity of the turning point (in our case $a \simeq 0$) and classical turning point (in our case $a \simeq H^{-1}$) behaves as an starting point for the arrow of time and is manifested through the matter field as if the geometry itself looks into the evolution through the matter field. (iv). In the standard derivation of SWD equation, an expansion of the Wheeler-DeWitt wavefunction with respect to the Planck mass leads to difficulties as discussed in [18]. Our derivation is basically an expansion with respect to \hbar and is justified through equations (30) and (31). (iv). The decoherence mechanism in our approach is the same as in ref. [3] and we do not repeat it here. (v). We also applied our approach to Starobinsky type R^2 -cosmology and find that the Gaussian ansatz correctly reproduces the wavefunction corresponding to the wormhole dominance proposal.

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5 Acknowledgment

A. Shaw acknowledges the financial support from ICSC World Laboratory, LAUS-SANE during the course of the work.