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Action and Reaction in the Theories of Direct Interparticle Action*

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Newton's third law of motion is examined in the context of the theories of direct interparticle action. In such theories, interactions between particles travel backward and forward in time at speeds not exceeding the speed of light. It is shown that while in the flat spacetime the equality of action and reaction can be clearly demonstrated, the situation is considerably more complicated in the curved spacetime. The phenomenon of gravitational scattering intervenes to destroy the equality of action and reaction. Nevertheless, when gravitation is taken into account, there is no violation of the conservation law of energy and momentum. These results are discussed in the framework of general relativity for the case of the electromagnetic interaction.

1. INTRODUCTION

Work on electromagnetic theory [1-5] and gravitation [6, 7] has shown that these interactions can be described in classical physics in terms of direct interparticle action. In the case of electromagnetism, the description has been extended into the quantum domain also [8, 9] and some progress has been made in describing the weak interaction as a direct particle theory [10].

One of the intriguing aspects of the direct particle theory is the mode of transfer of energy and momentum between interacting particles. In the conventional field theory, the exchange of energy and momentum between distant particles does not take place directly but through the agency of the field. The corresponding situation in the direct particle theory may be described with the help of Fig. 1. Here a typical point A on the world line of particle a is shown to be in interaction with points B_1 and B_2 on the world line of particle b. The rays AB_1 and AB_2 are null rays on the past and the future light cones from A. The loss or gain of energy and momentum of a at A resulting from its interaction with b can be traced to the sources at B_1 and B_2. If Newton's third law of motion is applied to this system it should be possible to argue that the action of B_1 on A is equal and opposite to

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that A on B_1 (and similarly for A and B_2). Does this property actually hold in direct particle theories? We will try to find an answer to this question in curved as well as flat spacetime.

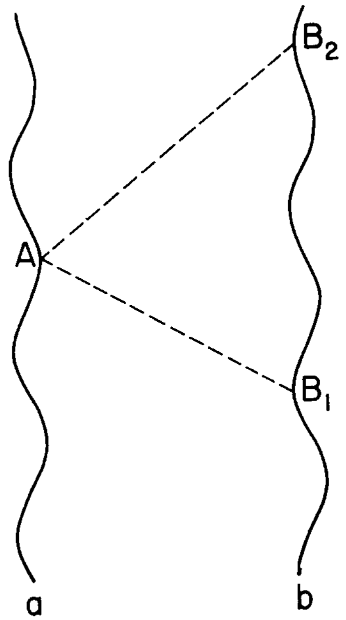

FIG. 1. The point A on the world line of a is in interaction with two points B_1 and B_2 on the world line of b. Both AB_1 and AB_2 are null rays, one on the past light cone and the other on the future light cone of A.

Associated with this question of action and reaction is the question of localization of energy and momentum of the interaction. In the above example, the energy exchanged between A and B_1 travels through spacetime between the instants corresponding to B_1 and A. Does it have a real existence during this interval? As far as dynamics of particles is concerned, this question need not be answered, because there we are concerned only at the ends B_1 and A. The question does, however, matter in general relativity, where energy affects the geometry of spacetime. Recent work has clearified the role of energy in direct particle theories (cf. [5]). We will relate the gravitational influence of the energy of interaction to the problem of action and reaction between a pair of particles.

Although it is possible to write down direct particle interactions between particles in an abstract fashion [11] it is instructive to consider the above question in the specific case of electromagnetic theory. It is then possible to make comparisons with the more familiar Maxwell field theory.

2. Flat Spacetime

In Minkowski space, the direct particle theory of electromagnetism is best described by the Fokker action principle. The action is given by

$$S = -\sum_a \int m_a \, da - \tfrac{1}{2} \sum_{a \neq b} \sum e_a e_b \iint \delta(s_{AB}^2) \, \eta_{ik} \, da^i \, db^k. \tag{1}$$

Here a, b, c,... label the particels, e_a and m_a being the electric charge and mass of the ath particle. The argument of the Dirac delta function in the double integral is the invariant square of the distance between typical points A (on the world line of a) and B (on the world line of b), so that

$$s_{AB}^2 = \eta_{ik}(a^i - b^i)(a^k - b^k). \tag{2}$$

In the above expression a^i and b^i are the four coordinates ($i = 1, 2, 3, 4$) of A and B, respectively, while η_{ik} is the diagonal metric $(-1, -1, -1, +1)$. The quantity da in the first term on the right-hand side of (1) measures the element of proper time at A along the world line of a. This term describes the inertial properties of the particles while the second term describes the electromagnetic interaction. Since $\delta(s_{AB}^2) = 0$ unless $s_{AB}^2 = 0$, we have the relativistically invariant statement that A and B can interact only if AB is a null ray. Further, since $a \neq b$, the self-action is specifically excluded.

Wheeler and Feynman [1], in their discussion of the Fokker action, arrived at the equality of action and reaction in the following way. Consider a two-particle system containing only the particles a and b. The change in the momentum of a over an infinitesimal proper time da is denoted by

$$dP^{(a)i}(A) = m_a \frac{d^2 a^i}{da^2} \, da, \tag{3}$$

where A is a typical point in the stretch da.

Now, the variation of S with respect to the world line of a gives the equation of motion of a. Using this, we get

$$m_a \frac{d^2 a^i}{da^2} = e_a e_b \frac{da^k}{da} \left\{ \eta^{il} \frac{\partial}{\partial a^l} \int \eta_{kn} \frac{db^n}{db} \delta(s_{AB}^2) \, db - \frac{\partial}{\partial a^k} \int \frac{db^i}{db} \delta(s_{AB}^2) \, db \right\}. \tag{4}$$

Substituting (4) into (3) and adding an extra term, which is identically zero, we get

$$\begin{aligned} dP^{(a)i}(A) = {} & e_a e_b \, da^k \left\{ \eta^{il} \frac{\partial}{\partial a^l} \int \eta_{kn} \frac{db^n}{db} \delta(s_{AB}^2) \, db - \frac{\partial}{\partial a^k} \int \frac{db^i}{db} \delta(s_{AB}^2) \, db \right\} \\ & + e_a e_b \, da^i \int_{-\infty}^{\infty} \frac{d}{db} \delta(s_{AB}^2) \, db = da \int_{-\infty}^{\infty} I^i(A, B) \, db, \end{aligned} \tag{5}$$

where

$$I^i(A, B) = 2e_a e_b \, \delta'(s_{AB}^2) \left\{ (a^i - b^i) \, \eta_{kn} \frac{da^k}{da} \frac{db^n}{db} - (a^n - b^n) \, \eta_{kn} \frac{da^k}{da} \frac{db^i}{db} \right.$$

$$\left. - (a^n - b^n) \, \eta_{kn} \frac{da^i}{da} \frac{db^k}{db} \right\}. \tag{6}$$

The purpose of the extra zero term introduced in (5) was to arrive at a manifestly antisymmetric expression for $I^i(A, B)$:

$$I^i(A, B) = -I^i(B, A). \tag{7}$$

In terms of (5), we can break down the influence of b on a into a continuous series of impulses $I^i(A, B)$ that change the momentum of a at every instant. Because of the delta function derivative in (6) the impulses at A come only from two points B_1, B_2 on the world line of b (see Fig. 1) The impulse function $I^i(A, B)$ therefore quantitatively describes the action of b on a while its antisymmetric nature as given in (7) implies that action and reaction are equal and opposite.

In the following section we generalize this analysis of Wheeler and Feynman to the Riemannian spacetime of general relativity.

3. Problems Arising in the Curved Spacetime

The Fokker action can be generalized to the Riemannian spacetime in more than one way. For example, an obvious generalization is

$$S = -\sum_a \int m_a \, da - \tfrac{1}{2} \sum_{a \neq b} \sum e_a e_b \iint \delta(s_{AB}^2) \, \bar{g}_{i_A k_B} \, da^{i_A} \, db^{k_B}. \tag{8}$$

In this case, the proper time element da is measured with respect to the spacetime metric g_{ik} (say). The quantity s_{AB}^2 is the square of the invariant distance

$$s_{AB} = \int_{\Gamma_{AB}} ds \tag{9}$$

between A and B measured along the geodesic Γ_{AB} joining them. We will assume that Γ_{AB} exists and is unique. The parallel propagator $\bar{g}_{i_A i_B}$ is required to form the invariant product of da^{i_A} at A and db^{i_B} at B. (The vector indices at different points henceforth will carry that point as a subscript). The properties of $\bar{g}_{i_A i_B}$ have been discussed by Synge [12].

This generalization, however, fails to reproduce Maxwell-type equations in curved space. On that count alone, (8) cannot be rejected, since the validity of Maxwell's equations in curved space has not been verified experimentally. It may well be that (8) gives an equally consistent description of electromagnetic phenomena within the available experimental data. However, although (8) is a simple looking expression, it leads to considerable difficulties of mathematical manipulation. This is apparent when, for example, one considers the covariant derivatives of parallel propagators. As shown by Synge [12] we have, for any vector V^{k_B} at B,

$$\bar{g}_{i_A i_B; k_B} V^{k_B} = \int_{\Gamma_{AB}} \bar{g}^{i}{}_{i_B} g^{m}{}_{i_A} V^k U^l R_{imkl}\, du. \tag{10}$$

Here u is an affine parameter along Γ_{AB}, U^l is the tangent vector to Γ_{AB} at a general point on it, and V^k is the geodesic deviation vector at u, corresponding to V^{k_B} at B. Thus, one is forced to deal with path-dependent quantities in even the simplest operations.

It is possible to bypass these difficulties by going over to another generalization of Fokker action which yields Maxwell type equations. This is formally given by

$$S = -\sum_a \int m_a\, da - \tfrac{1}{2} \sum_{a \neq b}\sum e_a e_b \iint D_{i_A i_B}\, da^{i_A}\, db^{i_B} \tag{11}$$

where $D_{i_A i_B}$ is a symmetric bivector at A and B, i.e.,

$$D_{i_A i_B} = D_{i_B i_A}\,. \tag{12}$$

$D_{i_X i_B}$ satisfies the vector wave equation

$$g^{kl} D_{i i_B; kl} + R_i{}^l D_{l i_B} = 4\pi\, \bar{g}_{i i_B} \{-\bar{g}(X, B)\}^{-1/2}\, \delta_4(X, B). \tag{13}$$

In (13), we have suppressed the subscript X for indices at X for convenience of writing. We will adopt this procedure whenever there is no ambiguity. The quantity $\bar{g}(X, B)$ is the determinant of the parallel propagators $\bar{g}_{i_X i_B}$. $\delta_4(X, B)$ is a delta-function in four dimensions.

This form has been used by Hoyle and Narlikar [4] in their discussion of direct particle electromagnetism in curved spacetime. For our discussion, we will use the same form. DeWitt and Brehme [13] have given the following expression for $D_{i_A i_B}$:

$$D_{i_A i_B} = \Delta^{1/2} \bar{g}_{i_A i_B} \delta(s_{AB}^2) + \bar{v}_{i_A i_B} \theta(s_{AB}^2), \tag{14}$$

where

$$\Delta = \det \mid \tfrac{1}{2} s^2_{AB; i_A i_B} \mid \cdot \{\bar{g}(A, B)\}^{-1}, \tag{15}$$

and $\theta(x)$ is the heaviside function: $\theta(x > 0) = 1$, $\theta(x < 0) = 0$. The function $\bar{v}_{i_A i_B}$ is responsible for the so-called "tail" of the propagation $D_{i_A i_B}$. It describes the part of the propagator that travels slower than light. The delta-function part of (14) continues to describe the disturbance that travels with the speed of light.

As in the flat space, we now write down the change in the 4-momentum of a over a small proper time interval da at a typical world point A. Using the equations of motion we get

$$dP^{i_A} = e_a e_b\, da^{k_A} \left\{ g^{i_A l_A} \frac{\partial}{\partial a^{l_A}} \int D_{k_A n_B} \frac{db^{n_B}}{db}\, db - \frac{\partial}{\partial a^{k_A}} \int D^{i_A}{}_{n_B} \frac{db^{n_B}}{db}\, db \right\}. \quad (16)$$

To arrive at a form analogous to (5) we may consider adding to the right-hand side the term

$$-e_a e_b\, da^{i_A} \int D^{k_A}{}_{n_B;k_A} \frac{db^{n_B}}{db}\, db, \quad (17)$$

provided it is identically zero. DeWitt and Brehme [13] have shown that

$$D^{k_A n_B}{}_{;k_A} = -D(A, B)^{;n_B} \quad (18)$$

where $D(X, B) = D(B, X)$ is the solution of the scalar wave equation in the coordinates of X:

$$g^{ik} D(X, B)_{;ik} = 4\pi[-\bar{g}(X, B)]^{-1/2}\, \delta_4(X, B). \quad (19)$$

Thus, (17) is zero, provided $D(A, B)$ vanishes, or has equal values at the ends of the world line of b. Assuming this to be the case, we get

$$dP^{i_A} = da \int I^{i_A}(A, B)\, db, \quad (20)$$

where

$$I^{i_A}(A, B) = e_a e_b \Big[D_{k_A n_B; l_A} g^{i_A l_A} \frac{da^{k_A}}{da} \frac{db^{n_B}}{db} - D^{i_A}{}_{i_B;k_A} \frac{da^{k_A}}{da} \frac{db^{i_B}}{db} - D^{k_A}{}_{n_B;k_A} \frac{da^{i_A}}{da} \frac{db^{n_B}}{db} \Big]. \quad (21)$$

Although (21) is the required generalization of (6) to curved spacetime, it does not share with it the simplicity of interpretation. The following complications are introduced by the transition to the curved spacetime.

1. Unlike the derivative of the delta function appearing in (6), the derivatives of the D-propagator have support inside the light cone as well as on it. Thus, the point A receives impulses not just from two points B_1, B_2 on the world line of b but from all points on it lying to the future of B_2 and in the past of B_1 (see Fig. 2).

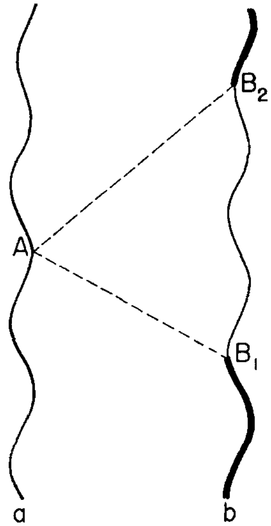

FIG. 2. The point A is in interaction with the portions of the world line of b shown by thick lines.

2. If we interchange the role of A and B in (21), we obtain an expression for $I^{i_B}(B, A)$. This is not equal and opposite of $I^{i_A}(A, B)$, as in (7). Nor can such a relation be obtained by a parallel propagation of $I^{i_A}(A, B)$ to B along Γ_{AB}, for (21) involves covariant derivatives of the parallel propagators and of the tail functions. These are, in general, very complicated and one cannot conclude that $I^{i_B}(B, A)$ cancels exactly the quantity $\bar{g}^{i_B}{}_{i_A} I^{i_A}(A, B)$.

It is possible, however, to show that the leading terms in these two expressions, which involve the derivative of the delta-function, do cancel completely. This follows from the relation

$$\delta(s_{AB}^2)_{;l_A} = 2\delta'(s_{AB}^2) \cdot s_{AB} U_{l_A} \tag{22}$$

and from the fact that

$$U_{n_B} = \bar{g}_{n_B}{}^{l_A} U_{l_A} . \tag{23}$$

Since $s_{AB} = -s_{BA}$, we get from (22) and (23)

$$\delta(s_{AB}^2)_{;l_A} \bar{g}_{n_B}{}^{l_A} = -\delta(s_{AB}^2)_{;n_B} . \tag{24}$$

Substituting (14) into (21) and using (24), the coefficient of $\delta'(s_{AB}^2)$ in $I^{i_B}(B, A)$ can be shown to be equal and opposite of the coefficient of $\delta'(s_{AB}^2)$ in $\bar{g}^{i_B}{}_{i_A} I^{i_A}(A, B)$.

Thus, the equality of action and reaction holds in the case of the leading term of (21). What interpretation can we give to the rest of the terms which do not seem to cancel in the same way? To understand this aspect we have to look to the role played by the non-Euclidean spacetime in the overall conservation of energy and momentum.

4. The Transport of Energy and Momentum

In the Maxwell theory, the transport and conservation of energy and momentum are discussed with the help of the electromagnetic energy tensor

$$T^{ik}(M) = \frac{1}{4\pi}\left\{\frac{1}{4} g^{ik}F_{mn}F^{mn} - F_{m}{}^{i}F^{mk}\right\} \tag{25}$$

where F_{ik} is the Maxwell field tensor. Equation (25) has the property that its divergence vanishes everywhere except on the world lines of electric charges. On these world lines, its divergence becomes singular in such a way that when integrated over a spacetime domain containing typical world line, it gives the Lorentz force on that charged particle.

Wheeler and Feynman [1] noted that in the direct particle theory, an analogous role is played by two energy tensors: the Frenkel tensor given by

$$T^{ik}(F) = \frac{1}{8\pi}\sum_{a\neq b}\sum\left\{\frac{1}{2} g^{ik}F^{(a)}_{mn}F^{(b)mn} - F^{(a)}{}_{m}{}^{i}F^{(b)mk} - F^{(b)}{}_{m}{}^{i}F^{(a)mk}\right\} \tag{26}$$

and the canonical tensor given by

$$T^{ik}(C) = \frac{1}{8\pi}\sum_{a\neq b}\sum\left\{\frac{1}{2} g^{ik}R^{(a)}{}_{mn}S^{(b)mn} - R^{(a)}{}_{m}{}^{i}S^{(b)mk} - S^{(b)}{}_{m}{}^{i}R^{(a)mk}\right\}. \tag{27}$$

In the above expressions, the notation is as follows. The field produced by charge a at a point X is given by

$$F^{(a)}{}_{ik} = A^{(a)}{}_{k;i} - A^{(a)}{}_{i;k}\,, \tag{28}$$

where

$$A^{(a)}{}_{i}(X) = e_a\int D_{ii_A}\,da^{i_A}. \tag{29}$$

(As in the previous section, the subscript X on the vector indices at X is suppressed.) $R^{(a)}$ and $S^{(a)}$ are respectively twice the retarded and advanced components of $F^{(a)}$, i.e.,

$$F^{(a)}{}_{mn} = \tfrac{1}{2}\{R^{(a)}{}_{mn} + S^{(a)}{}_{mn}\}. \tag{30}$$

The two tensors differ from each other by a tensor of zero divergence everywhere, including on particle world lines. Therefore, in pure electrodynamics they will be indistinguishable. However, in general relativity, they will produce different results and it becomes necessary to arrive at a unique answer. This was provided recently by Narlikar [5] and the result is in favor of the canonical tensor. It is this tensor that emerges when the Fokker action, Eq. (11), is varied with respect to geometry. Moreover, with this tensor there are no advanced Poynting fluxes in a completely absorbing universe (cf. [5]). In the following work we will use this tensor.

The divergence of $T^{ik}(C)$ can be related to the impulse function I^i of (21) by a trick used by Wheeler and Feynman [1] in their calculations in flat spacetime. We give below a discussion applicable to curved space. At a general point X, write

$$^{\dagger}A^{(a)}{}_i = e_a D_{ii_A} \frac{da^{i_A}}{da} \tag{31}$$

so that

$$A^{(a)}{}_i = \int {}^{\dagger}A^{(a)}{}_i \, da. \tag{32}$$

Similarly, other quantities are defined with a dagger superscript. Thus,

$$^{\dagger}F^{(a)}{}_{ik} = \tfrac{1}{2}\{^{\dagger}R^{(a)}{}_{ik} + {}^{\dagger}S^{(a)}{}_{ik}\}. \tag{33}$$

Using (18) we get

$$^{\dagger}A^{(a)i}{}_{;i} = -e_a D(X, A)_{;i_A} \frac{da^{i_A}}{da}. \tag{34}$$

Thus, the divergence of $^{\dagger}A^{(a)}{}_i$ is not zero, although that of $A^{(a)}{}_i$ vanishes identically. This illustrates the difference in the handling of daggered and undaggered quantities. Taking these into account, we find that the divergence of $^{\dagger}T^{ik}(C)$ is not zero but equals

$$\begin{aligned} ^{\dagger}T^{ik}(C)_{;k} = \sum_{a \neq b}\sum \frac{1}{2} e_a e_b \Big[&\{D^k{}_{n_B}{}^{;i} - D^i{}_{n_B}{}^{;k} + D_{;n_B} g^{ik}\} \\ &\times \frac{\delta_4(X, A)}{[-\bar{g}(X, A)]^{1/2}} \bar{g}_{km_A} \frac{da^{m_A}}{da} \frac{db^{n_B}}{db} \\ &+ \text{terms obtained by interchanging } a \text{ and } b \Big]. \end{aligned} \tag{35}$$

Consider (35) for a system of two particles a and b only. If we integrate (35) over a small spacetime region containing the point A, we recover I^{i_A} of (21). Similarly, if we integrate it over a small region surrounding B, we will get I^{i_B}. In either case we can interpret I^{i_A} or I^{i_B} as the exchange of energy between the particle and the direct particle field produced by the rest of the particles in the universe. Thus, although I^{iA} does not seem to cancel I^{iB} exactly, the whole scheme is consistent with the conservation of energy and momentum. It would be tempting to integrate (35) over an extended region containing both a and b. This, however, involves adding vectors at different spacetime points and is not a covariant procedure, except in flat space. Thus, we cannot give a global expression for the energy and momentum of the system in curved spacetime although such an expression in a flat spacetime was given by Wheeler and Feynman [1].

The difficulty of ascribing a global 4-momentum to the system in curved spacetime is not peculiar to action at a distance. A similar difficulty exists in Maxwell's field theory, or for field theories in general. Thus in flat spacetime, if σ is a spacelike hypersurface stretching to infinity in all spacelike directions, we can define a 4-momentum vector for the physical system over σ by

$$G^i(\sigma) = \int_\sigma T^{ik}\, d\sigma_k\,, \tag{36}$$

where T^{ik} is the energy tensor of the system. If instead of σ we choose another spacelike hypersurface σ', also extending to infinity in all spacelike directions, then for a bounded physical system

$$G^i(\sigma') = G^i(\sigma). \tag{37}$$

In a general Riemannian spacetime, we cannot give a covariant definition for $G^i(\sigma)$ nor can we establish its constancy with respect to spacelike hypersurfaces.

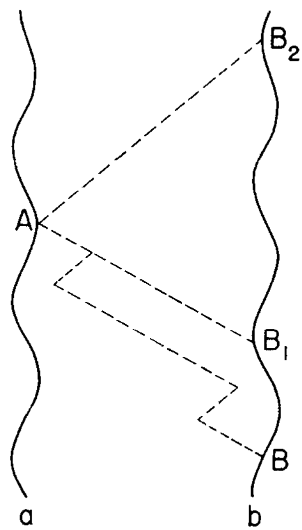

FIG. 3. The disturbance from B can arrive at A through a series of scatterings. Between two successive scatterings the disturbance travels along null lines.

The reason for this, both in the field theory and in the action at a distance picture, lies in the fact that the curved spacetime does not play a passive role. It also represents the presence of gravitation and its exchange of energy and momentum with the other physical entities. The apparent nonequality of action and reaction in direct particle theories also can be understood in terms of active interference by gravitation.

In Fig. 3, we see that the leading term in I^{iA} conveys the influence of B_1 and B_2 to A. There are, however, slower moving disturbances, which come from the past stretch of B_1 and the future stretch of B_2. The slow disturbance from a typical point B in the past of B_1 may be looked upon as a resultant of multiple scattering in the Riemannian spacetime, as pointed out by DeWitt and Brehme [13]. Between successive scatterings the disturbance is along null rays. As a result, the point B is not only linked with A but to all the points in the world line of a lying in the light cone of B. This many-to-one correspondence is responsible for the non-equality of action and reaction.

5. Conclusion

To summarize, the clear-cut equality between action and reaction in direct particle theories which exists in flat spacetime disappears in the curved spacetime. This is because the action and reaction travel not just along null lines but also inside the null cones. The portions traveling the "fastest" can be shown to be equal and opposite, but it is not possible to establish a similar result for slower portions. The reason for this is that slower portions of action and reaction result from scattering by the gravitation in curved spacetime. This destroys the one to one correspondence between any two world points of the interacting world lines.

This apparent inequality of action and reaction does not violate the overall conservation law of energy and momentum. This can be demonstrated by relating the instaneous action on a, I^{iA}, to the divergence of the instantaneous canonical energy tensor ${}^{\dagger}T^{ik}(C)$.

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