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Does inflation solve the horizon problem?

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Abstract. We stress the distinction between 'observed region of the universe' and 'observable region of the universe'. We show that (i) the observable region of the universe could not have evolved out of a causally connected domain in the past, even in inflationary models, and (ii) the homogeneity of CMBR is due to the fact that the universe is still young.

Consider a spatially flat Robertson-Walker universe with an expansion factor $S(t)$. We shall assume that the universe 'began' in a singularity at $t=0$, inflated from $t=t_i$ ($\approx 10^{-34}$ s) to $t=t_f$ ($\approx 60t_i$), was radiation dominated until $t=t_{rec}$ ($\approx 10^{13}$ s) and matter dominated from $t=t_{rec}$ until the present: $t=t_0$. The maximum *coordinate* distance which any signal could have travelled up to now is

$$r=\int_0^{t_0}\frac{dx}{S(x)}. \tag{1}$$

Therefore, the proper radius of the 'observable universe' (i.e. the region from which we can, in principle, receive signals at present) is $d_H(t_0)$, where

$$d_H(t)=S(t)\int_0^{t}\frac{dx}{S(x)}. \tag{2}$$

In practice, however, we only observe a smaller domain. Most of our observations are via electromagnetic radiation and hence are limited to $t=t_{rec}$ ('the surface of last scattering', SLS) which is at the proper distance:

$$d_{SLS}(t_0)=S(t_0)\int_{t_{rec}}^{t_0}\frac{dx}{S(x)}=d_H(t_0)-(1+z_{rec})d_H(t_{rec}). \tag{3}$$

The fact that the 'observed region' is smaller than the 'observable region' is merely due to technological limitations. For example, if cosmic background neutrinos were detected then one could probe all the way up to $Z\approx 10^{10}$. Thus $d_H(t_0)$ is the proper causally limited region of the 'observable universe'.

Let $\lambda(t_0)\equiv\lambda_0$ be any *proper* length scale in the present day universe, which would correspond to the size

$$\lambda(t)\equiv\lambda_0\frac{S(t)}{S(t_0)}=\frac{\lambda_0}{S_0}S(t) \tag{4}$$

at some other epoch t. We are interested in the ratio

$$R(\lambda;t)\equiv\frac{d_H(t)}{\lambda(t)}=\frac{S_0}{\lambda_0}\int_0^{t}\frac{dx}{S(x)}\qquad (t\leqslant t_0). \tag{5}$$

A particular scale λ_0 is said to be 'inside the horizon' if $R(\lambda) > 1$ and 'outside the horizon' if $R(\lambda) < 1$. From (5) it follows that, for $t \leq t_0$,

$$R(\lambda; t) = \frac{S_0}{\lambda_0}\left(\int_0^{t_0} \frac{dx}{S(x)} - \int_t^{t_0} \frac{dx}{S(x)}\right)$$

$$= R(\lambda; t_0) - \frac{S_0}{\lambda_0}\int_t^{t_0} \frac{dx}{S(x)} \leq R(\lambda; t_0). \quad (6)$$

(Note that λ and λ_0 have been used interchangeable in the sense that, for a given λ_0, there is a unique λ at any previous time.)

The proper length scale corresponding to the observable region of the universe (at present) is $\lambda_u(t_0) \equiv d_H(t_0)$. For this scale $R(\lambda_u; t_0) = 1$. From (6) we immediately get, for $t < t_0$,

$$R(\lambda_u; t) \equiv R_u(t) = 1 - \frac{S_0}{\lambda_u(t_0)}\int_t^{t_0} \frac{dx}{S(x)} < 1. \quad (7)$$

Notice that $\lambda_u(t)$ represents the size of that region, at time t, which expands to form the currently observable region of the universe. Equation (7) shows that the horizon size will always be smaller than the size of the region which evolves to form the observable universe. In other words, it is impossible for the observable universe to have evolved out of a single causally connected domain in the past.

It also follows from (6) that if $R(\lambda; t_1) > 1$ (for some λ and t_1) then for any later time $t > t_1$, $R(\lambda; t) > 1$, i.e. if a wavelength is inside the horizon at a particular time, it remains inside the horizon for all later times. There is no way some scale can grow bigger than the horizon and 'leave the horizon' (see also Ellis and Stoeger 1988). Any scale λ which enters the horizon at some time t_1 must have been outside the causal domain at $t < t_1$. (Usually, when the inflationists talk about perturbations 'leaving the horizon' and 're-entering the horizon', the reference is to an entirely different length scale, viz the inverse hubble distance

$$h(t) \equiv (\dot{S}/S)^{-1}. \quad (8)$$

This length scale has nothing to do *a priori* with the maximum distance signals can travel since the big bang. For a review of these concepts see, for example, MacCallum (1983).)

Thus no amount of inflation can homogenise the entire observable region of the universe. The impressions like 'the universe has evolved out of a single causally connected domain in the past' are simply wrong (Turner 1983, Linde 1984).

Having settled the situation regarding the observable region of the universe, let us ask: can inflation make the observed region of the universe homogeneous at all times? The answer again turns out to be 'no'.

Before we work out the details, it is worth pointing out that there is something inherently unsatisfactory in making the 'observed' universe homogeneous by inflation. This is because the 'observed' size of the universe is not a fundamental entity but an offshoot of technology. As we said before, development of neutrino astronomy will increase the observed size by seven orders of magnitude and graviton astronomy (if gravitons decouple at the Planck time) will increase it by thirty orders of magnitude. Thus the size of the 'observed' universe could be a wildly varying quantity with no fundamental significance (Padmanabhan and Seshadri 1987).

Even if we reconcile ourselves to work with such a concept, we are still not free from trouble. To see this, let us consider the size of the observed universe to be the proper distance $d_{SLS}(t_0)$ to the surface of last scattering. We shall take $S(t)$ to be

$$S(t)=\begin{cases} S_i(t/t_i)^{1/2} & 0<t\leqslant t_i \quad (9)\\ S_i\exp[H(t-t_i)] & t_i\leqslant t\leqslant t_f \quad (10)\\ S_iZ(t/t_f)^{1/2} & t_f\leqslant t\leqslant t_0 \quad (11)\end{cases}$$

where $Z=\exp[H(t_f-t_i)]$. The coordinate distance up to the SLS is

$$l(t_0, t_{rec})=\int_{t_{rec}}^{t_0}\frac{dx}{S(x)} \tag{12}$$

and the coordinate horizon distance at $t=t_{rec}$ is

$$l(t_{rec}, 0)=\int_0^{t_{rec}}\frac{dx}{S(x)}. \tag{13}$$

Hence, the number of causally disconnected volumes in CMBR is about N^3, where

$$N=\frac{l(t_0, t_{rec})}{l(t_{rec}, 0)}=\frac{l(t_0, 0)}{l(t_{rec}, 0)}-1. \tag{14}$$

Using equations (9)-(11) we get

$$l(t, 0)=\frac{1}{S_iZ}[(2t_i+H^{-1})Z-H^{-1}+2t_f^{1/2}(t^{1/2}-t_f^{1/2})] \tag{15}$$

$$\simeq\frac{1}{S_iZ}[4t_iZ+2(tt_f)^{1/2}] \tag{16}$$

where we have used the usual assumptions regarding inflation: $2t_i\approx H^{-1}$ ($\approx(10^{10}\,\text{GeV})^{-1}$), $z\gg 1$ and $t\gg t_f$. Using (14) and (16), and under the approximation $t_0\gg t_{rec}$, we get

$$N\simeq\frac{(t_ft_0)^{1/2}}{2t_iZ+(t_ft_{rec})^{1/2}}. \tag{17}$$

To ensure that the whole of CMBR is within a causally connected patch, we demand that $N\leqslant 1$. Hence

$$t_f^{1/2}(t_0^{1/2}-t_{rec}^{1/2})\approx(t_ft_0)^{1/2}<2t_iZ \tag{18}$$

or equivalently (Guth 1981)

$$Z\geqslant\left(\frac{t_f}{t_i}\right)^{1/2}\left(\frac{t_0}{t_i}\right)^{1/2}. \tag{19}$$

In conventional models, the right-hand side of (19) is $\sim 3\times 10^{27}$. So the usual model for inflation, with $Z\simeq 10^{29}$, will make the CMBR homogeneous.

It should be clear from (19) that we have not really made CMBR homogeneous at all times. Equation (19) can be rewritten as

$$t_0<t_{crit}\equiv\frac{t_i^2}{t_f}\exp[2H(t_f-t_i)]. \tag{20}$$

In other words CMBR will appear isotropic to us only as long as the age of the universe is below a critical value which is completely predetermined at $t \simeq 10^{-34}$ s! (Notice that t_{crit} is entirely fixed by microscopic (particle) physics theory, producing the inflation, through H, t_i and t_f.) There is no apparent, fundamental, reason why our universe should be young enough to satisfy (20). Essentially (20) tells us that if we wait long enough CMBR will appear anisotropic!

The universe was radiation dominated from $t = t_f$ to $t = t_{rec}$. Hence the scale factor S evolves as $t^{1/2}$ during this period. From $t = t_{rec}$ till today the universe was matter dominated. The scale factor evolves as $t^{2/3}$ for such a universe. In our analysis, we have assumed $S \propto t^{1/2}$ for $t = t_f$ up to the present time (equation (11)). This will not, however, affect the conclusions in any essential way.

We repeat that the conventional solution to the horizon problem is precisely the one given above. It is usual to state a 'condition for sufficient inflation' in the form of (19), but it is more transparent for our purpose to state it as in (20).

From the above observations it is clear that inflation does not—and cannot—really solve the horizon problem. In the first place, no amount of inflation can make the whole observable universe to arise out of a causally connected domain. As technology improves and we probe deeper anisotropies can surface. Secondly, even part of the observable region cannot sustain its look of homogeneity for ever. The conventional values for z, t_i, t_f, etc, can make the CMBR look isotropic today, but at some finite time in future this effect will cease to operate. One may take the point of view that the *only* problem about the horizon is the observed isotropy of the CMBR and that the inflation is only expected to solve this problem today; we feel that the horizon problem can—and should be—looked at in a much broader perspective.

Thus we conclude that (i) inflation can never make the whole of the *observable* universe homogeneous and (ii) it cannot make even a part of the observable universe homogeneous *for ever* (i.e. for all $t > 0$). These can only be achieved in models which are strictly horizonless. This happens in many quantum gravitational models (Narlikar and Padmanabhan 1983, 1985).

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