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ON THE COUNTING OF RADIO SOURCES IN THE STEADY-STATE COSMOLOGY, II

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Summary

The existence of large scale condensations introduces a discrete element into the steady state theory. This discreteness assumes crucial importance where the counting of an age-correlated property is involved—as it may do in the case of radio sources. The discrete nature of the problem makes it awkward to deal with analytically. A different method using a high speed computer was therefore tried recently. This approach and the results obtained from it are discussed in this paper. There is a good agreement between the results and those obtained earlier by an analytical method.

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1. *Introduction.*—In a previous paper ((1) referred to hereinafter as I) the number-flux density relation for radio sources was considered on the basis that the intrinsic emission of the majority of radio sources falls in the range 3×10^{26} to 3×10^{27} watts $(c/s)^{-1}$ at 160 Mc/s, i.e., one to ten per cent of Cygnus A. In I it was found that if groups of galaxies condense in a suitable way and if the probability of a galaxy being a radio source depends on its age in a suitable way, the radio source count can rise more steeply with distance than is the case for sources distributed uniformly in a Euclidean space.

Recently, Hanbury Brown has presented evidence (2) suggesting that the intrinsic emission of a typical radio source is about 10^{-3} of Cygnus A. On this basis the problem of the source count becomes a much more local one, hardly relevant to cosmology. It is shown that a single local irregularity of the general size considered in I, 30 to 100 mpc, is sufficient to explain the observations reported by Ryle and his colleagues. No relation between the probability of a galaxy being a radio source and its age is then required. On this simplified basis the complexities of I are unnecessary, as are the considerations of the present paper.

Without prejudice to the issues raised by Hanbury Brown, the present paper has been written to generalize the treatment adopted in I, where continuous theory was used to derive a basic $\log N - \log P$ distribution and where the effects of discreteness were added as fluctuations. Strictly, the whole problem should be treated in discrete terms. Although for sources at sufficiently large distances the results for a discrete model must tend to those of the continuous theory, some uncertainty arises in fitting the local discreteness to the distant continuum. This uncertainty was fully discussed in I and reasons were given for believing that our fitting was free from subjective judgments. This belief is confirmed by the objective calculations now being reported. In these, a discrete model

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has been used throughout, so that no question of fitting to a continuum arises. To obtain an adequate selection of observers, more than 10^5 sources must be counted, their distances and positions on the sky must be worked out, and for this automatic computation is necessary. An IBM 7090 was used in our work.

The nature and behaviour of condensations in the steady state theory, described in detail in I, will now be summarized.

2. *A brief restatement of the condensation picture.*—Gravitational forces alone are inadequate to condense matter into galaxies (**3, 4, 5**). Gold and Hoyle (**6, 7**) therefore suggested that non-gravitational forces, in particular pressure gradients, may be present. Such forces can arise from variations of temperature within an intergalactic medium. It was suggested that condensation may take place in two stages: a primary stage in which the density increases from $\sim 10^{-29}\,\mathrm{gm\,cm^{-3}}$ to $\sim 10^{-27}\,\mathrm{gm\,cm^{-3}}$, and a secondary stage from $\sim 10^{-27}\,\mathrm{gm\,cm^{-3}}$ to $\sim 10^{-24}\,\mathrm{gm\,cm^{-3}}$. The first of these requires a kinetic temperature of $\sim 10^9\,^\circ\mathrm{K}$. In the steady state theory creation of neutrons leads to a temperature of this order. The size of a primary condensation turns out to be $\sim 30\,\mathrm{mpc}$, sufficiently large to provide for the eventual creation of some 10^5 galaxies. Further, the energy dissipation during primary condensation is small, so that most of the thermal energy is converted to dynamical energy, with the result that in the main the resulting galaxies expand apart from each other—they eventually take part in the universal expansion.

It was shown in I that the large size of the primary condensations plays an important role in the counting of any age-correlated property. Perhaps the simplest example is the counting of galaxies of a particular specified age. An observer living inside the fragments of a comparatively old primary condensation will find that the average age of the galaxies near to him is substantially different from the average age of all galaxies, $\frac{1}{3}H^{-1}$. It was suggested that this indeed is the case for ourselves, that we live inside an age-correlated region which formed a time H^{-1} ago. If the probability of a galaxy being a radio source depends on its age, a similar situation may exist in the counting of radio sources. Provided the probability rises by a factor $\sim 10^2$ for galaxies with ages from H^{-1} to about $2{\cdot}5H^{-1}$, the radio source count can rise more steeply than is the case for sources distributed uniformly in Euclidean space.

3. *The discrete model.*—It is well known that in the steady state universe the average length of a generation of galaxies is $\frac{1}{3}H^{-1}$. For the sake of simplicity it is convenient to assume that the primary condensations are formed at intervals of $\frac{1}{3}H^{-1}$. It is then possible to regard the universe as made up of discrete generations of condensations. Condensations of the same generation have the same age at any given cosmic time. It must be noted however that this discreteness with respect to time is artificial—not one present in the actual universe.

Consider now the "world-map" of the universe at a given cosmic time. The members of the n^{th} generation form an irregular lattice with an average separation length l_n for two nearest neighbours. Further, let a_n denote the characteristic size at the time in question of this generation. Then, counting n so that the youngest generation has $n = 1$ we can write

$$(a_n, l_n) = e^{(n-1)/3}(a_1, l_1) \qquad (1)$$

it being supposed that the condensations take part in the universal expansion.

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In I it was also assumed that the lattice is cubical in structure with $a_n = \frac{1}{3}l_n$. This artificial requirement in space is removed in the present work. Instead it is assumed that the centres of mass of the lattice units are generated at random, but with the condition that the distance between any two of them shall exceed b_n where b_n also increases with the general expansion, so that b_n varies with n according to

$$b_n = e^{(n-1)/3} b_1. \tag{2}$$

If there are k_n condensations (lattice units) of generation n per unit volume we can estimate l_n by

$$l_n \sim (k_n)^{-1/3}. \tag{3}$$

It is further assumed that the positions of the condensations of the n^{th} generation are to be anti-correlated with those of the $(n+1)^{\text{th}}$ generation. Once the positions of the lattice units have been determined in accordance with these conditions, radio sources are to be distributed at random within them. The number of sources to be so distributed depends on what hypothesis is made for the dependence on age—i.e., on n—of the probability of a galaxy being a radio source.

Having obtained a discrete model of the universe in the above way it is then a straightforward matter to place random observers in it and do the counting. The details of this are described in the next section.

4. *The machine computations.*—As random distributions are required at various stages of the model a sub-routine was used to generate random numbers. The formula

$$x_{n+1} = \alpha x_n \pmod{\beta} \tag{4}$$

with $\alpha = 2^{18}+3$, $\beta = 2^{35}$ generates a sequence of pseudo random numbers $\{x_n\}$ and is particularly suited for use on the IBM 7090. Then sequences $\{y_n\}$ and $\{z_n\}$ were formed using

$$y_n = 2^{-35} x_n, \quad z_n = 2y_n - 1 \tag{5}$$

so that $0 < y_n < 1$, $-1 < z_n < 1$ and $\{y_n\}$, $\{z_n\}$ have uniform distributions in these ranges.

It is convenient to choose units such that $c = 1$, $H = 1$, and such that $t = 0$ is the present moment of time. In this case the luminosity distance $D = r \exp Ht$, given by usual astronomical procedures, is simply the coordinate distance r at the present moment, while $r = z$ at the present, so that the luminosity distance is given simply by

$$D = z \tag{6}$$

where z is the redshift of the object. Clearly $D = 1$ corresponds to the radius of the observable universe. The distances a_n, b_n, l_n were all expressed in these units.

From a computational point of view there are two ways in which the problem of counting can be handled. (i) After distributing radio sources in the universe according to a specified probability law, and in accordance with particular choices for a_1, l_1, b_1, observers can be placed at random and surveys can be carried out for each of them. (ii) Take an observer at the origin (0, 0, 0) and after distributing radio sources around him as in (i) carry through the survey

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for this particular observer. But then repeat as many times as required, distributing sources differently in each case. While (i) and (ii) are in principle equivalent, it is more convenient from the point of view of computer storage and speed to use the latter method. Accordingly the observer is taken to be at (0, 0, 0) and the region of survey is confined to a cubical box of size $D=1$ with the observer at the centre. The random generation of the positions of the centres of mass of the lattice units is done as follows.

Suppose the probability function $K(\tau)$ is such that only the generations with $n_1 \leqslant n \leqslant n_2$ contribute to the counting. After distributing $p-1$ condensations of the n^{th} generation, to find the coordinates of the centre of the p^{th} condensation a trial is made. The trial consists in generating three random numbers lying in the range $[-(\cdot 5+a_n),\ +(\cdot 5+a_n)]$. These numbers are taken to represent the three coordinates of the centre. The trial is a success if the newly obtained point is at distances exceeding b_n from the $p-1$ points already obtained, and also if it is at a distance exceeding (a_n+a_{n+1}) from each of the centres of the $(n+1)^{\text{th}}$ generation. (The latter restriction, arising out of anticorrelation of course does not apply when $n=n_2$—which is the oldest generation of importance.) If the trial is a success the coordinates of the newly created point are accepted for further calculations; otherwise a fresh trial is made. This procedure is carried out until the number of trials reaches a certain number previously assigned.

Suppose it is possible to place k condensations in a volume V without overlapping. For V large compared to the volume of a condensation $k \propto V$. Probability theory shows that the expected number of trials before k successes are obtained is

$$k \sum_{r=1}^{k} \frac{1}{r} \sim k \ln k \quad \text{for large } k. \tag{7}$$

The number of trials is therefore taken to be of that order. If the number of trials is substantially increased above this value the corresponding number of successes is increased by only a small number.

Having obtained the coordinate of the centres of condensations of the n^{th} generation, the next step is to consider the distribution of sources in each condensation. Suppose the number in an n^{th} generation cluster is given by the probability function to be N_n. Then N_n sources are to be distributed at random inside each condensation. However, care must be taken in counting these sources. N_n is the number existing in the condensation at the moment of observation. As radio waves take time to travel, if the age of an object with redshift z is τ_n at the moment of observation, its age τ at the moment of emission is

$$\tau = \tau_n - \ln(1+z). \tag{8}$$

Therefore all sources for which $\tau_n - \ln(1+z) < 0$ must be rejected from observation. Moreover, N_n is also an overestimate, since $k(\tau)$ is taken to increase with τ. If all the N_n sources were situated at the same redshift z, the overestimation would be by a factor

$$f = \frac{K(\tau_n)}{K(\tau_n - \ln 1+z)}. \tag{9}$$

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In general, however, the sources are not all at the same z. The situation is met by imposing a probability control on the selection process. Thus a source which has $\tau_n - \ln(1+z) \geqslant 0$ is selected if

$$f y_n < 1 \tag{10}$$

where y_n is a random number between (0, 1) generated once for each source.

A source accepted for observation was measured for its redshift and for its angular coordinate. A discrete set of values of z was chosen

$$z = \cdot 05, \cdot 10, \cdot 15, \ldots, \cdot 50 \tag{11}$$

and a count was made in the following way for each value. Starting initially at zero, the count was increased by unity whenever the z value of a source was exceeded by that of the member of (11). Otherwise the count was not changed. This was done for each member of (11), so that ten separate z-counts were made. In addition, the sky was divided into 108 equal areas and a count was kept for each area—the count for any particular area was increased by unity whenever a source fell in that area.

Assuming the spectrum of the radio source to be of the form dv/v the flux level corresponding to the redshift z is

$$P = \frac{L}{4\pi z^2 (1+z)^2} \tag{12}$$

where L is the intrinsic emission of the source. Also assuming no dispersion in intrinsic emission, the slope of the $\log N - \log P$ curve between two successive values of z is given by

$$\frac{\Delta \log N}{\Delta \log P} = -\frac{\Delta \log N}{2\Delta \log (z \,.\, \overline{1+z})}. \tag{13}$$

This was computed from the counts at the various values of z given in (11). Comparison with $-1{\cdot}5$, the value for a uniform distribution in a Euclidean universe, could then immediately be made.

5. *Results.*—In I the probability law was taken to be

$$K(\tau) = \begin{cases} (\text{constant}) & \exp(4\tau) \quad \text{for } 1 \leqslant \tau \leqslant 2{\cdot}5 \\ 0 & \text{otherwise} \end{cases}.$$

Also a_n/l_n was taken as $\frac{1}{3}$. These values of the parameters were therefore first tried on the computer. As predicted in I, the number count showed considerable fluctuation down to $z = \cdot 3$. Beyond this value the fluctuation is dominated by the redshift effect. This is to be expected since at this stage regions are getting large enough to smooth out the effects of discreteness. Two extreme cases of fluctuation are shown in Table I, while Table II shows that about half the cases considered gave $\log N - \log P$ slopes steeper than Euclidean out to $z = \cdot 3$.

For a uniform distribution of sources in a steady state universe N, P are given as functions of z by

$$N = (\text{constant}) \int_0^z \frac{z_1^2 dz_1}{(1+z_1)^3}, \quad P = \frac{(\text{constant})}{z^2(1+z)^2}. \tag{14}$$

From this it is possible to calculate $-d\log N/d\log P$. Of the 50 cases of Table II some gave number counts steeper than this while others gave a flatter rise. If all cases are combined together, the distribution is close to that obtained from (14).

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TABLE I

Two extreme cases of fluctuation in the log N − log P curve

z	N	$-\Delta \log N/\Delta \log P$	z	N	$-\Delta \log N/\Delta \log P$
·05	21		·05	0	
		1·35			—
·10	154		·10	3	
		1·14			2·40
·15	419		·15	104	
		·92			1·87
·20	770		·20	349	
		·91			1·50
·30	1889		·30	1457	
		·90			1·15
·40	3527		·40	3246	
		·67			·70
·50	5171		·50	4888	

TABLE II

Redshift z	Number of cases with super-Euclidean slope	Number of cases with sub-Euclidean slope
·2	30	20
·3	28	22
·4	11	39
·5	0	50

TABLE III

Two extreme cases of fluctuation when the probability $K(\tau) = constant$

z	N	$-\Delta \log N/\Delta \log P$	z	N	$-\Delta \log N/\Delta \log P$
·05	2		·05	4	
		1·55			1·12
·10	20		·10	21	
		1·44			1·35
·20	187		·20	176	
		1·20			1·25
·30	616		·30	589	
		·88			·92
·40	1131		·40	1138	
		·46			·44
·50	1477		·50	1461	

In many examples of steep slope, the slope first falls in magnitude and then rises again. The reason for this is that the observer actually lies inside a condensation old enough to contribute an appreciable number of local sources. Cases of this sort were discussed in I.

Trials with different values of the parameters a_1, b_1, l_1 were also made. Their variation within a factor of 2 turned out not to affect the above conclusions. Some cases with less steep rise in probability were also tried. The fluctuation about the mean curve, obtained from (14), was less in these cases and was found to extend to smaller distances from the observer. It is interesting to note that some fluctuation is present even in the case $K(\tau) = \text{const.}$, as is shown by the two extreme examples given in Table III.

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TABLE IV

Five cases of source distributions over the sky divided equally into 18 *areas* (*i.e. each area* $N=\frac{2}{3}$ *steradian*) *and the corresponding number counts* (*arbitrary scale for* P)

(i) $K(\tau)\propto \exp(4\tau)$

Source distribution at $z=\cdot 3$

62 67 55 52 56 58 56 50 67 59 67 68 50 52 66 67 53 61

Number count

z	·1	·2	·3	·4	·5
N	6	278	1066	2242	3364
$\log NP^{3/2}$	·65	1·30	1·26	1·00	·90

(ii) $K(\tau)\propto \exp(4\tau)$

Source distribution at $z=\cdot 3$

68 80 79 68 77 95 70 79 82 80 83 73 61 75 60 60 85 85

Number count

z	·1	·2	·3	·4	·5
N	1	186	1360	3034	7670
$\log NP^{3/2}$	·88	2·13	2·38	2·23	2·25

(iii) $K(\tau)\propto \exp(4\tau)$

Source distribution at $z=\cdot 3$

60 58 61 59 51 48 58 55 52 40 46 59 45 59 56 55 64 54

Number count

z	·1	·2	·3	·4	·5
N	26	330	980	1775	2998
$\log NP^{3/2}$	1·29	1·38	1·22	1·05	·85

(iv) $K(\tau)\propto \exp(3\tau)$

Source distribution at $z=\cdot 3$

39 46 40 34 35 28 37 26 29 26 31 32 37 46 45 35 46 32

Number count

z	·1	·2	·3	·4	·5
N	69	327	644	1136	1586
$\log NP^{3/2}$	1·71	1·37	1·03	·81	·54

(v) $K(\tau)=$ const.

Source distribution at $z=\cdot 3$

59 77 60 54 69 63 69 87 90 76 57 62 60 69 65 82 72 80

Number count

z	·1	·2	·3	·4	·5
N	51	429	1251	2287	2934
$\log NP^{3/2}$	1·58	1·49	1·32	1·11	·84

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In all the cases studied the distribution of sources over the sky was obtained. As departure from isotropy would be—if at all—over large areas (~ 1 steradian) it was decided to consider only areas greater than $20° \times 20°$. Accordingly, numbers lying in each of the 108 equal divisions of the sky were studied. If one takes the view that the majority of sources are weak compared to Cygnus A, it is necessary to consider only the distributions of sources with redshifts up to $z = \cdot 3$. This view is supported by the recent work at Jodrell Bank (**2**) where a large proportion of sources are found to be of the order 10^{-3} that of Cygnus A.

In most of the cases the departure from isotropy does not appear to be significant. A few typical cases are given in Table IV where the sky has been divided into 18 equal areas, each area being $\cong \frac{2}{3}$ steradian. In a few cases some areas of the sky contain significantly more points than others. It is interesting to note in this connection that an effect of this type was reported by Mills (**8**). The case for anisotropy becomes stronger at higher redshifts. However, at flux levels $> 10^{-26}\,\mathrm{w.m^{-2}\,(c/s)^{-1}}$ the numbers of strong sources will be small and the effect will be hidden in the general isotropic distribution of the weak sources.

By using a sense switch arrangement it was possible to count the number of random numbers generated in a typical survey. This number was $\sim 10^5$ and the sequence did not get into a cycle during any survey. Further, a small subset was tested for randomness and was found to satisfy the appropriate statistical test.

6. *Conclusion.*—The above results show a general agreement with the analytical approach of I. In a recent paper (**9**) Davidson has argued that the analytical approach contains too many arbitrary parameters. It is not proposed to give justification for their values—this has been given in I. The present work, however, shows that the results are valid even when there is a fairly substantial variation in the values of the parameters. Moreover, the recent studies of source distributions over the sky by Mills (**8**) over large areas and Leslie (**10**) over small areas appear to be consistent with the above theory.

The type of fluctuations found does not contradict the cosmological principle, because of the inherent discreteness of the cosmology. The cosmological principle cannot be expected to hold for a property unless it is applied over regions large enough to make that property sufficiently frequent. Such regions are much smaller for ordinary galaxy counts than for the radio source counts. Indeed, for sufficiently large distances, the number count of radio sources does show a similarity for all observers, as required by the cosmological principle.

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