

CONSTRAINTS ON THE SHAPE OF THE DENSITY SPECTRUM FROM COBE AND GALAXY SURVEYS

T. Padmanabhan^{1*} and D. Narasimha^{2*}

2673

¹Inter University Centre for Astronomy and Astrophysics
Pune University Campus, Postbag no. 4, Ganeshkhind,
Pune 411 007, INDIA

²Astronomy Unit, School of Mathematical Sciences
Queen Mary and Westfield College
Mile End Road, London E1 4NS



ABSTRACT

Many popular models for the galaxy formation are based on power spectra which behave as $P(k) = Ak$ for small k and flattens at large k . The resulting mass fluctuation $(\delta M/M)_R^2 \equiv \sigma_{gal}^2(R)$ can be constrained by galaxy surveys up to $R \approx 60h^{-1} Mpc$. On the other hand, the recent COBE results constrain the dark matter spectrum at very large scales ($R \gtrsim 10^3 h^{-1} Mpc$). The COBE results $(\Delta T/T)_{rms} = (1.1 \pm 0.2) \times 10^{-5}$ and $(\Delta T/T)_Q = (0.4 \pm 0.15) \times 10^{-5}$ are consistent with a scale invariant spectrum, $\sigma_{DM}(R) = (R_0/R)^2$ at large scales with $R_0 \simeq (23.9 \pm 2.1)h^{-1} Mpc$. This is consistent with APM data and large scale streaming velocity measurements (both of which require $\sigma_{gal}(50h^{-1} Mpc) \simeq 0.2$) provided the scale invariant spectrum is extrapolated from $3000h^{-1} Mpc$ to $50h^{-1} Mpc$. The fact that the extrapolated COBE result matches with galaxy survey results at $50h^{-1} Mpc$ suggests that biasing is not significant at $R \gtrsim 50h^{-1} Mpc$. On the other hand a CDM spectrum, normalised to COBE value, will overshoot galaxy survey results at small scales. Such a spectrum can be consistent with observations only if the biasing varies with scale and $b < 1$ at small scales. Comparing the shape of $\sigma(R)$ at $R \lesssim 60h^{-1} Mpc$, determined from galaxy surveys (IRAS, CfA, and APM), with the COBE result, we find that a relatively rapid bend in $\sigma(R)$ around $R \approx (40 - 60)h^{-1} Mpc$ is needed. We show that simple models to describe this bend based on spectra of the form $P(k) = Ak[1 + (k/k_c)^n]^{-1}$ are severely constrained. The implications of this result are discussed.

IUCAA-16/92 Oct 92; revised form of TIFR-TAP-3 April 92
Submitted for publication

¹email: paddy@iucaa.ernet.in

²email: dna@tifrvax.bitnet

* On leave of absence from: T.I.F.R., Homi Bhabha Road, Bombay 400 005, India.

1. Introduction

Standard models for galaxy formation assume that the inhomogeneities observed in the universe grew out of small perturbations in the past, through gravitational instability (see e.g. Peebles, 1980; Padmanabhan, 1992). Given the initial power spectrum $P_i(k) = |\delta_k(t_i)|^2$, where $\delta_k(t)$ is the Fourier transform of the density contrast $[\delta\rho(\mathbf{x}, t)/\rho_b(t)]$, one can - in principle - find the power spectrum of fluctuations today. In practice, however, this task can be reliably achieved only in the linear regime, where $(\delta\rho/\rho) \ll 1$. Using linearized Einstein equations, one can predict the form of $P_R(k)$ at recombination epoch, given the initial spectrum. If the initial spectrum is a power law $P_i(k) \propto k^n$, $[-3 < n \leq 1]$ then the spectrum at recombination will behave as k^n for small k (large scales) and will flatten at large k . [We ignore effects due to free streaming since we shall *not* be concerned with HDM models in this paper]. During further evolution, small scales (large k) will go nonlinear first and structure will form hierarchically. Numerical simulations show that the nonlinear evolution (i) will steepen the spectrum at small scales but (ii) will not significantly affect the form of the spectrum at large scales. Thus the spectrum evolved by linear theory does contain useful information about the universe at large scales, say, at $R \gtrsim 20h^{-1} Mpc$. Even at small scales, the slope of the linear theory spectrum provides a lower bound to the actual slope and - as we shall see - offers interesting constraints.

At large scales, the spectrum is constrained by the recent COBE observations of MBR anisotropy (Smoot et al., 1992; Wright et al., 1992). If we make the natural assumption that the spectrum is scale invariant [$P(k) \propto k$] at small $-k$, the amplitude of the spectrum is completely determined by the COBE result. Combining the COBE results with those from galaxy surveys leads to an interesting overall picture about the spectrum which we shall discuss in this paper.

2. Comparison of models with COBE and galaxy surveys

The theoretical models are best compared with observation using the *rms* fluctuations in mass, $\sigma(R)$, where

$$\begin{aligned} \sigma^2(R) = \langle (\delta M/M)_R^2 \rangle &= \int_0^\infty \frac{dk}{k} \left[\frac{k^3 |\delta_k|^2}{2\pi^2} \right] \left[\frac{3 \sin kR - 3kR \cos kR}{(kR)^3} \right]^2 \\ &= \int_0^\infty \frac{dk}{k} \Delta^2(k) W_S(kR)^2 \end{aligned} \quad (1)$$

The quantity $\Delta^2(k)$ represents contribution from each logarithmic interval in k to the mean-square fluctuations in mass and W_S is the Fourier transform of the spherical window function.

Let us now consider the constraints on $\sigma(R)$ from observations. The most interesting constraint is from the recent COBE measurements, (Smoot et al., 1992):

$$\left(\frac{\Delta T}{T} \right)_{rms} = (1.1 \pm 0.2) \times 10^{-5} \quad (2)$$

$$\left(\frac{\Delta T}{T} \right)_Q = (0.48 \pm 0.15) \times 10^{-5} \quad (3)$$

Theoretical estimates for the above quantities can be made from the standard formulas (eg. Peebles, 1982):

$$\left(\frac{\Delta T}{T}\right)_{\text{rms}}^2 = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1) C_l \exp\left(-\frac{l^2 \theta_c^2}{2}\right) \quad (4)$$

$$\left(\frac{\Delta T}{T}\right)_Q^2 = \frac{5}{4\pi} C_2 \exp(-2\theta_c^2) \quad (5)$$

where

$$\langle |a_{lm}|^2 \rangle = C_l = \frac{H_0^4}{2\pi} \int_0^{\infty} dk \frac{|\delta_k|^2}{k^2} |j_l(k\eta)|^2 \quad (6)$$

Here θ_c is the 'smearing angle' due to the beam size of COBE [$\theta_c \approx 0.425 \theta_{FWHM} \approx 0.0519$ (rad) if $\theta_{FWHM} \approx 7^\circ$]; $\eta = 2H_0^{-1} = 6000h^{-1}\text{Mpc}$ and j_l is the spherical Bessel function of order l . In arriving at the above expression we have assumed that the Sachs-Wolfe term dominates the anisotropy and that $\Omega = 1$. [In models with zero cosmological constant, spatial curvature becomes important at $\theta > \theta_{\text{curv}} \simeq \Omega/(1-\Omega)^{1/2}$. There is no natural definition of scale invariance for $\theta > \theta_{\text{curv}}$. We will see later that COBE results are consistent with scale invariance, thereby offering additional support for $\Omega = 1$.]

Since COBE is essentially probing large angular scales, it is legitimate to use the asymptotic form $P(k) \simeq Ak$ in computing C_l . Then we find that $C_2 = AH_0^4/24\pi$ and $C_l = [6C_2/l(l+1)]$. Substituting these relations into (4) and (5) we get $(\Delta T/T)_Q^2 = (5.28 \times 10^{-3})(AH_0^4)$ and $(\Delta T/T)^2 = 0.03(AH_0^4)$. The quantity (AH_0^4) is directly related to the fluctuations in the gravitational potential $\Phi^2 = (k^3|\phi_k|^2/2\pi^2)$ at large scales. Since $\phi_k = (4\pi G\rho_b)(\delta_k/k^2) = (3/2)H^2(\delta_k/k^2)$ we find that $AH_0^4 = (8\pi^2/9)\Phi^2$ and we can reexpress the anisotropies as,

$$\left(\frac{\Delta T}{T}\right)_Q \cong 0.22\Phi; \quad \left(\frac{\Delta T}{T}\right)_{\text{rms}} \cong 0.51\Phi \quad (7)$$

We can now compare the theoretical results with the COBE observations. To begin with, notice that $(\Delta T_{\text{rms}}/\Delta T_Q) \cong 2.3$ if the spectrum has $n = 1$. The COBE results allow this ratio to fall between 1.43 and 3.94 with a mean value of 2.29. This is quite consistent with the assumption of $n = 1$. We shall, hereafter, assume that $P(k) \propto k$ for small k . The parameter Φ and the amplitude A can now be determined by comparing (7) with COBE result. Within the error bars, we get $\Phi \cong 2.2 \times 10^{-5}$ and $A = (24h^{-1}\text{Mpc})^4$. Also note that the quadrupole result gives $C_2^{1/2} = (4\pi/5)^{1/2} (\Delta T/T)_Q = (0.76 \pm 0.24) \times 10^{-5}$; thus the maximum permitted value for $C_2^{1/2}$ is about 10^{-5} .

A spectrum of the form $P(k) = Ak$ will lead to a mass fluctuation of the form $\sigma_{DM}(R) = (R_0/R)^2$ provided the spectrum flattens at large k leading to a convergent integral in (1). [The subscript 'DM' is added to emphasise the fact that COBE (Sachs-Wolfe) result probes the dominant gravitating component, viz. the dark matter]. In this case $R_0 = fA^{1/4}$. For a wide class of spectra which we will consider later $f \cong 1$ to sufficient accuracy. In this case, the bound on A implies that $\sigma_{DM}(R) = (R_0/R)^2$ at large scales

with $R_0 \simeq 24h^{-1}Mpc$. This $\sigma_{DM}(R)$ is shown at the right-bottom edge of figure 1. The error bounds are also shown by two adjacent lines. Strictly speaking, COBE result covers only scales which subtend $\theta \gtrsim 7^\circ$ in the sky; the extrapolation of COBE result to smaller scales is shown by a dashed line. Since the spectra in most theoretical models bend down at small R , this dashed line represents an upper bound to $\sigma_{DM}(R)$.

The galaxy survey results constrain the form of $\sigma_{gal}(R)$ at small scales. [A priori, we cannot expect σ_{gal} to be the same as σ_{DM}]. We have plotted in fig. 1 the $\sigma_{gal}(R)$ obtained from three galaxy surveys (CfA, IRAS and APM) based on the analysis by Hamilton et al. (Hamilton et al., 1991; the original data is from Huchra et al., 1983; Strauss et al., 1990; Maddox et al., 1990) The CfA and IRAS are three dimensional surveys and the published results can be directly converted to give $\sigma_{gal}(R)$. The APM result is published in the form of the angular correlation function which can be inverted using Limber's equation to give $\sigma_{gal}(R)$ using the selection function suggested in Maddox et al. (1990). [For more details regarding the determination of $\sigma_{gal}(R)$, see Hamilton et al., 1991].

For the sake of completeness, we have marked a few other constraints on $\sigma(R)$ in the same diagram. The POTENT reconstruction of velocity fields (Bertschinger et al. 1990) gives a *lower* bound to $\sigma_{DM}(R)$ in the range $(40 - 60)h^{-1}Mpc$. The fact that masses in the range $(10^{10} - 10^{11})M_\odot$ should have gone nonlinear by $z \simeq 4.5$ leads to another lower bound on $\sigma(R)$ shown near the top-left. (Kashlinsky and Jones, 1991; Padmanabhan, 1991). The small scale anisotropy of MBR in models with reionisation provides a (weak) upper bound to $\sigma(R)$. We have also shown by a thick dashed line the slope corresponding to the galaxy-galaxy correlation function.

Several conclusions can be drawn from this constraint diagram. To begin with, notice that the COBE line denoting $\sigma_{DM}(R)$, *extrapolated linearly* agrees with the $\sigma_{gal}(R)$ estimated from APM and large-scale-streaming at around $50h^{-1}Mpc$. This consistency is easily verified by the following simple estimate: We know from theory that $v_{pec} \simeq (1/3)v_H\sigma_{DM}$ where v_H , v_{pec} are Hubble and peculiar velocities at a given scale. If we take $v_{pec} \simeq 330km s^{-1}$ at $50h^{-1}Mpc$, we find that $\sigma_{DM}(50h^{-1}Mpc) \simeq 0.2$. To estimate the APM result, note that the 3-dimensional correlation function $\xi(R)$ is related (approximately) to the angular correlation function $W(\theta)$ by $\xi(R) \simeq W(\theta)(D/R)$ where $D \simeq 200h^{-1}Mpc$ is the sample depth and $\theta = (R/D)$. From the published APM result, $W(14^\circ) \simeq (1 - 5) \times 10^{-3}$; since 14° at the depth of $200h^{-1}Mpc$ corresponds to $R \simeq 50h^{-1}Mpc$, we find $\xi(50h^{-1}Mpc) \simeq (200h^{-1}Mpc/50h^{-1}Mpc) W(14^\circ) \simeq (0.004 - 0.020)$. Hence $\sigma_{gal}(50h^{-1}Mpc) \simeq \xi^{1/2} \simeq (0.06 - 0.14)$; a more precise analysis gives the slightly larger value of $\sigma_{gal} \simeq 0.2$ seen in fig.1, which is consistent with the peculiar velocity estimate. On the other hand COBE result suggests $\sigma_{DM}(50h^{-1}Mpc) \simeq (24/50)^2 \simeq 0.2$. *These observations, therefore, seem to suggest that $\sigma_{DM}(50h^{-1}Mpc) \simeq \sigma_{gal}(50h^{-1}Mpc) \simeq 0.2$.* Thus within the error bars, it is consistent to assume that the bias factor $b(R)$ is unity at $R \simeq 50h^{-1}Mpc$.

Figure 1 also suggests that the magnitude of the slope of $\sigma_{gal}(R)$ is lower at small scales (say, $(5 - 50)h^{-1}Mpc$) compared to its value at larger scales. A spectrum of the form $P(k) \simeq Ak$ at large scales, extrapolated to $50h^{-1}Mpc$ has to bend rather sharply at smaller scales to account for the observation, if biasing factor is not scale dependent. To make these conclusions more quantitative, we have fitted (in fig. 2) a series of simple test

spectra to the constraints. The simplest type of spectra, which behaves as $P \propto k$ at small k and exhibits a sharp bend at large k , is given by the family of functions:

$$P(k) = \frac{Ak}{1 + (k/k_c)^n}. \quad (8)$$

(These spectra were studied earlier in Peacock, 1991; Padmanabhan, 1991.) Figure 2 shows the behaviour of $\sigma(R)$ for these functions, for different choices of k_c and n . In all the cases, we normalize the spectrum to give $C_2 = 10^{-5}$ which is the maximum value allowed by COBE results. A good fit is achieved for the dotted curve which corresponds to $n = 2.4$ and $k_c = 0.047hMpc^{-1}$. We have also shown in the diagram two curves ($n = 2.2, k_c = 0.057h$ and $n = 2.4, k_c = 0.084hMpc^{-1}$) which overshoots the best fit curve and two curves ($n = 2.2, k_c = 0.025h$ and $n = 2.4, k_c = 0.025hMpc^{-1}$) which undershoots the best fit curve. The result shows that a sharp bend at $k_c = 0.047hMpc^{-1}$ (corresponding to $R_c \simeq 21h^{-1}Mpc$) is needed even to come anywhere close to fitting the data. [Even in this case, the fit is far from good in detail. To illustrate this point we have calculated the angular correlation function, scaled to the Lick depth for the best fit spectra. This is shown in figure 3 along with some representative maximum and minimum points based on APM result. The large errors in the $W(\theta)$ determined by APM survey makes it difficult to judge how good is this fit, but it does not seem to be very satisfactory.] Notice that these spectra are characterized by pure power laws for both large and small k . In contrast, a CDM spectrum *with the same COBE normalization* shows much more curvature. We have shown in figure 4 such a CDM spectrum along with the curves A-E. The CDM spectrum agrees fairly well with the best fit curve at large scales but has a very different shape at small scales. If we normalize the CDM spectrum using the COBE results, we will get an amplitude of $A = 5.3 \times 10^6$. [In contrast, notice that if we normalise the CDM spectrum by setting $\sigma_8 \equiv \sigma(8h^{-1}Mpc) = 1$ we find that $P \simeq A_0 k$ at small k with $A_0 = 4.4 \times 10^6$ for a universe with $\Omega = 1, \Omega_{vac} = 0, h = 0.5$.] This will make $\sigma_{DM}(8h^{-1}Mpc)$ overshoot the value of unity. Thus we would require $b(8h^{-1}Mpc) < 1$ to obtain the correct σ_{gal} . What is more, the shape of the spectrum will be highly curved in the range of $(2 - 20)h^{-1}Mpc$ thereby leading to a $\xi_{gg}(R)$ which is in contradiction with observations. Non linear effects could steepen the small scale spectrum, thereby reducing the curvature; but this will also *increase* the amplitude to still higher values.

3. Conclusions

The above result clearly shows that it is not easy to explain both the large and small scale power in a natural fashion. Before the COBE results, it was generally felt that all that one required was an enhancement of large scale power to bring the theory in consonance with APM results. COBE has added a new dimension to the problem, namely, the shape of the power spectrum. Large scale streaming motions and APM suggest that $\sigma(50h^{-1}Mpc) \simeq 0.2$. The COBE result is consistent with this fact provided the scale invariant spectrum is extended straight down to $50h^{-1}Mpc$. But if this is done, one requires a relatively sharp bend to account for the small scale observations and presumably a new length scale in the theory.

The $\sigma_{gal}(R)$ in the above analysis has been obtained from the raw data in a fairly simple minded way. [For example, there could be a systematic shift between infrared and optical selection of samples; the redshift space distortion can affect the shape of the curve; such effects have not been included in this analysis.] But our basic result seems to be reasonably independent of the *detailed* behaviour of $\sigma_{gal}(R)$; it only depends on the overall trend. We also stress the fact that most of these conclusions can be reliably based on the linear theory and that non-linear effects will only evolve the spectrum in a direction which will make the conclusions even stronger. It may be possible to directly probe the spectrum around $(20h^{-1} - 50h^{-1})Mpc$ by concentrating on MBR anisotropies at $6'$ to 1° scale. Such observations may offer an important clue to the shape of the density spectrum.

After this work was completed we came to know of a preprint (Saunders et al., 1992) which determines the $\sigma_{gal}(R)$ from QDOT-QIGC data. It turns out that QDOT-QIGC data shows evidence for structure almost corresponding to the k_c we derived. Though it is too early to draw any firm conclusions, it is tempting to identify this scale as the length upto which the effects of the interaction between the already formed smaller scale structures are significant. In principle, such an hypothesis can be tested if we study the angular distribution function for a small region in the sky corresponding to objects at a narrow range of redshift at say, around 0.3 or 0.5 and test whether the derived k_c is larger. However, getting reliable distribution function from such a data may be difficult. But if this hypothesis is not totally incorrect, it gives an idea of the new problems in the study of the formation and evolution of large-scale structures (length scales larger than, say, $10h^{-1}Mpc$). Initial small - k perturbations are still evolving linearly when new structures appear to be formed also because of the non-linear interaction between smaller scale features.

While this paper was being processed for publication, several papers have appeared working out the constraints from COBE for galaxy formation scenarios (see eg., Efstathiou et al., 1992; Taylor et al., 1992; Davis et al., 1992). We shall briefly comment on the difference in perspective between this paper and the others. In the present work, we have put together the observations at various scales and attempted to extract from them two effective parameters k_c and n . The spectrum in (8) is not directly related to the spectra in any of the standard theoretical models. In contrast, the papers cited above all deal with specific models for structure formation (CDM models with or without cosmological constant, mixed dark matter models etc.) and attempts to either constrain the models or obtain a 'best-fit' scenario. In some sense, these two approaches are complementary to each other and have

their own advantages and disadvantages. Using a specific class of spectra arising from certain dark matter models has stronger physical motivation than fitting the data to (8). However, such an analysis - by its very nature - is too strongly tied to the models which are being studied. As a consequence, it is somewhat difficult to draw model-independent conclusions which are hiding in the available data. We have attempted to fit the data to a test spectra of the form in (8)- rather than to any class of dark matter models - to see what general conclusions can be drawn from the observations at various scales. In fact, any specific model which is in good agreement with the data will have an *effective* k_c and n which are close to the best fit values we have obtained.

There is another motivation for keeping the analysis model independent at this stage. It is clear, both from the present work and the references cited above, that the simplest models (with a single component dark matter, $\Omega_{vac} = 0, \Omega = 1$) are incapable of accounting for all data *if the bias factor is independent of scale*. Most of the authors cited above attempt to keep the bias factor constant but introduce more complicated combinations of dark matter. However, if the biasing is due to any sensible physical mechanism then it is very likely to be scale dependent. In such a case, there is not much point in comparing a theoretical spectra (however well motivated the model is) with observations at different scales without understanding the various biasing mechanisms which operate at different scales. Unfortunately, our knowledge of biasing processes is quite primitive at present. We, therefore, feel that it is indeed worthwhile not to tie oneself down to specific theoretical models but concentrate on more general conclusions which can be drawn from the data. In that sense, any theoretical model which - after the inclusion of biasing effects - leads to an effective k_c and n close to the best fit values will be acceptable. Only when the biasing effects are sufficiently well understood, we will be able to really constrain the theoretical models. We plan to investigate the evolution of these parameters due to various biasing mechanisms in different models in a future publication.

Acknowledgments

We thank Michel Rowan-Robinson for critical comments on the manuscript and useful discussions. We have benefited from discussions with Ramesh Sinha and K Subramaniam. D.N. is grateful to I. W. Roxburgh, Director of Astronomy Unit, Queen Mary College for a Visiting Research Fellowship supported by the Science and Engineering Research Council of U.K.

REFERENCES

- Bertsinger, E. et al, 1990, Ap J **364**, 370.
Davis, M. et al., 1992, Nature, **359**,393.
Efstathiou, G.P et al., 1992, MNRAS, **258**,1P.
Hamilton, A.J.S. et al., 1991, Ap. J. **374**, L1.
Huchra, J. et al., 1983, Ap. J. Supp. **52**, 89.
Kashlinsky, A. and Jones, BJT., 1991, Nature, **349**, 753.
Maddox, S. et al., 1990, MNRAS, **242**, 43P; **243**, 692.
Padmanabhan, T., 1991, invited talk at the Annual meeting of the Indian Academy of Sciences (8-11 Nov); to be published.

Padmanabhan, T., 1992, *Structure formation in the universe*, Cambridge University Press (in press).

Peacock, J.A., 1991, MNRAS, **253**, 1P.

Peebles, P.J.E., 1980. *The Large Scale Structure of the Universe*, Princeton University Press.

Peebles, P.J.E., 1982, Ap. J. Lett. **263**, L1.

Strauss, M.A. et al., 1990, Ap. J. **361**, 49.

Smoot, G.F. et al., (1992) Ap.J., **396**, L1-L6.

Saunders, W et al., (1992) OUAST/92/4 preprint.

Taylor, A.N. and Rowan-Robinson, M, 1992, Nature **359**, 396.

Wright, E.L. et al., (1992) Ap.J., **396**, L13-L18.

Figure Captions

- Figure 1: Various constraints on the *rms* fluctuations in mass, $\sigma(R) = \langle (\delta M/M)_R^2 \rangle^{1/2}$, are plotted against the scale R . Within a factor of order unity, this $\sigma^2(R)$ also represents the power in each octave, $[k^3 P(k)/2\pi^2]$, at $k = R^{-1}$. At the right-bottom, we have marked the result from COBE based on $(\Delta T/T)_{rms}$ assuming a spectrum which is scale invariant at large scales. The $\sigma(R)$ determined from CfA, IRAS and APM surveys are shown by different symbols. In the case of APM, $W(\theta)$ was inverted to give $\xi(r)$. The lower bound on $\sigma(R)$ - marked 'high- z objects' - arises from requiring that mass scales in the range $(10^{10} - 10^{11})M_\odot$ should go nonlinear by $z = 4.5$. The bound marked 'large scale streaming' arises from the POTENT reconstruction of velocity fields. We have also shown the (rather weak) upperbound from small angle anisotropies in MBR in the reionised models and the slope of the galaxy-galaxy correlation function. ($h = 0.5$)
- Figure 2: The results of galaxy surveys and COBE anisotropy are fitted using simple test spectra of the form $P(k) = Ak[1 + (k/k_c)^n]^{-1}$. All the spectra are normalised using the maximum permitted value of a_2 from COBE, viz. $a_2 = 10^{-5}$. There is reasonable agreement for $n = 2.4$, $k_c = 0.047h$ [The fit can be marginally improved by choosing $n = 2.41$ and $k_c = 0.047h$]. This suggests a length scale of $k_c^{-1} = 21h^{-1}Mpc$ in the theory. We have set $h = 0.5$.
- Figure 3: The angular correlation function $W(\theta)$, computed for the best fit spectrum $n = 2.4$, $k_c = 0.047h$ is compared with the $W(\theta)$ from APM survey. For simplicity, we have only indicated the maximum and minimum points allowed by the APM survey for a representative sample of θ values. All results are scaled to Lick depth and $h = 0.5$.
- Figure 4: The model spectra in figure 2 are shown in an enlarged scale and compared with CDM (dash-dot-dot curve): All curves are normalised by COBE result. Note that CDM spectra shows much more significant curvature compared to the model spectra.

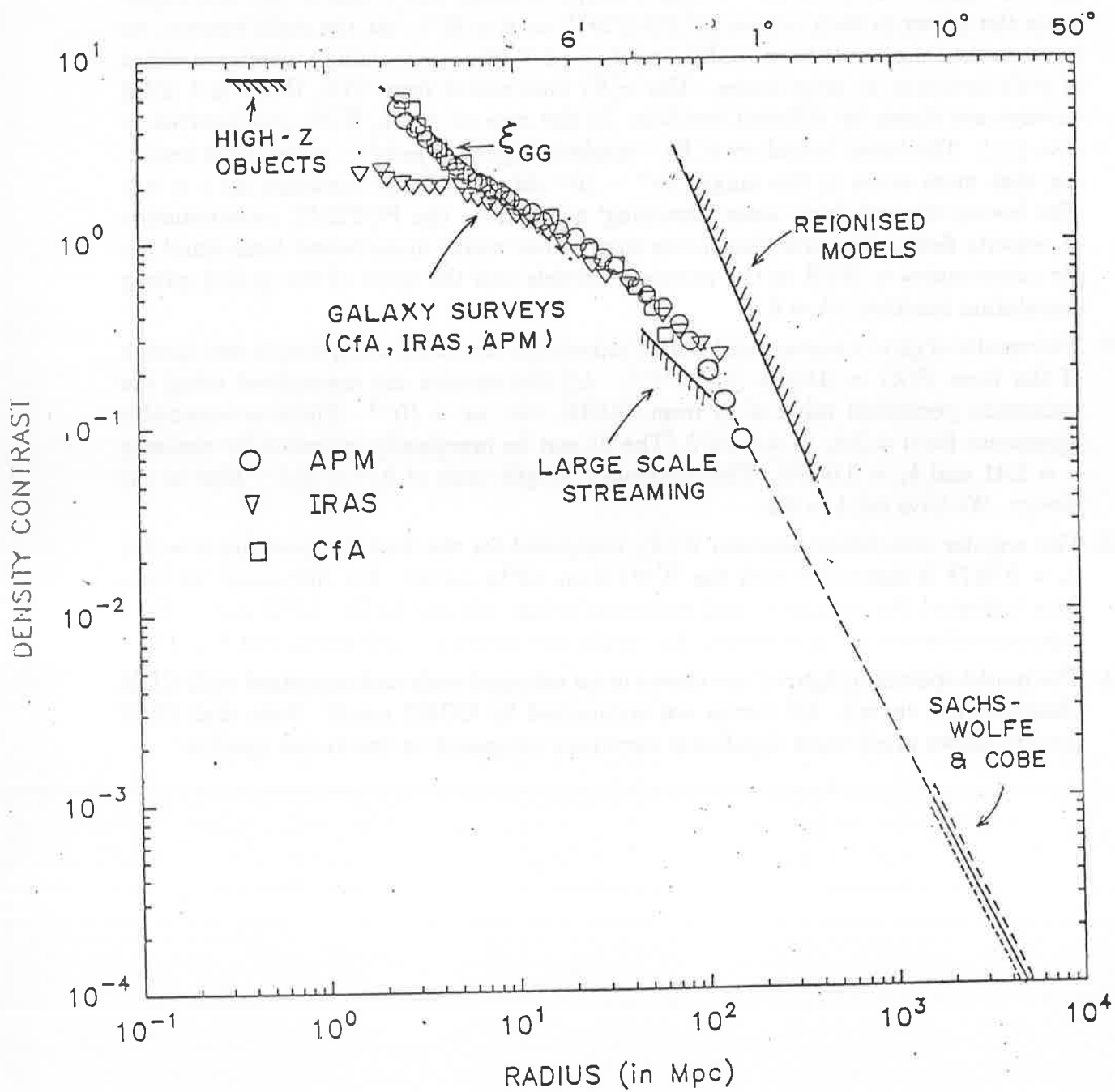


Fig 2

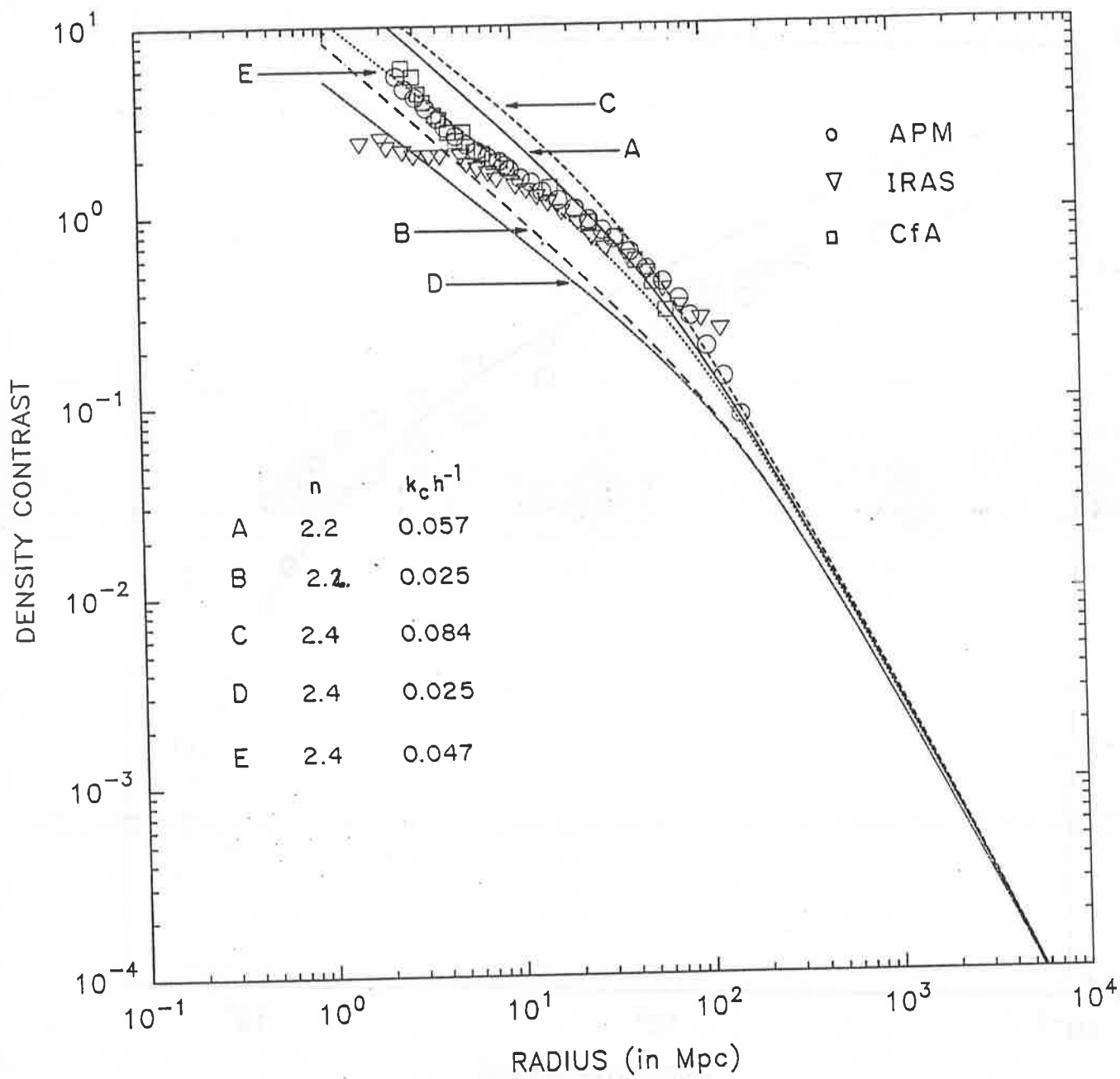


Fig 2

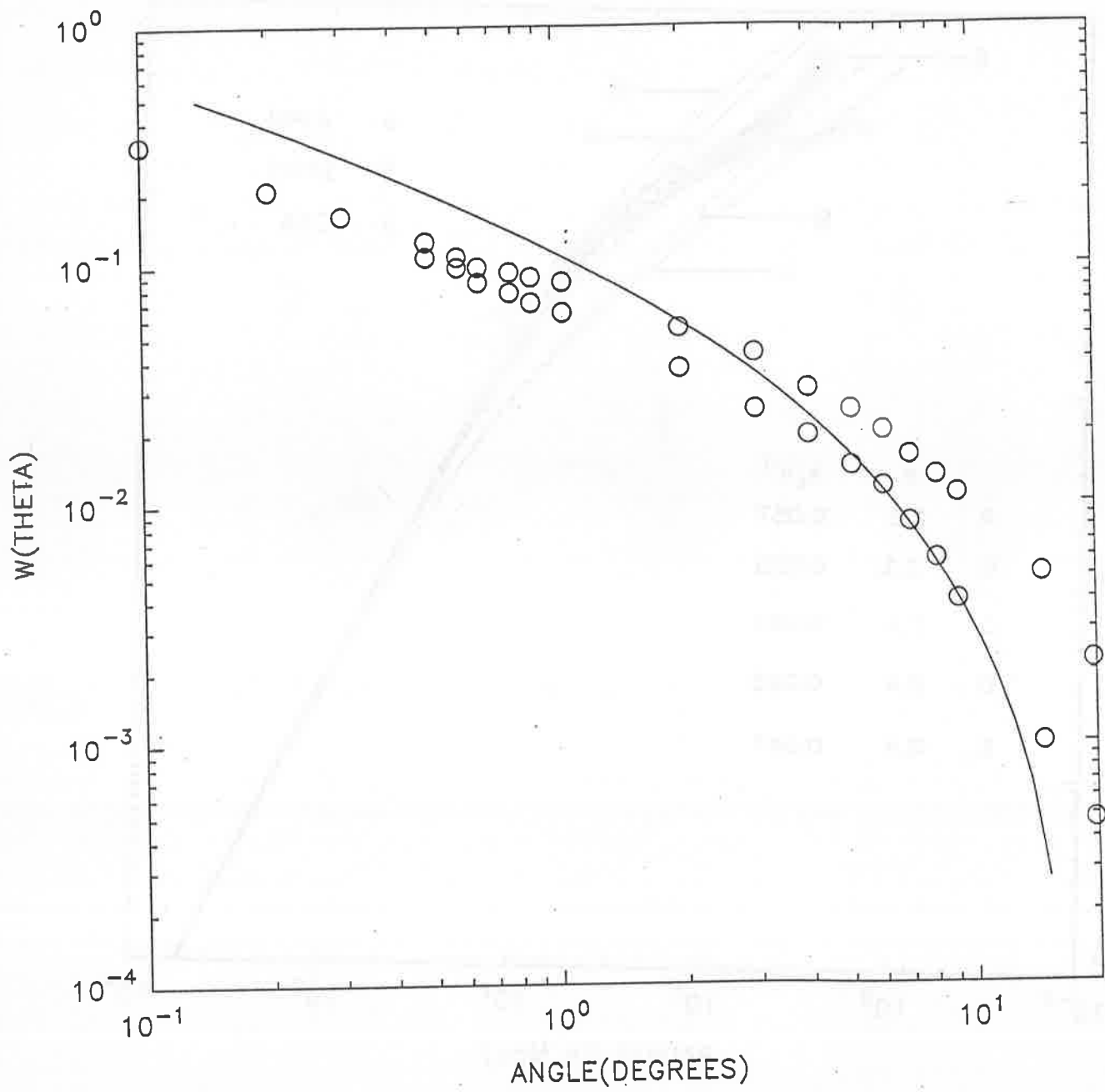


Fig. 3

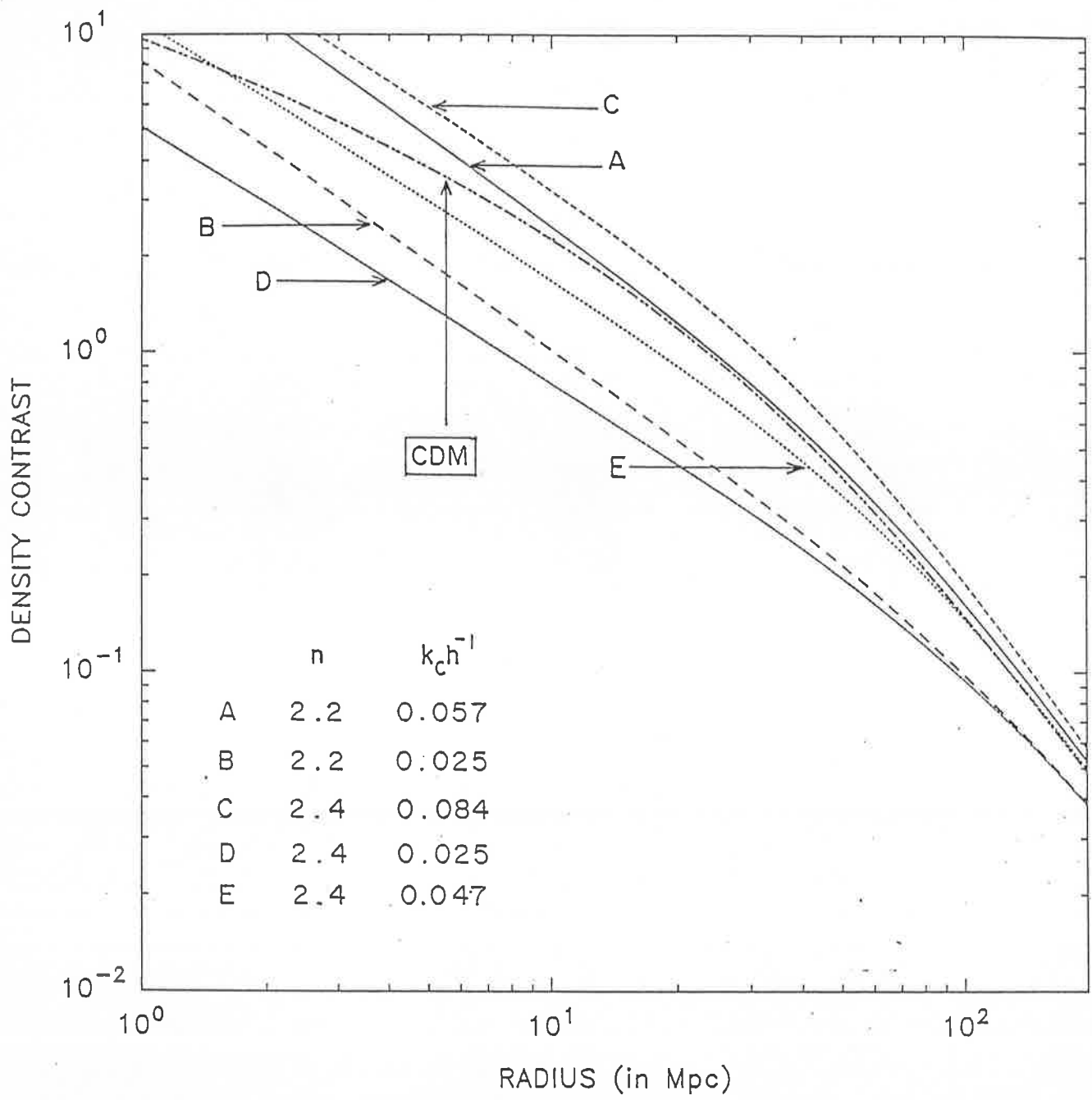


Fig. 4

