Chandra, Newton and the *Principia*

Jayant V. Narlikar

Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune 411 007, India

IN this special section dedicated to Chandra, experts have tried to highlight his multi-faceted contributions. This short article is devoted to his last love, namely Isaac Newton, especially the three-volume work, the *Principia* by Newton. True to his style Chandra worked hard on the *Principia* for several years in order to write a commentary on it. This he wrote in the form of a book which was published by the Oxford University Press only a few weeks before his death. Following are the thoughts that came to my mind on perusing this last book by Chandra.

Newton's *Principia* for the common reader

In the humanities, commentaries are written on classic texts. A few commentaries and the commentators in due course themselves become classic. Thus Mallinatha's commentaries on the texts of Kalidasa are read avidly by scholars who wish to derive the maximum enjoyment and appreciation of the poet's work.

This, however, is rare in science. So when a distinguished scientist with several accolades including the Nobel Prize, writes on the *Principia*, a scientific classic

authored by no less a person than Isaac Newton, and its title states that it is for the 'common reader' great expectations are roused. Even more so with Chandra, whose record of writing comprehensive scientific monographs was legendary.

For, Chandra did pathbreaking work in several parts of theoretical astrophysics and general relativity. He would work at a terrific pace in his chosen field regardless of whether it was a popular one amongst his fellow scientists. At the end of a decade's (or so) work he would come out with an authoritative monograph that was treated as the 'last word' on the subject. As an author he would make no compromises with exactness and his works were directed at researchers in the field. It was therefore something of a surprise that his last work *Newton's Principia for the Common Reader* was an altogether different type of exercise.

A few years prior to the publication of the book, Chandra had started giving lectures on his appreciation of Isaac Newton's contributions as depicted in the *Principia*. When he visited the IUCAA for the Centre's dedication ceremonies in December 1992, he delivered a talk entitled, 'Newton's *Principia*: Its relevance for a student of today'. This would, perhaps, have been a more appropriate title for this book.

For, the 'common reader' will be disappointed with the fare served to him. If after reading (or having on his shelf) Stephen Hawking's much publicized *A Brief History of Time*, he has persuaded himself that he can understand science and appreciate Newton's work as described here he will be in for disillusionment. For, to get the maximum benefit out of this book he would need to have a knowledge of dynamics, calculus and geometry at the undergraduate level! Only with such accomplishments can he dare take a plunge into the myriad of figures, propositions and problems that make up this book.

A brief history of the *Principia*

To get the historical perspective right, Isaac Newton wrote his magnum opus entitled *Philosophiae Naturalis Principia Mathematica* (The Mathematical Principles of Natural Philosophy) in three parts during 1684–86. In this volume he described his work on the law of gravitation, dynamics as applied to planetary motion, optics and also many other investigations. He also used calculus, both integral and differential, but in order to make his work more readable (and more credible) he resorted to the then familiar geometrical proofs in preference to the new methods of calculus that he himself had invented.

Newton had a taciturn nature. He preferred to work alone. We do not know of any student or co-worker of Newton, let alone of any school established by him. But his solo achievements far surpassed several schools put together. But for them to be made widely known to the contemporary scientists, it was necessary to have them published as a book. Left to himself, it is doubtful if Newton would have taken upon himself the writing of *Principia*. The credit for motivating him and pushing him towards this task goes to one of his few friends, Edmund Halley (well known to the common man for 'his' comet!).

The motivation

Principia as the book is called in short had been the guiding light for mathematicians and physicists for over a century. But for the modern reader the language (even though translated to English from Latin) and style of arguments are archaic. I myself recall opening the book in several places just for selective reading, then putting it down. Yet, as Chandra said in his lecture and mentions in the opening parts of his book, he undertook the task of explaining and elaborating on Newton's work in the modern framework so that 'With the impediments of language and syntax thus eliminated, the physical insight and mathematical craftsmanship that invariably illuminate Newton's proofs come sharply into focus . . .'.

It is certainly a very worthwhile attempt to look at the classic from a modern standpoint, armed with the modern techniques of maths that were unfamiliar to the Newtonians. Professor Chandrasekhar goes through select portions of the *Principia*, describing in places how Newton argued and then explaining it in modern jargon. Thus what to a common reader may appear obscure and antiquated arguments, turn out to be examples of ingenious geometrical constructions and brilliant reasoning. The author quotes the Cambridge mathematician Professor Littlewood at one place: ' . . . (this construction) must have left its readers in helpless wonder'.

Chandra himself admits that when he worked through the various propositions of the *Principia* he did so without looking at Newton's solutions. Only subsequently when he compared his methods with Newton's did he realize how far superior they were compared to his own. Coming from one whom modern theoreticians held in awe as an expert in mathematical techniques this is high praise indeed! The book is motivated by the desire to share with others the author's awe of Newton's intellect and dexterity in identifying and solving problems posed by nature.

The contents

Without going into technical details the following is the fare served in this feast on (or, more correctly, by) Newton. Chandra obviously could not and did not wish to cover the entire canvas painted by Newton. Thus,

some propositions are extensively discussed, others briefly referred to and some not at all. In making a selection, he has largely restricted his attention to those parts that deal with the law of gravitation and its dynamical consequences. Thus, 24 out of 28 chapters deal with such topics. Starting from basic concepts and axioms on laws of motion, followed by ideas of limit and continuity and notions of calculus, Kepler's laws and their derivation from the law of gravitation follow. There are discussions of celestial mechanics, lunar theory, three-body problem, etc. and then what Chandra calls 'superb theorems' on gravitational forces produced by spherical bodies. (The adjective 'superb', as Chandra explains was actually taken over by him from J. W. L. Glaisher.)

Next come portions of Book III, which deal with wider applications of the law of gravitation, including figures of the Earth and planets, generation of tides, more of lunar theory, the precession of equinoxes and comets. This menu is then supplemented by a few miscellaneous topics like air drag on falling bodies, the solid of least resistance, the velocity of sound and of long waves in canals and the problem of brachistochrone. The reader may have heard of this last episode wherein Newton solved a challenge problem set by Johann Bernoulli to mathematicians of Europe and Britain. It is good to see a detailed and authentic version of the story along with several solutions (including Newton's) of the brachistochrone problem. I strongly recommend the student to read this chapter.

The reaction

Chandra indeed succeeds in sharing with his readers, his excitement at the elegance of geometrical proofs in the *Principia*. I myself recall how pleased I used to be when finding the proof of a problem by geometry as compared to by algebra or calculus. There is something aesthetically pleasing about intrinsic methods with analytical reasoning as opposed to brute force number crunching. This fact is amply demonstrated for the *discerning* and *capable* reader in this book. The adjectives in italics are important: and in this sense Chandra's 'common' reader will have to be rather uncommon by normal standards.

Chandra himself in his other books laid stress on analytical approaches as opposed to abdication to the computer. His book on stellar structure, written in the era when computers had not invaded and taken over the subject, is a delight to read for its mathematical derivations using differential equations, the integral calculus, special functions and so on. While these methods have proved useful in understanding the basic physics of the astronomical problem, the real world, alas, cannot be adequately described by these methods; which is why the area of stellar structure and evolution had ultimately to be dealt with by the brute force of computer programming.

The same could be said about algebra and calculus *vis-à-vis* geometry as applied to dynamics or to physics in general. Newton's method of fluxions, later to be called *Calculus*, despite being unfamiliar, had indeed come to stay and was to become a major tool for theoretical physics. Although Newton chose the medium of geometry in preference to calculus so as to be more understandable and credible to his contemporaries, he must have seen the writing on the wall. That is why to the student of today the proofs of propositions in the *Principia* appear quaint and unfamiliar, requiring a Chandra to explain and elucidate them.

Having said all this, one cannot help remarking that this last of Chandra's works does not measure up to the high standard he himself set through his earlier works. Whether it was *An Introduction to the Theory of Stellar Structure*, or his *Mathematical Theory of Black Holes* there was an aura of perfection about them. The reader was assured that whatever was worked out was meticulously checked and all conclusions were backed up by fully worked-out analytical arguments. Here in this book there are glaring examples of numerical errors and also some errors of reasoning. An example of the former is the error in calculating the time period of a simple pendulum. Of the latter is a mix-up right at the beginning (pages 4–5) when calculating the acceleration of the Moon.

On some important aspects the author's commentary would have been welcome, but alas it is left out. An example is the *Scholium* in which Newton discusses the well-known bucket experiment with the introduction of the concept of inertial forces. This topic has been and still is debated by physicists and philosophers with earlier inputs from Bishop Berkeley, Ernst Mach and Albert Einstein. The celebrated *Mach's Principle*, which arose out of Mach's discussions in the 1890s, is related to this discussion in Newton's *Principia* but the common reader here does not encounter it at all.

A modern scholar will miss the index at the end and a bibliography. Surely it would have been of great use to be able to locate a topic in the text and to know about works of others in the field.

As I mentioned, Chandra passed away within a few weeks of the publication of this book. At 85 he was more active as a scientist than many half his age. It is a pity that he is not alive to respond to constructive criticism which would surely have made a revised edition far better. Even as it is, the 'common' reader interested in knowing how the subject of mechanics evolved from a great mind, will find this re-presentation of Newton's outstanding contributions to the subject most rewarding to read.