

Gravitational wave astronomy - astronomy of the 21st century

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Abstract. An enigmatic prediction of Einstein's general theory of relativity are gravitational waves. With the observed decay in the orbit of the Hulse-Taylor binary pulsar agreeing within a fraction of a percent with the theoretically computed decay from Einstein's theory the existence of gravitational waves was firmly established. Currently there is a worldwide effort to detect gravitational waves with interferometric gravitational wave observatories or detectors and several such detectors have been built or being built. The initial detectors have reached their design sensitivities and now the effort is on to construct advanced detectors which are expected to detect gravitational waves from astrophysical sources. The era of Gravitational Wave Astronomy has arrived. This article describes the worldwide effort which includes the effort on the Indian front - the IndIGO project -, the principle underlying interferometric detectors both on ground and in space, the principal noise sources that plague such detectors, the astrophysical sources of gravitational waves that one expects to detect by these detectors and some glimpse of the data analysis methods involved in extracting the very weak gravitational wave signals from detector noise.

Keywords :

Gravitational waves, blackholes, binary stars, laser interferometric detectors

1. Introduction

In the past half a century or so, astronomy has been revolutionised by several unexpected discoveries because of the plethora of windows being opened in various bands of the electromagnetic spectrum. To name a few, the cosmic microwave background and the discovery of pulsars in radio band, gamma-ray bursts, X-ray objects, all go to show that whenever a new window has been

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opened, startling discoveries have followed. Yet another window to the universe should soon open up in few years time - the gravitational wave (GW) window. Not only will this window test Einstein's general theory of relativity, but also provide direct evidence for blackholes, more generally test general relativity in the strong field regime. Just as the other windows to the universe have brought in unexpected discoveries, it is not unreasonable to expect the same in this case also - and more so, because this involves changing the physical interaction from electromagnetic to gravitational.

The existence of gravitational waves predicted by the theory of general relativity, has long been verified 'indirectly' through the observations of Hulse and Taylor [1, 2]. The inspiral of the members of the binary pulsar system named after them has been successfully accounted for in terms of the back-reaction due to the radiated gravitational waves - the observational results and the theory agree with each other within a fraction of a percent. However, detecting such waves directly with the help of detectors based either on ground or in space has not been possible so far.

The key to gravitational wave detection is the very precise measurement of small changes in distance. For laser interferometers, this is the distance between pairs of mirrors hanging at either end of two long, mutually perpendicular vacuum chambers. A GW passing through the instrument will shorten one arm while lengthening the other. By using an interferometric design, the relative change in length of the two arms can be measured, thus signalling the passage of a GW at the detector site. GW detectors produce an enormous volume of output consisting mainly of noise from a host of sources both environmental and intrinsic. Buried in this noise will be the GW signature. Sophisticated data analysis techniques are needed to optimally extract the GW signal from the interferometric data. IUCAA has made significant contributions in this area.

2. Interferometric detection of GW

Historically with pioneering efforts of Weber in the 1960s, the detectors were resonant bar detectors which were suspended seismically isolated aluminium cylinders. The later versions were cooled to extremely low temperatures - ultracryogenic - to suppress the thermal noise. There are also spherical resonant mass detectors being constructed/operating. However, these ideas although interesting and useful in their niche, have their limitations. In this article we will confine ourselves to the interferometric detectors.

2.1 The principle of interferometric detection

A weak GW is described by a metric perturbation $h_{\mu\nu}$ in general relativity. Typically, for the astrophysical GW sources which are amenable to detection, $h_{\mu\nu} \sim 10^{-22}$. In the transverse-traceless gauge, the $h_{\mu\nu}$ can be expressed in terms of just two amplitudes, h_+ and h_\times , called the 'plus' and 'cross' polarisations. If a weak monochromatic gravitational wave of + polarisation is incident on a ring of test-particles, the ring is deformed into an ellipse as shown at the top of Fig.1. Phases, quarter cycle apart, of the GW are shown in the figure. For the \times polarisation

the ellipses are rotated by an angle of 45° . A general wave is a linear combination of the two polarisations.

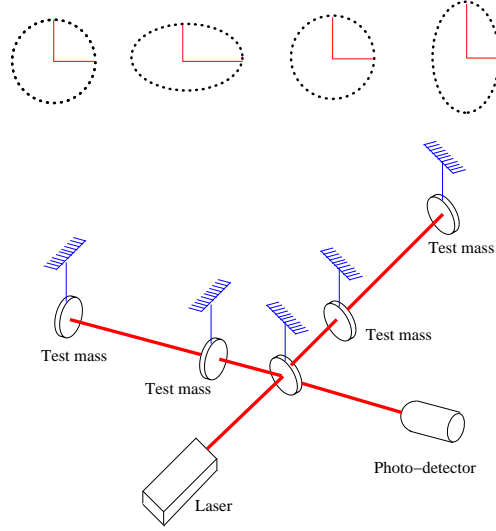


Figure 1. A circular ring test particles is deformed into an ellipse by an incident GW. Phases, quarter of a cycle apart are shown for the + polarisation. The length change in the interferometric arms is also shown schematically. Below, a schematic diagram of an interferometer is drawn.

At the bottom of Fig.1 a schematic of the interferometer is depicted. If the change in the armlength L is δL , then,

$$\delta L \sim hL, \quad (1)$$

where h is a typical component of the metric perturbation.

For a GW source, h can be estimated from the well-known Landau-Lifschitz quadrupole formula. The GW amplitude h is related to the second time derivative of the quadrupole moment (which has dimensions of energy) of the source:

$$h \sim \frac{4}{r} \frac{G}{c^4} E_{\text{nonspherical}}^{\text{kinetic}}, \quad (2)$$

where r is the distance to the source, G is the gravitational constant, c is the speed of light and $E_{\text{nonspherical}}^{\text{kinetic}}$ is the kinetic energy in the *nonspherical* motion of the source. If we consider $E_{\text{nonspherical}}^{\text{kinetic}}/c^2$ a fraction of a solar mass and the distance to the source ranging from galactic scale of tens of kpc to cosmological distances of Gpc, then h ranges from 10^{-17} to 10^{-22} . These numbers then set the scale for the sensitivities at which the detectors must operate. The factor of 4 is also important given the weakness of the interaction and the subsequent signal extraction from detector noise.

2.2 Ground-based interferometric detectors

There are a host of noise sources in ground based interferometric detectors which contaminate the data. At low frequencies there is the seismic noise. The seismic isolation is a sequence of stages consisting of springs/pendulums and heavy masses. Each stage has a low resonant frequency about a fraction of a Hz. The seismic isolation acts as a low pass filter, attenuating high frequencies, but low frequencies get through. This results in a 'noise wall' at low frequencies and marks the lower end of the detector bandwidth. It is about 40 Hz for initial detectors but will go down to 10 Hz for advanced detectors increasing the bandwidth. At mid-frequencies upto few hundred Hz, the thermal noise is important and is due to the thermal excitations both in the test masses - the mirrors - as well as the seismic suspensions. Currently, this seems to be the noise hardest to suppress. The natural modes of the mirrors and the suspension are driven by the thermal excitations. One 'solution' is to cool the mirrors/suspensions, but this has its own problems. Nevertheless, the Japanese have planned a detector doing just this - the Large-scale Cryogenic Gravitational-wave Telescope (LCGT) which has been funded recently. At high frequencies the shot noise from the laser dominates. This noise is due to the quantum nature of light. From photon counting statistics and the uncertainty principle, the phase fluctuation is inversely proportional to the square root of the mean number of photons arriving during a period of the wave. So increasing the laser power and hence the mean number of photons during a given period of the wave tends to reduce this noise. Apart from these main noise sources there are other noise sources, the important among them is the gravity gradient noise which cannot be screened and occurs only at low frequencies - these are slowly changing gravity gradients due to natural causes (such as clouds moving in the sky, changes in atmospheric density) or manmade. Thus long arm lengths, high laser power, and extremely well-controlled laser stability are essential to reach the requisite sensitivity. The figure 2 shows the sensitivity achieved by the initial LIGO detectors by March 2007 when the actual noise in the detectors reached theoretical design sensitivity (shown by the bold curve). The sensitivity has continued to improve with time.

2.3 The worldwide network of ground-based interferometric gravitational wave observatories

US has been at the forefront in building large scale detectors. The LIGO project [3] has built three detectors, two of armlength 4 km and one of armlength 2 km at two sites about 3000 km apart at Hanford, Washington and at Livingston, Louisiana. The 2 km detector is at Hanford. These initial detectors have had several science runs and the design sensitivity has not only been reached but surpassed. The goal of this initial stage was to mainly vindicate the technologies involved in attaining the design sensitivities. Now the next phase is to build advanced detectors with state of the art technologies which will be capable of observing GW sources and doing GW astronomy. With these future goals a radical decision has been taken by the LIGO project, that of building one of its detectors in Australia - that is LIGO will build two advanced detectors in US and partially fund a full scale detector in Australia with advanced design. This detector is called LIGO-Australia and will be built in collaboration with the Australians who already have a

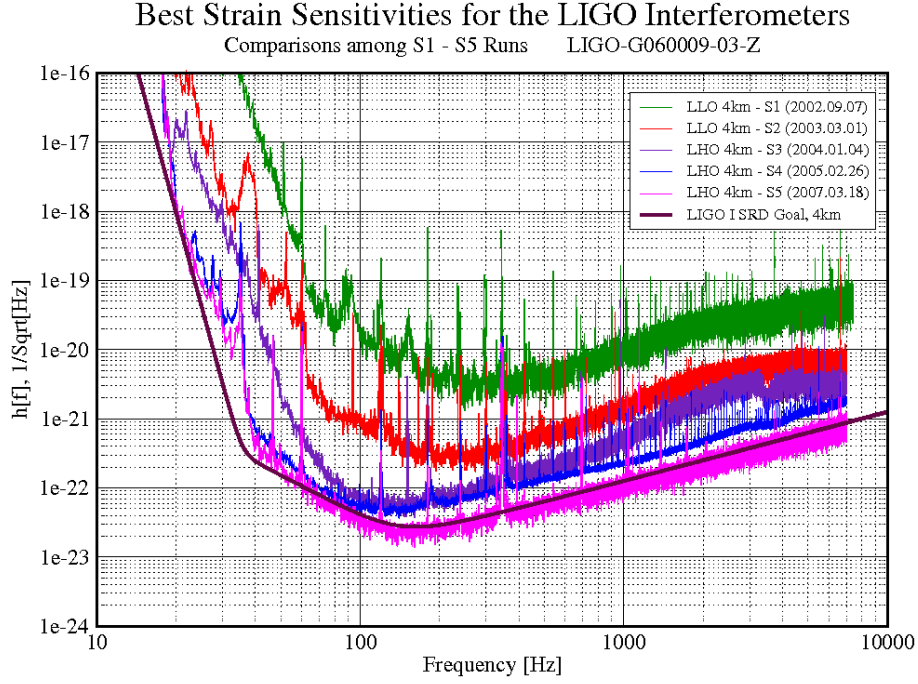


Figure 2. The figure shows the sensitivity achieved by LIGO detectors by March 2007. This sensitivity level has been surpassed in later operations. Courtesy LIGO website.

interferometric facility at Gingin near Perth - the AIGO (Australian Interferometric Gravitational-wave Observatory) project. The reason for this decision by the US is clear - it is to increase the baseline and have a detector as far removed from other detectors on Earth, which has several advantages, such as improving the pointing accuracy to the GW source.

In Europe the large scale project is the VIRGO project [4] of Italy and France which has built a 3 km armlength detector. After commissioning of the project in 2007, it had also science runs. The GEO600 [5] is a German-British project and whose detector has been built near Hannover, Germany at Saxony with armlength 600 metres. The idea of GEO600 is to develop advanced technologies required for the next generation detectors with the goal of achieving higher sensitivity.

Japan was the first (around 2000) to have a large scale detector of 300 m armlength - the TAMA300 detector under the TAMA project [6] - operating continuously at high sensitivity in the range of $h \sim 10^{-20}$. Now Japan plans to construct a cryogenic interferometric detector called the LCGT (Large Scale Gravitationalwave Telescope) [7] which has been recently funded. The purpose of the cryogenics is to quell the thermal noise. But this technology is by no means straight forward and will test the skills of the experimenters.

Because of LIGO-Australia, Australia in turn is looking for international partners. Given the twenty year old legacy in GW data analysis at IUCAA, Pune and waveform modelling at RRI, Bangalore, Australia would welcome the Indians as partners in this endeavour. Recently, about two years ago an Indian Initiative in Gravitational Wave Astronomy (IndIGO) has begun whose goal is to promote and foster Gravitational Wave Astronomy in India and join in the worldwide quest of observing gravitational waves. Apart from the data analysis this initiative includes the all important experimental aspect. Accordingly a modest beginning has been made by IndIGO with TIFR, Mumbai approving a 3 metre prototype on which Indian experimenters can get first hand experience and develop expert manpower. This project has already been funded. Concurrently, an MOU with Australia has been signed which purports to ask for funding from Indian agencies parallelly with Australia. An IndIGO consortium has been formed with scientists from leading insitutions such as TIFR, RRCAT, RRI, IUCAA, IISERs, Delhi University, CMI etc. and also including scientists (mainly Indian) working abroad. The current strength of the consortium is about 25 scientists. In order to further this effort the first goal is to muster up sufficient expert and skilled manpower which will launch this activity. It will mean India getting into this worldwide challenging experiment.

Besides the current projects, studies have begun for third generation detectors which will include further advanced technologies to enhance the sensitivities of GW detectors to reach out farther in the sky; the Einstein Telescope (ET) is just such a future goal.

2.4 Space-based detectors: The LISA project

A natural limit occurs on decreasing the lower frequency cut-off of 10 Hz because it is not practical to increase the arm-lengths on ground and also because of the gravity gradient noise which is difficult to eliminate below 10 Hz. Thus, the ground based interferometers will not be sensitive below the limiting frequency of ~ 10 Hz. But on the other hand, there exist in the cosmos, interesting astrophysical GW sources which emit GW below this frequency such as the galactic binaries, massive and super-massive blackhole binaries etc. If we wish to observe these sources, we need to go to lower frequencies. The solution is to build an interferometer in space, where such noises will be absent and allow the detection of GW in the low frequency regime. **LISA** - *Laser Interferometric Space Antenna* - is a proposed ESA-NASA mission which will use coherent laser beams exchanged between three identical spacecraft forming a giant (almost) equilateral triangle of side 5×10^6 kilometers to observe and detect low frequency cosmic GW [8]. The ground based detectors and LISA complement each other in the observation of GW in an essential way, analogous to the way optical, radio, X-ray, γ -ray etc., observations do for the electromagnetic waves. As these detectors begin to operate, a new era of *Gravitational Astronomy* is on the horizon and a radically different view of the universe is expected to be revealed. There are also further space projects being considered.

LISA consists of three spacecraft, flying five million kilometers apart, in an equilateral triangle. The spacecraft are maintained drag-free by a complex system of accelerometers and micro-propellers. Each spacecraft will carry two optical assemblies that contain the main optics and

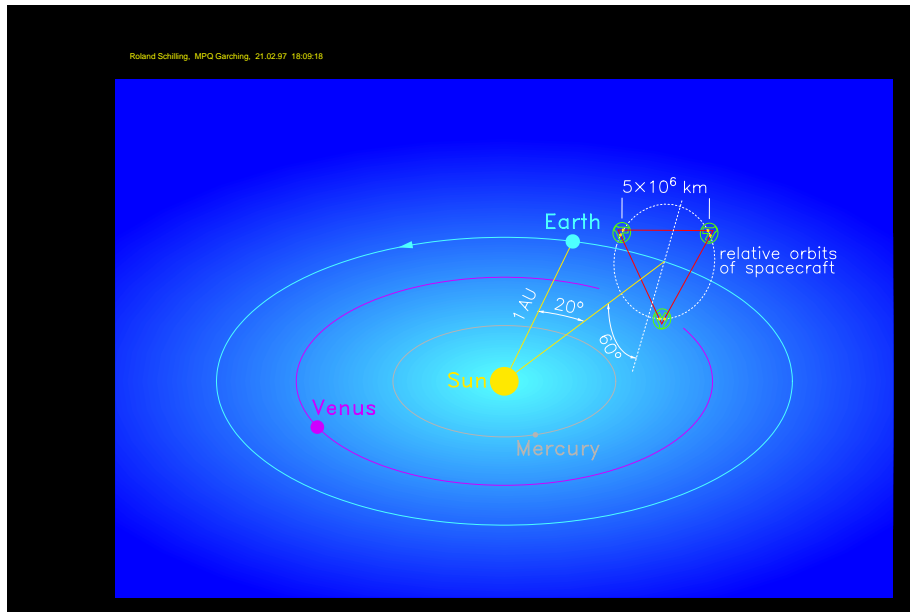


Figure 3. LISA orbital configuration around the Sun, describing a cone with 60° half opening angle. The centroid of the triangle follows an Earth-like orbit trailing 20 degrees behind.

a free-falling inertial sensor. The light sent out by a laser in one spacecraft is received by the telescope on the distant spacecraft. The incoming light from the distant spacecraft is then mixed with the in-house laser and the differential phase is recorded. This defines one elementary data stream. There are thus six elementary data streams which are formed by going clockwise and anti-clockwise around the LISA triangle. Suitable combinations of these elementary data streams can be used to optimally extract the GW signal from the instrumental noise. In other words, LISA is basically a giant Michelson interferometer placed in space, with a third arm added to give independent information on the two gravitational wave polarisations, and for redundancy. The distance between the spacecraft - the interferometer arm-length - determines the frequency range in which LISA can make observations; it was carefully chosen to allow for the observation of most of the interesting sources of gravitational radiation. Each spacecraft revolves in its own heliocentric orbit. The centre of LISA's triangle will follow Earth's orbit around the Sun, trailing 20 degrees behind. It will maintain a distance of 1 AU (astronomical unit) from the Sun, the average distance between the Earth and the Sun (Fig. 3). The spacecraft rotate in a circle drawn through the vertices of the triangle and the LISA constellation as a whole revolves around the Sun. LISA's operational position was chosen as a compromise between the need to minimise the effects on the spacecraft of changes in the Earth's gravitational field and the need to be close enough to the Earth for easy communication.

LISA will observe low frequency GW in the range 0.1 mHz to 0.1 Hz. Since astrophysical systems are generally large and in spite of high velocities do not change their quadrupole moment

too quickly, the universe is richly populated with sources in this frequency band. Also the masses that produce GW in this frequency band are generally large and thus produce stronger GW than those in ground-based detectors, leading to high signal-to-noise ratios (SNR). The signals for LISA arise from a large variety of phenomena, merging massive and supermassive blackholes, vibrating blackholes (quasi-normal modes), stellar mass objects falling into massive and supermassive blackholes, GWs of cosmological origin etc. The high SNRs of these signals imply detailed and accurate information which can test general relativity and its ramifications to unprecedented accuracies. Astrophysics of various objects like compact binaries, stellar remnants can be studied and LISA observations can provide useful clues to events in the early universe [9, 10, 11, 12].

LISA sensitivity is limited by several noise sources. A major noise source is the laser phase (frequency) noise which arises due to phase fluctuations of the master laser. Amongst the important noise sources, laser phase noise is expected to be several orders of magnitude larger than other noises in the instrument. The current stabilisation schemes estimate this noise to about $\Delta\nu/\nu_0 \simeq 3 \times 10^{-14} / \sqrt{\text{Hz}}$, where ν_0 is the frequency of the laser and $\Delta\nu$ the fluctuation in frequency. If the laser frequency noise can be suppressed then the noise floor is determined by the optical-path noise which causes fluctuations in the lengths of optical paths and the residual acceleration of proof masses resulting from imperfect shielding of the drag-free system. The noise floor is then at an effective GW strain sensitivity $h \sim 10^{-21}$ or 10^{-22} . Thus, cancelling the laser frequency noise is vital if LISA is to reach the requisite sensitivity.

In ground based detectors the arms are chosen to be of equal length so that the laser light experiences identical delay in each arm of the interferometer. This arrangement precisely cancels the laser frequency/phase noise at the photodetector. However, in LISA it is impossible to achieve equal distances between spacecraft and also the data are taken at a phasemeter as beat note between the local oscillator and the incoming beam coming from a spacecraft 5 million km away. In LISA, six data streams arise from the exchange of laser beams between the three spacecraft - it is not possible to bounce laser beams between different spacecraft, as is done in ground based detectors. The technique of time-delay interferometry (TDI) is used [13, 14] which combine the recorded data by suitable time-delays corresponding to the three arm-lengths of the giant triangular interferometer. An original approach to this problem was taken by IUCAA. A *systematic method* based on modules over polynomial rings has been successfully formulated which is most appropriate for this problem [15, 16]. The method uses the redundancy in the data to suppress the laser frequency noise.

3. General discussion of GW sources

3.1 GW sources

Several types of GW sources have been envisaged which could be directly observed by Earth-based detectors: (i) Burst sources – such as binary systems consisting of neutron stars and/or black holes in their in-spiral phase or merger phase; supernovae explosions – whose signals last for a

time much shorter, between a few milli-seconds and a few minutes, than the typical observation time; (ii) stochastic backgrounds of radiation, either of primordial or astrophysical origin, and (iii) continuous wave sources – e.g. rapidly rotating non-axisymmetric neutron stars – where a weak sinusoidal signal is continuously emitted. As one sees from the discussion that follows, the strengths of these sources is usually well below or even way below, the mean noise level in the detectors either currently operating or even for those planned in the near future - the advanced detectors. This situation makes the expert data analysis all the more vital, in firstly detecting the source, and secondly and more importantly in extracting astrophysical information from it.

Inspiring binaries have been considered highly promising sources not only because of the enormous GW energy they emit, but also because they are such ‘clean’ systems to model; the inspiral waveform can be computed accurately to several post-Newtonian orders adequate for optimal signal extraction and parameter estimation. The typical strength of the source is:

$$h \sim 2.5 \times 10^{-23} \left(\frac{\mathcal{M}}{M_{\odot}} \right)^{5/3} \left(\frac{f}{100 \text{ Hz}} \right)^{2/3} \left(\frac{r}{100 \text{ Mpc}} \right)^{-1}, \quad (3)$$

where \mathcal{M} is the chirp mass equal to $\mu M^{2/3}$, μ and M being respectively the reduced and the total mass of the system, r is the distance to the source - it is given at the scale of 100 Mpc because such events would be rare and therefore to obtain a reasonable event rate, sufficient volume of the universe needs to be covered - and f is the instantaneous fiducial frequency of the source as the source evolves adiabatically during the inspiral stage. Since the phase of the waveform, apart from the amplitude, can be computed accurately by post-Newtonian methods, the optimal extraction technique of matched filtering is used. In the recent past, numerical relativity has been able to make a breakthrough by continuing the waveform to the merger phase and eventually connect it with the ringdown of the final blackhole. It is here that Chandrasekhar’s contribution stands out as he pointed out that a blackhole rings like a bell if it is subjected to a perturbation [17]. In the current context this occurs in the final stages of the merger when a blackhole is formed. Quasi-normal modes were first discovered by Vishveshwara [18] while examining the stability of the Schwarzschild blackhole.

Another important burst source of GW is the supernova. It is difficult to reliably compute the waveforms for supernovae, because complex physical processes are involved in the collapse and the resulting GW emission. This limits the data analysis and optimal signal extraction.

Continuous wave sources pose one of the most computationally intensive problems in GW data analysis [37, 20, 21]. A rapidly rotating asymmetrical neutron star is a source of continuous gravitational waves. There are some astrophysical systems known from electromagnetic observation which might be promising sources of continuous GWs. Surveys for continuous GWs have so far not led to a direct detection, but the searches are now become astrophysically interesting. We mention the result about the Crab pulsar in the next subsection. These searches for known systems are not computationally intensive since they target a known sky position, frequency and spindown rates. On the other hand, blind all-sky and broad-band searches for previously unknown neutron stars are a different matter altogether. Long integration times, typically of the order of a few months or years are needed to build up sufficient signal power. The reason for this is that the

signal is very weak and lies way below the detector noise level. We give a typical example:

$$h \sim 10^{-25} \left(\frac{I}{10^{45} \text{gm.cm}^2} \right) \left(\frac{f}{1 \text{kHz}} \right)^2 \left(\frac{\epsilon}{10^{-5}} \right) \left(\frac{r}{10 \text{kpc}} \right)^{-1}, \quad (4)$$

where I is the moment of inertia of the neutron star, r the distance to the source and f the GW frequency and ϵ is a measure of asymmetry of the neutron star. The asymmetry of a neutron star can occur in various ways such as crustal deformation, intense magnetic fields not aligned with the rotation axis or the Chandrasekhar-Friedman-Schutz instability [22, 23]. This instability is in fact driven by GW emission and consists of strong hydrodynamic waves in the star's surface layers. This phenomenon results in significant gravitational radiation. Earth's motion Doppler modulates the signal, and this Doppler modulation depends on the direction of the GW source. Thus, coherent extraction of the signal whose direction and frequency is unknown is an impossibly computationally expensive task. The parameter space is very large and a blind survey requires extremely large computational resources.

To detect stochastic background one needs a network of detectors, ideally say two detectors preferably identically oriented and close to one another. The stochastic background arises from a host of unresolved independent GW sources and can be characterised only in terms of its statistical properties. The strength of the source is given by the quantity $\Omega_{\text{GW}}(f)$ which is defined as the energy-density of GW per unit logarithmic frequency interval divided by ρ_{critical} , the energy density required to close the universe. The typical strength of the Fourier component of the GW strain for the frequency bandwidth $\Delta f = f$ is:

$$\tilde{h}(f) \sim 10^{-26} \left(\frac{\Omega_{\text{GW}}}{10^{-12}} \right) \left(\frac{f}{10 \text{Hz}} \right)^{-3/2} \text{Hz}^{-1/2}, \quad (5)$$

The signal is extracted by cross-correlating the outputs. Two kinds of data-analysis methods have been proposed (i) full sky search - but this drastically limits the bandwidth [24], (ii) the radiometric search in which the sky is scanned pixel by pixel - since a small part of the sky is searched at a time, it allows for larger bandwidth, and more importantly includes the bandwidth in which the current detectors are most sensitive, thus potentially leading to a large SNR [25]. Moreover, by this method a detailed map of the sky is obtained.

Apart from these sources, there can be burst sources of GW from mergers or explosions or collapses which may or may not be seen electromagnetically but nevertheless deserve attention. In this case time-frequency methods are the appropriate methods which look for excess power in a given time-frequency box.

In the section on data analysis, we will focus on two of the above mentioned and prominent GW sources, namely, the compact binaries and the continuous wave sources. Before moving on to the data analysis we would like to briefly describe the astrophysically interesting results so far obtained in GW astronomy.

3.2 Astrophysical results from current GW data

Even in this initial stage of the detectors, it is important to note that astrophysically interesting results have been obtained from the data so far taken from the LIGO detectors, more specifically, the data from the S5 run. The data have set astrophysically interesting upper limits on the GW emanating from these astrophysical sources. We mention few of the important results below.

The S5 data have constrained the cosmological GW background in which the upper limit surpasses the previous upper limit set by nucleosynthesis [26]. This result has excluded several string theory motivated big bang models.

The GW data analysis from the S5 run shows that less than 4% of the energy can be radiated away in GW from the Crab pulsar [27]. This is because the spindown rate is $\sim 3.7 \times 10^{-10}$ Hz/sec, while no GW signal was observed even as low as $h \sim 2.7 \times 10^{-25}$.

Since no GW signal was detected from the GRB source 0702012, this implies that a compact binary progenitor with masses in the range $1M_{\odot} < m_1 < 3M_{\odot}$ and $1M_{\odot} < m_2 < 40M_{\odot}$ located in M 31 is excluded as a GW source with 99% confidence. If the binary progenitor was not in M 31, then it rules out a binary star merger progenitor upto a distance of 3.5 Mpc, with 90% confidence and assuming random orientation [28].

These are some of the salient astrophysical results which merely serve to indicate the revolutionary scientific impact that GW astronomy can bring to science.

4. Data analysis of GW sources

As can be seen from the foregoing, data analysis of interferometric data is a very important aspect in the quest for detection of gravitational waves. This is because the signal is weak and must be extracted from the noisy data - in fact the noise, in general, strongly overwhelms the signal. Thus sophisticated statistical techniques, efficient algorithms based on statistical analysis are vital for extracting the signal from the noise. The data analysis technique of course depends on the nature of the signal. We would like to discuss a couple sources and their data analysis in more detail. We first describe the matched filtering paradigm for the inspiraling binaries and then describe some current efforts in the so called 'All sky all frequency search' for GW from periodic or continuous wave sources, which are based on group theoretic methods. This does not mean that the sources not discussed here are unimportant in any way, but the idea here is to give a flavour of the data analysis methods employed in GW detection. Here we have chosen two such data analysis problems.

In this article we would like to emphasise the importance of the role of symmetries which play a crucial part in increasing the efficiency of an algorithm and in turn reducing the computational burden. The symmetries arise from the physical model of the GW source. The idea is to capture

the symmetries in terms of group representation theory and then use the representation theory to develop efficient search algorithms.

4.1 Inspiring/coalescing binaries

Here we will deal essentially with the inspiral waveform which is the first stage when the stars are relatively far apart, and the stage ends a little before the last stable orbit is reached. The last stable circular orbit for a test particle orbiting a Schwarzschild blackhole of mass M is at radial distance of $6MG/c^2$. Here we may take M to be the total mass of the binary components, and then the inspiral stage is the one before the orbit shrinks to around $10M$ or a little less. After the inspiral stage, comes the merger stage and the final stage is that when a single blackhole is formed (in the case the masses are two blackholes) and just before the final blackhole is formed it oscillates, emitting quasi-normal mode radiation finally settling into a stable configuration of a stationary blackhole. The merger waveform for blackholes can now be computed from numerical relativity; there was a recent breakthrough in 2007 [30] and this was followed by several groups actually implementing their numerical code [31, 32]. The inspiral waveform we will consider also holds for two neutron stars or a neutron star/blackhole pair. Here we will restrict ourselves to the binary inspiral and its data analysis.

4.1.1 Matched filtering

The appropriate technique to use, when one has the accurate knowledge of the waveform - especially of the phase - is matched filtering. First, it yields the maximum signal-to-noise (SNR) among all linear filters. Secondly, it is optimal in the Neyman-Pearson sense - in additive Gaussian noise, the matched filter statistic gives the maximum detection probability for a given false alarm rate. The matched filtering operation is defined as follows: if $x(t)$ is the data in the time domain, then the matched filter output $c(\tau)$ at the epoch τ is given by:

$$c(\tau) = \int x(t)q(t + \tau)dt, \quad (6)$$

where $q(t)$ is the matched filter. In stationary noise (the noise is independent of absolute time) q has a particularly simple form and is conveniently described in the Fourier domain as:

$$\tilde{q}(f) = \frac{\tilde{h}(f)}{S_h(f)}, \quad (7)$$

where $h(t)$ is the expected signal in the detector and $S_h(f)$ is the power spectral density of the noise. An illustration of the matched filtering paradigm is given in Fig.4.

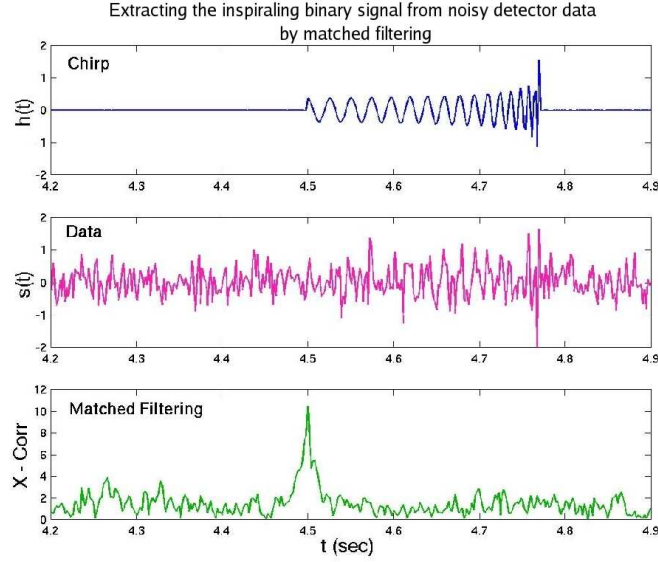


Figure 4. The top part of the figure shows the signal - the inspiral binary waveform usually called the chirp; the middle part shows the signal embedded in the detector noise while the bottom shows the plot of the output of the matched filter $c(\tau)$. By recognising the peak by thresholding, the signal can be detected.

4.1.2 Searching the parameter space: the spinless case

In this section we will be considering only the point mass approximation which is valid for black-holes and to a large extent for neutron stars if they do not deform. The problem would have been simple if there was a single signal waveform. But the signal depends on several parameters. Thus one is actually searching through a family of signals. The signal has the form:

$$h(t; \mathcal{A}, t_a, \phi_a, \tau_0, \tau_3) = \mathcal{A}a(t - t_a, \tau_0, \tau_3) \cos[\phi(t - t_a, \tau_0, \tau_3) + \phi_a], \quad (8)$$

where \mathcal{A} is the amplitude, t_a is the time of arrival of the signal, τ_0 and τ_3 as defined below are functions of the individual masses m_1, m_2 of the binary and ϕ_a the phase at arrival of the wave. The signal described in Eq.(8) is what is called the restricted post-Newtonian waveform in which the amplitude is of the Newtonian waveform which is slowly varying with time, while the phase is given to as much as accuracy is possible, that is upto the 3.5 post-Newtonian order which is deemed sufficient because it gives a phase accuracy to better than a cycle for the stellar mass objects inspiraling in the bandwidth of the current or even advanced detectors. It is most important for the technique of matched filtering that the phase is known as accurately as possible, because even half a cycle can put the signal waveform out of phase with the template waveform which can lead to substantial decrease in the output of the matched filter. The amplitude \mathcal{A} depends on the chirp mass parameter $\mu M^{2/3}$, on the fiducial frequency f_a of the wave at the time of arrival t_a . Instead of the masses, it has been found useful to use the chirp times τ_0 and τ_3 as signal parameters - these parameters appear in a simple way in the Fourier transform of the

signal, namely, they appear *linearly* in the phase of the Fourier transform in the stationary phase approximation. The final search algorithm becomes simple in terms of these parameters. They are related to M and $\eta = \mu/M$ by the relations:

$$\tau_0 = \frac{5}{256 \eta f_a} (\pi M f_a)^{-5/3}, \quad \tau_3 = \frac{1}{8 \eta f_a} (\pi M f_a)^{-2/3}. \quad (9)$$

τ_0 is the Newtonian time of coalescence and the chirp time τ_3 is related to the 1.5 PN order. As one can see from Eq.(8), both the amplitude a and phase ϕ depend on these parameters. We do not give the explicit forms of these functions here because they are unimportant to the discussion here, but they can be found in the literature, eg. [35].

We use the maximum likelihood approach, that is, the likelihood ratio must be maximised over the signal parameters, namely, $\{\mathcal{A}, t_a, \phi_a, \tau_0, \tau_3\}$. The maximum likelihood method shows that the matched filter is the simpler surrogate statistic than the likelihood ratio, and it is sufficient to maximise the output of the matched filter over the search parameters. The amplitude \mathcal{A} is readily extracted from the signal by normalising the template waveform. The parameters t_a and ϕ_a are searched over by using the symmetry of the signal family. The signal is *translationally invariant* in time, that is, a signal at another time of arrival is just obtained from translation. This symmetry can be exploited by using the Fast Fourier Transform (FFT), that is, writing Eq. (6) in the Fourier domain:

$$c(t_a) = \int \frac{\tilde{x}^*(f) \tilde{h}(f)}{S_h(f)} e^{2\pi i f t_a} df + c.c., \quad (10)$$

where the integral in the Fourier domain is carried out essentially over the bandwidth of the detector and where the signal cuts off at the upper frequency end. This is a family of integrals parametrised by all the signal parameters, in particular, t_a . We have suppressed other parameters to avoid clutter, since we now focus on t_a . But the $c(t_a)$ can be obtained for each t_a by just using the FFT algorithm. This saves enormous computational effort because now the number of operations reduce to order $N \log_2 N$ rather than N^2 , where N is the number of samples in the data segment. Typically, for a 500 sec. data train sampled at 2 kHz, $N \sim 10^6$, which implies a saving of computational cost of more than 10^4 !

In the phase parameter ϕ_a also, there is a symmetry - changing ϕ_a to say $\phi'_a = \phi_a + \phi_0$ involves just adding a constant phase to the signal and the waveform still remains within the family. This is the so called S^1 symmetry. The search over phase can be carried out by using just two templates say for $\phi_a = 0$ and $\phi_a = \pi/2$. If we call the correlations so obtained by c_0 and $c_{\pi/2}$ respectively, where we have computed these correlations from the corresponding templates $h(f; \phi_a = 0)$ and $h(f; \phi_a = \pi/2)$, the $c(\phi_a)$ at arbitrary ϕ_a is then given by,

$$c(\phi_a) = c_0 \cos \phi_a + c_{\pi/2} \sin \phi_a, \quad (11)$$

where we have suppressed other parameters to avoid clutter. Moreover, the maximisation of $c(\phi_a)$, the surrogate statistic, over ϕ_a can be done analytically. Thus,

$$\max_{\phi_a} c(\phi_a) = (c_0^2 + c_{\pi/2}^2)^{1/2}. \quad (12)$$

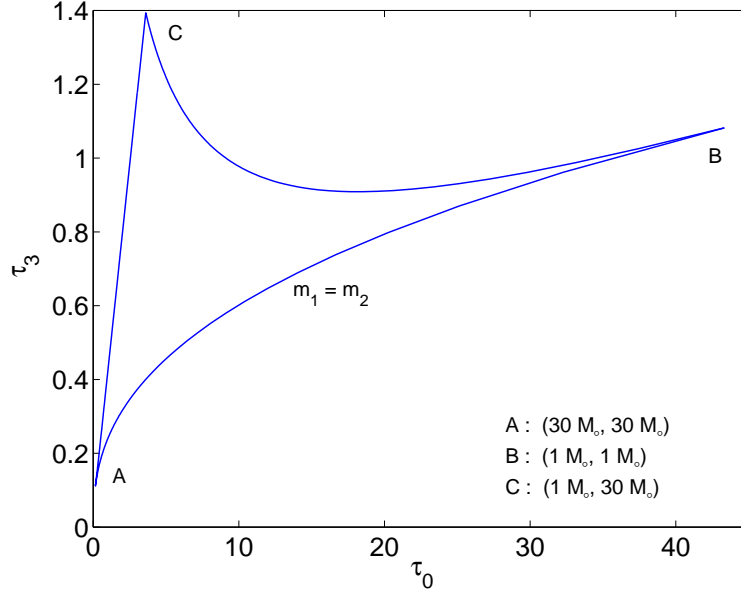


Figure 5. Parameter space in terms of the parameters τ_0, τ_3 for the mass range $1M_\odot \leq m_1, m_2 \leq 30M_\odot$ and $f_a = 40$ Hz.

Thus the kinematical parameters t_a and ϕ_a in the signal waveform are efficiently dealt with; the search over the masses, which are the dynamical parameters, now remains. There does not seem to be any efficient way, for example of using symmetries, to search over these parameters. The figure (5) shows the parameter space for $1M_\odot \leq m_1, m_2 \leq 30M_\odot$ in the parameters τ_0 and τ_3 . Since the waveform is symmetric in m_1, m_2 , one needs to only search the space $m_1 \leq m_2$. This gives roughly a triangular shape to the search region of the parameter space which is topologically equivalent to the triangle in the (m_1, m_2) space. One now spans the parameter space densely with a bank of templates. The templates are arranged so that the maximum mismatch between a signal and a template never exceeds a small fixed amount. The usual number taken is 3% which corresponds to a maximum loss of 10% in the event rate of the signals. With this criterion, in the parameters τ_0, τ_3 the templates are approximately uniformly spaced. The idea is to *tile* the parameter space so that (i) there are no ‘holes’ and (ii) have minimal overlap so that the number of templates is reduced to a minimum which then in turn reduces the computational burden. The best such scheme happens to be hexagonal tiling as shown in figure (6). One does this template placement elegantly by defining a metric [33, 34] over the parameter space. Then one finds that the metric coefficients are nearly constant when the parameters τ_0, τ_3 are used; the parameters in fact play the role of coordinates on a signal manifold and the above statement can be reworded as saying that τ_0, τ_3 are like Cartesian coordinates, while m_1, m_2 are curvilinear. For this level of mismatch the number of templates required is $\sim 10^4$ for the noise PSD of the initial LIGO. If the signal is cut-off at a little less than 1 kHz, so that the sampling rate is 2 kHz, then a simple computation shows that the online search for these signals is little more than 3 GFlops.

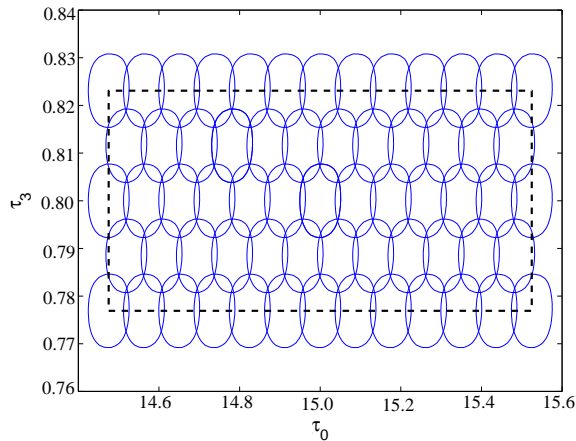


Figure 6. Hexagonal tiling of the parameter space.

One now takes the maximum of the matched filter output over all the parameters and compares this maximum with a threshold. The threshold is set by the noise statistics and the false alarm rate that one is prepared to tolerate. Clearly, the false alarm rate must be much less than the expected event rate. If the noise is Gaussian, the c maximised over ϕ_a has a Rayleigh distribution in the absence of the signal. Assuming a false alarm rate of 1 per year, gives a false alarm probability of $\sim 10^{-14}$ for one year observation period, which in turn sets the threshold at 8.2 (this is in units of the standard deviation of the Gaussian noise). Detection is announced if the c maximised over the parameters exceeds the threshold. However, in order to achieve good detection probability one must have c well over the threshold - thus $c > 8.9$ gives a detection probability better than about 95%.

The foregoing describes the general matched filtering paradigm. It clearly holds for a larger mass range or a larger bandwidth. If one lowers the lower limit of the band to 10 Hz as will be the case for advanced detectors and increases the mass range to begin from say $0.2M_\odot$, the online search requirement increases by a hundred times. Also if one includes spins, the waveform now must depend on 6 more parameters, namely, the spin vectors \vec{S}_1, \vec{S}_2 , the computational burden increases roughly by three orders of magnitude.

In order to deal with the rather large computational cost, hierarchical schemes have been proposed [35, 36] which can reduce this cost. The idea is to look for triggers with a low threshold with a coarse bank of templates, and then follow only the trigger events by a fine search and then use the high threshold as in the regular search described above. This method can reduce the cost considerably and theoretical factors of few tens in reducing the cost have been obtained in stationary Gaussian noise. These factors of course will be significantly less in the presence of real detector noise. Recently Cannon et al [29] have shown that singular value decomposition can be used to reduce the computational cost. They have shown that for neutron star/neutron star

inspiral where the parameter space is smaller, the computational gain can be by almost an order of magnitude.

4.1.3 Merger and ringdown

For the blackhole merger, one must solve the Einstein's equations with complex initial and boundary conditions. Due to nonlinearity of Einstein's equations, the problem is highly complex and work had been going on for few decades until Pretorius [30] made a breakthrough. This was followed by other successful numerical solutions [31, 32]. Clearly, the first such solutions corresponded to nonrotating blackholes which now are not too difficult to obtain. The rather surprising fact was that the merger phase is a smooth continuation of the inspiral phase, contrary to what had been expected. There are results also for spinning blackholes. Work is in progress of obtaining numerical relativistic solutions spanning the entire parameter space for different mass ratios and spins. There are also interesting effects such as 'kicks' in the general case; the final blackhole has a residual linear momentum.

The final ring-down phase consists of a superposition of quasi-normal modes (QNM) with their amplitude depending on the details of the perturbation. But each QNM is uniquely given by the blackhole mass and the angular momentum parameter. The 'no hair' theorem for blackholes in general relativity states that a blackhole is completely characterised by its mass and angular momentum. The above mentioned property of QNMs is a consequence of this theorem. Thus observing QNMs would unambiguously show that the source is a blackhole and also confirm the no hair theorem of general relativity.

The important point for data analysis is that the inspiral waveform can be continued into the merger phase and to the ring-down phase of the final blackhole to obtain a single stitched waveform, thus yielding a higher SNR. The mass range can now go upto $100M_{\odot}$ and the distance by about a factor of 2 which would then correspondingly increase the event rate by about an order of magnitude.

4.2 The all sky all frequency search for GW from rotating neutron stars

We will consider the simple model of an isolated rapidly rotating neutron star and ignore spindown. We will show here, how the group theory and other algebraic methods can be used to elegantly formulate the problem by exploiting the symmetries in the physical model. In this endeavour, we will make use of the *stepping around the sky method* - a method proposed by Schutz twenty years ago [37], which gives an apt framework for this approach. There have been a host of methods proposed, notably the Hough transform, the stack and slide, resampling methods etc. [38] which reduce the computational cost over the straight forward search over the sky directions, frequency and spindown parameters. Although these methods significantly reduce the computational burden, it is not reduced to the point where the search can be performed with the current

computer resources available in reasonable time. Therefore, it becomes necessary to explore novel approaches which address this problem.

Consider a *barycentric frame* in which the isolated neutron star is at rest or moving with uniform velocity. Ignoring spindowns the signal in this frame is assumed to be a pure sinusoid - monochromatic of constant frequency say f . The detector however, takes part in an accelerated motion - in general a superposition of simple harmonic motions of varying amplitudes and phases. Thus the signal in the detector is not a pure sinusoid but is modulated by Doppler effects - the Doppler correction depending on the direction to the source, relative to the motion of the detector. Since the detector moves in a complicated way relative to the barycentre, a complex Doppler profile is generated which depends on the direction to the source. If the direction to the source and the frequency is unknown, the Doppler profile is unknown and then one must face the problem of scanning over all directions in the sky and also the frequency. From astrophysical considerations usually the maximum frequency f_{\max} is taken to be 1 kHz. The *stepping* method gives a direct way for obtaining the Fourier transform in the barycentric frame of the demodulated signals connecting two different directions say $\hat{\mathbf{n}}$ and $\hat{\mathbf{n}}'$.

The signal expected is so weak that one typically needs to integrate the signal for several months or an year before one can build up significant SNR. So if the observation time needed is $T \sim 10^7$ sec or more and if the maximum frequency f_{\max} to be scanned is taken about a kHz, then the number of samples in a data train are $N \sim 2f_{\max}T \sim 10^{10}$. Since the detector orbits the Sun in this time, the ‘aperture’ of the ‘telescope’ is the diameter D of the Earth’s orbit; $D \sim 3 \times 10^8$ km while the minimum GW wavelength is $\lambda_{\text{GW}} \sim 300$ km corresponding to a frequency of 1 kHz. Thus the resolution is $\Delta\theta = \lambda_{\text{GW}}/D \sim 10^{-6}$ radian, fraction of an arc second - a Fourier transform of such a signal spreads into a million Fourier bins, and consequently the signal is lost in the noise of the detector. One therefore needs to demodulate the signal first and then take its Fourier transform in order to collect the signal power in a single frequency bin. This means one needs to scan or demodulate over $N_{\text{patches}} \simeq 4\pi/\Delta\theta^2 \sim 10^{13}$ directions or patches in the sky. So even this naive calculation gives the number of operations for the search to be $N_{\text{ops}} \sim 3N_{\text{patches}}N \log_2 N \sim 10^{25}$ if one were to perform the FFT of the data train after demodulating in each direction. A machine with a speed of few teraflops would need several thousand years to perform the analysis! Moreover, this estimate excludes overheads, ignores spindown parameters etc. Including these would increase the cost of the search by several orders of magnitude. Thus the search is highly computationally expensive and novel and original ideas should be explored, if this search has to be brought within the capabilities of current resources or those envisaged in the near future. The approach outlined here is based on group theory and is one such attempt towards finding a solution to this problem.

Moreover, there exists also the possibility of using this approach in tandem with the previous approaches which have been aimed at reducing the computational cost. It is envisaged that a judicious combination of several approaches may go towards alleviating the computational burden.

4.2.1 The formulation of the problem

Let the motion of the detector be described in general by $\mathbf{R}(t)$ in the barycentric frame (X, Y, Z) ; for circular motion $\mathbf{R}(t) = R(\cos \Omega t \hat{\mathbf{X}} + \sin \Omega t \hat{\mathbf{Y}})$ - we take the detector motion in the (X, Y) plane - where $\hat{\mathbf{X}}$ and $\hat{\mathbf{Y}}$ are unit vectors along the X and Y axes respectively, and Ω is the angular velocity of the detector in the barycentric frame. We will treat $\mathbf{R}(t)$ generally for now until later when we specialise to circular motion. The key defining equation which describes the transformation between barycentric time t and detector time t' is:

$$s'(t') = s(t). \quad (13)$$

The detector time coordinate t' , which is in fact a retarded or advanced time, is given by $t' = t - \mathbf{R}(t) \cdot \hat{\mathbf{n}}/c$ and is related to the barycentric time coordinate t . From our assumptions, the signal in the barycentric frame can be taken to be monochromatic. So after demodulation a Fourier transform is all that is necessary to extract the signal from the detector noise. It is in fact the matched filter!

We write the Doppler modulation in an abstract form in terms of an operator. The signal at the detector coming from the direction $\hat{\mathbf{n}}$ is related to the signal in the barycentric frame by the equation:

$$s'(\hat{\mathbf{n}}) = M(\hat{\mathbf{n}}) \mathbf{s}, \quad (14)$$

where $M(\hat{\mathbf{n}})$ is the modulation operator which is defined via Eq. (13). This operator has explicit representations in the time as well as Fourier domain [39]. Note that in the all sky all frequency search we do not know $\hat{\mathbf{n}}$. Therefore we need to scan over the directions. A trial demodulation is performed for some general direction $\hat{\mathbf{n}}'$ given by $\hat{\mathbf{n}}' = (\sin \theta' \cos \phi', \sin \theta' \sin \phi', \cos \theta')$, which is not necessarily $\hat{\mathbf{n}}$. Thus we try the direction $\hat{\mathbf{n}}'$ and have a trial demodulated signal,

$$\mathbf{s}_{\text{trial}}(\hat{\mathbf{n}}'; \hat{\mathbf{n}}) = M^{-1}(\hat{\mathbf{n}}') s'(\hat{\mathbf{n}}). \quad (15)$$

If $\hat{\mathbf{n}}' \neq \hat{\mathbf{n}}$, then the demodulation is incorrect and we must try again with a different $\hat{\mathbf{n}}'$ until we get to $\hat{\mathbf{n}}$ or at least get close enough. If $\hat{\mathbf{n}}' \simeq \hat{\mathbf{n}}$, we must observe a peak in Fourier domain. Using these formulae we can now *step directly* to a direction $\hat{\mathbf{n}}'$ as follows:

$$\mathbf{s}_{\text{trial}}(\hat{\mathbf{n}}'; \hat{\mathbf{n}}) = Q(\hat{\mathbf{n}}', \hat{\mathbf{n}}) \mathbf{s}, \quad (16)$$

where the *stepping* operator is defined by:

$$Q(\hat{\mathbf{n}}', \hat{\mathbf{n}}) = M^{-1}(\hat{\mathbf{n}}') M(\hat{\mathbf{n}}). \quad (17)$$

This was the approach suggested by Schutz, now expressed in our formulation, so that one may directly *step* from the direction $\hat{\mathbf{n}}$ to the direction $\hat{\mathbf{n}}'$ in the space of demodulated waveforms. This formulation was expected to enhance the efficiency of the search for example by using the sparseness of the matrices. The approach here builds upon this formulation. Apart from the sparseness of matrices, the idea is to use symmetries in the problem for stepping efficiently in the sky. The symmetry is made manifest via the language of group theory.

In order to get a group structure and go beyond the method advocated by Schutz, it is necessary to expand the scope of the direction vectors $\hat{\mathbf{n}}$ to the full three dimensional Euclidean space \mathcal{R}^3 . It is clear that this is required because even a step in the sky namely, $\hat{\mathbf{n}}' - \hat{\mathbf{n}}$ will not be of unit length. Thus it is necessary as also convenient to ‘unwrap’ the space of directions, which is a projective space, to its universal covering space \mathcal{R}^3 . We then define the operators $M(\mathbf{a})$, where \mathbf{a} is an arbitrary vector in \mathcal{R}^3 , and \mathbf{a} is used in the ‘retarded time’ instead of $\hat{\mathbf{n}}$. We can then show that these operators now form a group, atleast approximately, well within the physical requirements [39]. It is important to note that these operators M act on functions, namely signals, and map them to other signals - the signals are Doppler shifted. Such groups are called transformation groups in the literature [40].

4.2.2 Circular motion of the detector

For concreteness, we give an example of circular motion of the detector. This is a very simplified case because in reality the detector partakes of a complicated superposition of simple harmonic motions which have complex set of phases. This simple case is taken to see how the group theory helps. The group now is reduced to Euclidean group in 2 dimensions, usually denoted by $E(2)$. We consider the motion as above and consider the situation when the motion consists of exactly one orbit, i.e. $0 \leq t \leq T$ and $\Omega T = 2\pi$. Then in the Fourier space, where $n = f/T$ and $n' = f'/T$, we look at the action of $M(\mathbf{a})$ on the complete orthonormal basis of the Hilbert space of square integrable functions over $[0, T]$, namely, the set of functions $e^{2\pi i n t/T}$. The natural scalar product on this Hilbert space for the two functions g_1 and g_2 is defined by:

$$(g_1, g_2) = \frac{1}{T} \int_0^T g_1(t) g_2^*(t) dt. \quad (18)$$

In this basis, the matrix representation for M (we have chosen $\mathbf{a} = \hat{\mathbf{n}}$ a unit vector) is readily given:

$$\begin{aligned} M(\hat{\mathbf{n}}; n', n) &= (M(\hat{\mathbf{n}}) e^{2\pi i n t'/T}, e^{2\pi i n' t'/T}) \\ &= (e^{2\pi i n t'/T}, e^{2\pi i n' t'/T}) \\ &= \frac{1}{T} \int_0^T dt' e^{2\pi i n t'/T - 2\pi i n' t'/T}. \end{aligned} \quad (19)$$

where we have used the definition $M(\hat{\mathbf{n}})s(t') = s(t)$. An explicit expression for $M(\hat{\mathbf{n}}; n', n)$ can be obtained for the direction $\hat{\mathbf{n}} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$. Writing $\psi = \Omega t'$ and $\beta = R\Omega/c$ we obtain:

$$\begin{aligned} M(\hat{\mathbf{n}}; n', n) &= \frac{1}{2\pi} \int_0^{2\pi} d\psi e^{i(n-n')\psi + i n \beta \sin \theta \cos(\psi - \phi)} \\ &\equiv e^{i\chi(n-n')} J_{n-n'}(n\beta \sin \theta), \end{aligned} \quad (20)$$

where $\chi = \phi + \pi/2$ is the translated azimuthal angle.

From the form of $M(\hat{\mathbf{n}}; n', n)$ it is evident that when applied to the data vector x_n , the search in χ can be performed by a fast Fourier transform; the stepping in the azimuthal parameter is done in an efficient way. If there are B samples of the χ parameter, then the search over χ for a given θ and frequency n' can be performed in order of $B \log_2 B$ number of operations. It may be further possible to reduce the number of operations by similar methods, but this example underlines the role of symmetry and the group theory in developing efficient data processing algorithms.

5. Concluding Remarks

The era of gravitational wave astronomy has arrived. The initial detectors have not only reached their promised sensitivities but have surpassed them. The advanced detectors will start operating in few years time and the era of gravitational wave astronomy would then have truly begun. From the astrophysical knowledge that we possess as of now, one should expect fair rate of gravitational wave events that one should be able to observe. An important recent development has been LIGO-Australia where LIGO is planning to build one of its detectors in Australia with partial funding from Australia. A detector far away and out of the plane of other detectors in US and Europe would greatly benefit the search of gravitational waves. In this India is also thinking of chipping in, so that India also has a stake in this exciting world project. Already, India has a 20 year old legacy in gravitational wave data analysis at IUCAA, Pune and wave form modelling at RRI, Bangalore. And recently a three metre prototype detector at T.I.F.R., Mumbai has been funded. Apart from the groundbased detectors, there is also the prospect of the space-based ESA-NASA detector LISA which will bring in important astrophysical information at low frequencies complementing the ground-based detectors. The future looks bright for GW astronomy.

Acknowledgements

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