

SEARCHING FOR A SOLUTION TO THE AGE PROBLEM OF THE UNIVERSE

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We present here a phenomenological cosmological model under perfect fluid distribution with a stiff equation of state $p = \rho$. The erstwhile cosmological constant is assumed to be a time dependent variable, i.e., $\Lambda = \Lambda(t)$ in our study. It has been shown that the estimates of different cosmological parameters from this model are in good agreement with the experimental results, especially 13.79 Gyr as the age of the universe is quite satisfactory. The behavior and relation of Λ -stiff fluid model with dust, viscous fluid and variable G have also been investigated in detail.

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“The problem of the age of the observable universe - the time back to the ‘big bang’ - has puzzled laymen, churchmen and scientists for many years.” - W. A. Fowler (in *The Age of the Observable Universe*) [1]

1. Introduction

The cosmological picture that emerges after the observations of SNe Ia by HZT and SCP teams reveals that at present we are residing in an accelerating universe [2, 3] whose geometry is Euclidean in nature [4]. This speeding up of the universe started about 7 Gyr ago [5] and some kind of repelling force, termed as *dark energy*, is supposed to be responsible for catapulting the once decelerating Universe into an accelerating one. Using various observational data it has also been possible to pin down, to some extent, the ranges of various parameters of the Universe at the present era. For instance, the acceptable limit of H_0 , the Hubble parameter at present, is 72 ± 8 Km s⁻¹Mpc⁻¹ [6] and while taking into consideration of various ranges as proposed by different workers from varying standpoints [7] the currently accepted observational value of the age of the universe becomes 14 ± 0.5 Gyr [8] (an elaborated data sheet has been produced in ref. [9]).

Many variants of Λ , a dark energy representative, have been proposed by different workers to account the cosmological consequences. Since, supernovae Ia observations predict a small value of Λ at the present epoch [10], now-a-days Λ with dynamical character is preferred over a constant Λ [11]. This has been assumed to vary with time so that decrease of Λ from a large initial value to its present small value can easily be realized.

Mainly, in the theoretical point of view, the possibility of a nonzero and varying Λ came into picture in connection to the *age problem* of the universe. This is because of the fact that for large Λ the age of the universe can, in

principle, become infinite [11, 12]. Even with the time-decaying Λ , it is seen that the present age of the universe either suffers from low-age problem [11] or it is as large as 27.4 ± 5.6 Gyr [13]. In connection to the low age we can mention the results of Overduin and Cooperstock [11] where for the Hubble parameter $H_0 = 73 \pm 10$ kms⁻¹Mpc⁻¹ the age of the universe in its lower limit is 5.4 Gyr! The situation improves slightly for lower value of $H_0 (= 55 \pm 10$ kms⁻¹Mpc⁻¹) [14] when it becomes 7.6 Gyr only. This values are much lower than the estimated globular cluster ages which are thought to be in the range 9.6 Gyr [8] to 13 ± 3 Gyr [15] and very recent observations claim that it is most probably 12.5 ± 1.2 Gyr [16, 17]. However, in the standard model of a flat Λ -dominated universe the age of the universe is 13.7 ± 0.2 Gyr as obtained by Spergel et al. [18] whereas Kunz et al. [19] in the Λ -CDM case find out value as 13.55 ± 0.26 . For a slightly closed Λ -CDM universe with $H_0 = 66_{-6.4}^{+6.7}$ kms⁻¹Mpc⁻¹ Tegmark et al. [20] estimate an age of $14.1_{-0.9}^{+1.0}$ Gyr.

Under this background, in the present article we have investigated three types of widely used [21, 22] phenomenological forms of kinematical Λ , viz., $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim (\ddot{a}/a)$ and $\Lambda \sim \rho$, where a is the scale factor of the Robertson-Walker metric and ρ is the energy density of the universe (a detail account of these phenomenological Λ models is available in ref. [9]).

As a result of the present investigations under the different phenomenological models as mentioned above we have found out the equivalence of the Λ -models through the parameters α , β and γ involving in the models. Under the perfect fluid distribution, specially with a stiff equation of state $p = \rho$, the age and the other physical parameters have been estimated. Surprisingly, the age 13.79 Gyr is in good agreement with the present available experimental results. The behavior and relation of stiff fluid to dust and viscous fluid also have been critically investigated. It is also observed that Variable Λ and variable G models show some unique common features in connection to the present accelerating universe.

The article has been organized as follows: in Section 2 the standard general relativistic Einstein field equations are presented with the inclusion of time varying cosmological constant, viz., $\Lambda = \Lambda(t)$, in the

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energy-momentum tensors. Section 3 deals with the solutions of various phenomenological models within this framework of modified general relativity. In the Section 4 some salient features of the present model are discussed. Section 5 presents concluding remarks on the status of the stiff fluid and also the problems related to the age of the universe.

2. Einstein Field Equations

Let us consider the Robertson-Walker metric which is given by

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (1)$$

where k is the curvature constant which takes the specific values -1 , 0 and $+1$ respectively for open, flat and close models of the universe and a is the cosmic scale factor as mentioned earlier.

With the assumption that the so-called *cosmological constant* is time dependent here, viz., $\Lambda = \Lambda(t)$ and c , the velocity of light in vacuum is assumed to be unity in relativistic units, the Einstein field equations

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G \left[T^{ij} - \frac{\Lambda}{8\pi G}g^{ij} \right] \quad (2)$$

for the above spherically symmetric metric (1) yield, respectively, the Friedmann and Raychawdhuri equations

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}, \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad (4)$$

where G , ρ and p are, respectively, the gravitational constant, matter-energy density and fluid pressure.

Also, the energy conservation law can be written as

$$8\pi G(p + \rho)\frac{\dot{a}}{a} = -\frac{8\pi G}{3}\dot{\rho} - \frac{\dot{\Lambda}}{3}. \quad (5)$$

Let us now choose the barotropic equation of state in the form

$$p = w\rho, \quad (6)$$

where w is a constant, known as the equation of state parameter and can take the values 0 , $1/3$ and 1 respectively for the pressureless dust, electromagnetic radiation and stiff or Zel'dovich fluid.

Using this barotropic equation of state (6), in a straight forward way, the equation (4) can be written as

$$\frac{\ddot{a}}{a} + \frac{4\pi G}{3}(1 + 3w)\rho = \frac{\Lambda}{3}. \quad (7)$$

After performing differentiation of equation (3) with respect to time and using equations (4) - (7) for elimination of ρ , one obtains the following equation

$$\left(\frac{\dot{a}}{a} \right)^2 + \left[3 \left(\frac{1+w}{1+3w} \right) - 1 \right] \frac{\ddot{a}}{a} + \frac{k}{a^2} = \left(\frac{1+w}{1+3w} \right) \Lambda. \quad (8)$$

This is the general equation for our investigation in the presence of the curvature parameter k .

3. Solutions

At this point let us consider the *ansatz* $\Lambda = 3\alpha(\dot{a}/a)^2$, where α is a constant. Now, inflation theory predicts and CMB detectors such as BOOMERanG [23, 24, 25], MAXIMA [26, 27, 28], DASI [29], CBI [30] and WMAP [31, 32] confirm that the universe is spatially flat. Therefore, for the flat universe where $k = 0$, the equation (8) reduces to

$$2a\ddot{a} + (1 + 3w - 3w\alpha - 3\alpha)a^2 = 0. \quad (9)$$

The general solutions of the above equation (9) can be given as

$$a(t) = C_1 t^{2/3(1-\alpha)(1+w)}, \quad (10)$$

$$\rho(t) = \frac{1}{6\pi G(1-\alpha)(1+w)^2} t^{-2}, \quad (11)$$

$$\Lambda(t) = \frac{4\alpha}{3(1-\alpha)^2(1+w)^2} t^{-2} \quad (12)$$

where C_1 is an integration constant and $0 < \alpha < 1$ for physical validity.

Therefore, for stiff fluid distribution ($w = 1$) the proportional relation of the cosmic scale factor to the cosmic age becomes $t^{1/3(1-\alpha)}$. In a similar way, it can be shown that for the *ansatz* $\Lambda = \beta(\ddot{a}/a)$ and $\Lambda = 8\pi G\gamma\rho$ the cosmic scale factor $a(t)$ is, respectively, proportional to $t^{(\beta-1)/(\beta-3)}$ and $t^{(\gamma+1)/3}$, where β and γ are another two constants like α . For all these three models, as are evident from equations (11) and (12), the cosmic matter-energy density $\rho(t)$ and the cosmological parameter $\Lambda(t)$ both follow an inverse square law with t .

Now, in absence of any curvature, the cosmic matter-energy density parameter $\Omega_m (= 8\pi G\rho/3H^2)$ and cosmic vacuum-energy density parameter $\Omega_\Lambda (= \Lambda/3H^2)$ are related by

$$\Omega_m + \Omega_\Lambda = 1. \quad (13)$$

This result is consistent with the current constraint on cosmic density parameters [33]. Expressing the solutions of (8) for stiff fluid in terms of Ω_m , it has been found that for all the three Λ -models, $a(t)$ is proportional to $t^{1/3\Omega_m}$ whereas the expressions for $\rho(t)$ and $\Lambda(t)$ are identical. Moreover, α , β and γ are related to Ω_m and Ω_Λ by

$$\alpha = \Omega_\Lambda, \quad \beta = \frac{3\Omega_\Lambda}{\Omega_\Lambda - 2\Omega_m}, \quad \gamma = \frac{\Omega_\Lambda}{\Omega_m}. \quad (14)$$

Equation (14), with the help of equation (13), enables us to have interrelation between α , β and γ as follows

$$\alpha = \frac{2\beta}{3(\beta-1)} = \frac{\gamma}{1+\gamma}. \quad (15)$$

This clearly shows that the three forms, viz., $\Lambda = 3\alpha(\dot{a}/a)^2$, $\Lambda = \beta(\ddot{a}/a)$ and $\Lambda = 8\pi G\gamma\rho$, are equivalent and the three parameters α , β and γ are interconnected by the relation (15). Therefore, it is possible to search for the identical physical features of others if any one of the phenomenological Λ model is known.

4. Physical Features

4.1. Age of the universe from the stiff fluid model

The most striking and puzzling feature of this stiff fluid model is the fine agreement of the physical parameters, especially the present age of the universe, with the observational results. To get an explicit expression for age let us differentiate equation (10) and divide it by the cosmic scale factor a , which ultimately yields

$$t = \frac{2}{3(1-\alpha)(1+w)H}. \quad (16)$$

In the case of stiff fluid ($w = 1$) the above equation (16), by the use of the equations (13) and (14), reduces to

$$t = \frac{1}{3\Omega_m H}. \quad (17)$$

Recent measurements indicate that the value of Ω_{m0} , the cosmic matter-energy density parameter at present, is $\Omega_{m0} = 0.33 \pm 0.035$ [13]. Assuming $\Omega_{m0} = 0.295$, the lowest acceptable limit of this range, and $H_0 = 72 \text{kms}^{-1} \text{Mpc}^{-1}$, the present age of the universe, $t_0 (= 1/3\Omega_{m0}H_0)$ is found to be 13.79 Gyr which is in excellent agreement with the age of the universe as calculated on the basis of WMAP data [5] as well as the current accepted age mentioned earlier. A comparison between the ages for other values of ω shows a situation which goes in favor of stiff fluid (TABLE I).

Moreover, the values of $\rho_0 (= 1/24\pi G\Omega_{m0}t_0^2)$ and q_0 , respectively the present values of the cosmic matter-energy density ρ and the deceleration parameter q , as calculated on the basis of $\Omega_{m0} = 0.33 \pm 0.035$ and $H_0 = 72 \text{kms}^{-1} \text{Mpc}^{-1}$, confirm that the present models support the idea of an open as well as accelerating universe [34]. Also, the value of $\Lambda_0 (= (1 - \Omega_{m0})/3\Omega_{m0}^2 t_0^2) \sim 10^{-35} \text{s}^{-2}$ is in agreement with the current status of Λ with a small value [35].

4.2. The enigma of the stiff fluid model

It is worthwhile to mention that from the solutions of equation (9), one may arrive at the solutions obtained by Arbab [36] for the dust ($\omega = 0$) case under the *ansatz* $\Lambda = 3\alpha(\dot{a}/a)^2$. This is possible via the transformation

relation in between α and β in the form $\alpha = \beta/3(\beta-2)$. By the use of this one can easily arrive at the equations (10) - (12) which are the same as those of his equations (8) - (10). However, in the case of stiff fluid, the expected transformation relation can be obtained from our equation (15) and his equation (18) which yields the value for $\beta = 3$. It can be seen that this immediately makes the whole set of Arbab's equation to blow up. This, therefore, suggests that such transformation is forbidden which is also true in his own case as this makes the solution set singular. However, exploiting this transformation relation $\alpha = \beta/3(\beta-2)$ in the solution of Majerník [37] it has been shown by Arbab [36] that α can be expressed as having the form $\kappa/(1+\kappa)$, where $\kappa = \beta/2(\beta-3)$ is a free parameter. This yields $\alpha = -w_Q$ and $w_Q = p_Q/\rho_Q$ ($-1 < w_Q < 0$), being the Quintessence equation of state, Arbab [36] argued that w_Q is nothing but the vacuum energy parameter. One can easily find out that the κ of Majerník [37] is nothing but our γ under the *ansatz* $\Lambda = 8\pi G\gamma\rho$ and follows the same relation as shown in equation (15). It suggests that stiff fluid has a deep connection to *dark energy*.

It is also interesting to note that by assuming $n = \Omega_\Lambda$ (this assumption is not unreasonable since, $2/3 < n < 1$) in the equation (27) of the bulk viscous model of Arbab [38] with variable G , we get the same expression for the age of the universe, viz., $t_0 (= 1/3\Omega_{m0}H_0)$ in our model. Therefore, it is possible to obtain the same value of t_0 for the dust case of Arbab as that obtained in the present stiff fluid model. Now, it is known that bulk viscosity is associated with the inflationary universe scenario and after the decoupling of neutrino in the early universe matter did behave like viscous fluid [39]. This bulk viscosity, therefore, is similar to a variable cosmological term with a repulsive pressure. In the present investigation, variable Λ -based stiff fluid model with constant G and Arbab's viscous model with variable Λ and variable G produce the same result so far as the present age of the universe is concerned. This again suggests about the underlying relation between the stiff fluid and the viscous fluid which has a feature of *dark energy*.

In this context we would like to add here that Chakrabarty and Pradhan [40], with a time-dependent gravitational constant G , have shown that the cosmological constant Λ varies as t^{-2} , as in the present stiff fluid case with constant G . So, there seems to exist some connection between the models with variable G and the present stiff fluid-filled Zel'dovich universe.

5. Concluding Remarks

Although in the present cosmological scenario it is customary to choose the value of the barotropic index ω as zero (for pressureless dust) and sometimes as $1/3$ (for radiation), yet stiff fluid model which refers to a Zel'dovich universe have been selected by some authors for various situations such as cold baryonic universe [41], early hadron era [42], scalar field fluid [43], for the relativistic

TABLE I: Age of the universe from different ω models

| Ω_{m0} | H_0 (km s ⁻¹ Mpc ⁻¹) | t_0 (Gyr) | | |
|---------------|--|----------------|--------------|----------------|
| | | $\omega = 1$ | $\omega = 0$ | $\omega = 1/3$ |
| 0.295 | 64 | 15.53 | 31.07 | 25.91 |
| | 72 | 13.79 | 27.61 | 22.83 |
| | 80 | 12.43 | 24.86 | 20.70 |

situation prevailing during the early stages of the universe and LRS Bianchi I cosmological models [40]. There are recent applications and claims for equation of states in the various astrophysical realm, e.g. in neutron star RX J1856-3754 [44] and hyperon stars [45] which are very close to the stiff fluid limit (for some more astrophysical as well as cosmological applications, see [46] and refs. therein). In this connection we are interested to mention the work of Buchert [47] where it has been shown that in the spatially averaged inhomogeneous cosmologies the averaged equations show that the averaged scalar curvature must generically change in the course of structure formation and that an averaged inhomogeneous perfect fluid, in some cases, act like a free scalar field source which can be modelled by a stiff fluid.

It is seen that the present age of the universe as calculated in the standard Friedmann model as well as in some other models either suffers from low-age problem [11] or it is as large as 27.4 ± 5.6 Gyr [13] in the ‘favored’ dust case. In that respect the present stiff fluid model with variable cosmological constant demands some attention because the age of the model fits well with the modern accepted limit. But, the result of this Λ -stiff fluid model is very much surprising in the sense that we do not know how this value falls within the range of the present values of the age of the universe as obtained in the different Λ -CDM models. It is already discussed in the earlier subsection that there exists some kind of underlying relation in between either stiff fluid

and variable G model or stiff fluid and viscous fluid which has a feature of *dark energy* or with the both. If there is some relation between them then what is the possible mechanism - that is also not understood. One possibility may be that the effect of the *stiffness* of the stiff fluid through the process of evolution via radiation to the present epoch of matter-dominated universe is too weak to perceive and hence *dark energy* in terms of variable cosmological constant does play its definite role for the Λ -stiff fluid model. Thus, here also we are facing a kind of age problem of the universe which is really intriguing! In this respect the comment made by Born, on different context, seems appropriate to quote: “Whether one or the other of these methods will lead to the anticipated “world law” must be left to future research” [48].

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